COMFORTABLE SIMPLE ANALYSIS OF VERTICAL STRESSES AND SETTLEMENTS WITH THE COMPUTER PACKAGE B4

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Abstract

The paper presents some principal features of the computer package B4 – an advanced software primarily developed for analyses of vertical stresses and settlements of the half-space loaded with vertical loads. Besides that, the package gives also support to the computation of the ultimate bearing capacity of foundations and consolidation without or with vertical drains. The computation of settlements of the soil, improved with slopes is also included. All the obtained results, accompanied with essential equations are transmitted to a desktop publishing package, thus creating a powerful tool for the user who requires easy-to-do and fast solutions in a representative form without effort.

1 Introduction

The rapid and extensive development of computers allows not only extensive development of new computational procedures, but also permits revisions and upgradations of some established computational approaches. Furthermore, the computer programs have been replacing graphs and tables, and thus their popularity and credibility grow.

The task of the prediction of vertical stresses and especially settlements as a result of a vertical load is one of the most representative problems in the field of geomechanics. Although the recent development in this field is oriented towards the finite or boundary elements solutions, such programs are not frequently implemented in civil engineering practice due to their complexity and/or especially high prices. Therefore particularly in small and medium-sized enterprises simple solutions are still frequently implemented.

The first solutions for calculation of stresses and settlements in an elastic, homogeneous, isotropic half-space due to a concentrated vertical load were given by Boussinesq (1885). For the computation of vertical stresses beneath a rectangle loaded with a uniform load the first integration was given by Steinbrener (1934) but the results of other similar integration are also well known. The limitation of all these solutions is that they are given for the corner of a base loaded with a uniform load while for the nonuniform load or points not coinciding with the corner a superposition is required.

A very wide collection of the results for several load cases is given also by Poulus and Davis (1974). Vitone and Valsangkar (1986) presented equations for some loads with nonuniform distribution over a rectangular base, however again for the computational point beneath the corner point of the base. For more general cases of the load some authors (Bowles 1996) proposed numerical approximations dividing the base into small areas with uniform distribution of the load over each area. The accuracy of the results increases with number of rectangles used in the discretisation of the load.

The disadvantage of all precedent solutions was the necessity to place the point of the interest strictly beneath a corner of the load to use the equations directly, otherwise the superposition principle must be used successively. These problems have been overcome by obtaining several unique equations for stresses and deformations in an arbitrary point of the half-space loaded with several base types. The first two equations based on the Boussinesq theory considered a half-space loaded with the variable load distributed over a rectangular base where the load is described with general polynomial functions (Skrinar and Battelino, 1995). The assumption of no lateral deformations, i. e. neglection of Poisson's ratio, was implemented for the estimation of the vertical deformations. This lack has also been overcome by a new equation that takes the Poisson's ratio into account. All the solutions obtained are very complex due to their generality and an adequate computer program is recommended to implement them.

Programs written for stress and settlement analysis have usually focussed on the analysis itself, rather than on the design process as a whole. As for example, older versions of such programs (still widely used in some engineering companies) were usually written in FORTRAN without any graphical supervision options available to the users. The output of the results in such programs was thus given as a set of numbers and the user did not receive any immediate feedback if the input data were correct or not.

The computer package B4 was in the first phase actually introduced as an educational tool in the form of two separate modules for the computer package Mathematica. The modules covered the first two unique direct equations for the computation of stresses and settlements of an arbitrary point of the half-space, loaded with a nonuniform load over the rectangle. The modules were not able to produce their own graphical results but they have used the already built-in graphical capatibilies of the Mathematica package. Very soon the limitations of this decision become evident as this required from the students to obtain the Mathematica package to implement the prepared modules. Therefore the code was transformed into the Visual Basic environment that is capable to produce self standing executable files. As Visual Basic supports object oriented programming the file rouge to writing the program for design was to create a program for the convenience of the user rather than the convenience of the programmer. The main principles for programming the results were:

it should be possible for the user to define the problem with minimum data available but also to put in all the remaining data available, the data should be read in the form in which they are presented to the user and the program should support any necessary conversion of the input data,

options should be provided to allow the user to manipulate the already defined data directly from the program and to immediately perform new analysis,

the numerical results must be presented on the screen to the user immediately after the analysis of each step is completed and the user should select among various graphical representations of the results obtained,

the output of the results in the output file must be clear and representative, and the details presented must provide the user with enough data for a full numerical check of an analysis. If possible, all the main equations should be presented.

The package has very soon expanded over the originally settled margins with the second release where a complementary computational approach based on similar Westergaard's equations has been introduced. Those equations, perhaps less recognised than Boussinesq's, yield the designer a comparative platform for the confrontation of the results obtained with the two methods.

The next step in the evolution of the package introduced the computation of stresses and settlements of an arbitrary point of the half-space, loaded with a uniform load over an arbitrarily shaped quadrilateral (Skrinar, 1997). The integration over the area of the arbitrarily shaped quadrilateral from the global coordinates has been transformed into the integration over the bi-unit square in local coordinates using transformation. The first integral has been computed symbolically whereas the second integration requires numerical evaluation.

Recently, new equations were constructed that allow computation of vertical stresses and settlements beneath the corner of a triangular base, loaded with a nonuniform load. Although for points that do not coincide with the corner of a triangle the superposition principle must still be implemented it is possible to obtain results for a general polygonal base with the linear load distribution in very fast fashion as all solutions are presented as symbolically evaluated integrals that are much faster and more stable as numerically evaluated.

2 Main Modules of the Package

The present version 3.0 of the package supports user interfaces in four different languages : Slovenian, Croatian, English and Italian. If the interface language is changed during the analysis all menus are immediately transformed in the selected language. Further languages can also be added upon a request.

The program can be considered as an assemblage of five entireties:

- preparation of the input data
- verification of the ultimate bearing capacity
- estimation of vertical stresses and settlements
- computation of consolidation
- soil improvement with slopes

2.1 Data Preparation

Currently four accessory modules are available for the data preparation.

2.1.1 Determination of Underlying Layers And Corresponding Thickness Using Geophones

Using geophones it is possible to determine the number of underlying layers and their corresponding thicknesses. The procedure allows the identification of a single layer or two layers. The in situ investigations are performed with series of geophones mounted on selected distances, and on the geophones the time delay between the initiation of the noise signal and its recognition on the geophones is measured. The analysis data are therefore the time delay measured on each geophone, and also the distance of the geophone from the centre of the noise signal. As the time delays are usually very short their units are *msec*.

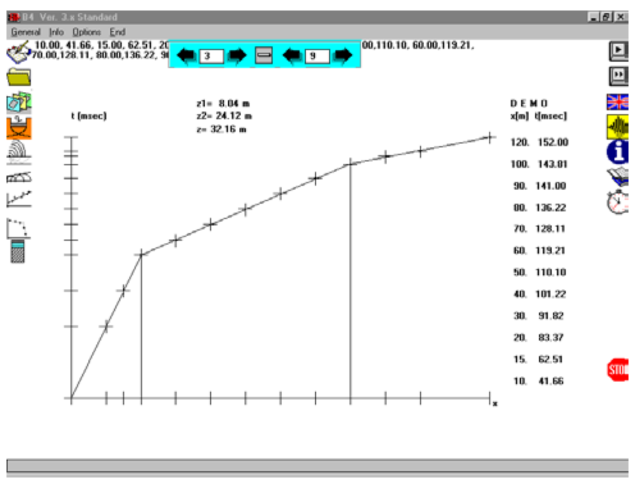

Figure 1: Determination of number and corresponding thicknesses of the underlying layers

When the data needed for the analysis are transmitted in pairs to the program, it is possible to compute the thickness for up to two layers. The procedure, then, on the basis of the least square method computation fits the lines to the given data and determines the thicknesses (Figure 1).
The solution of the problem is performed with automatic adjustment of two (or three) lines to the given data. If three lines can be adjusted to the given data then two layers can be determined, and with two lines only one layer can be determined. The algorithm adjusts the lines optimally but also the manual adjustment by the user is available using the simple menu provided.

2.2.2 Determination of Cohesion and Friction Angle using either Direct Shear Tests or Triaxial Tests

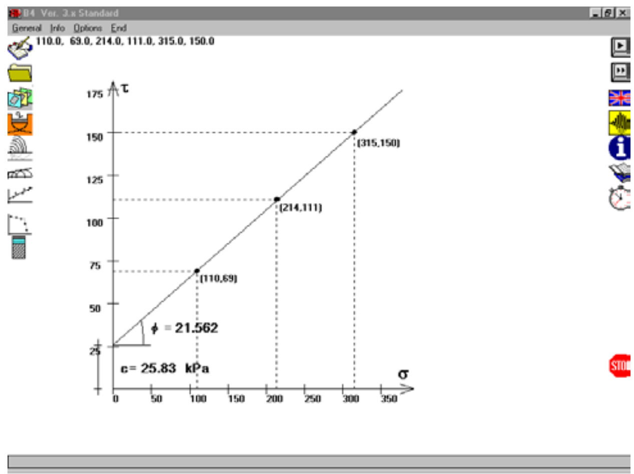

Figure 2: The results obtained from the direct shear test

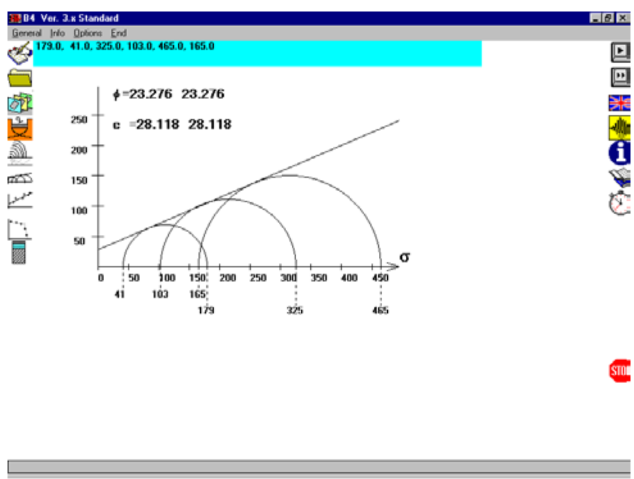

Figure 3: Determination of cohesion and shear angle from triaxial test

The modules calculate the cohesion and the friction angle on the basis of experimentally obtained results obtained either from direct shear tests or triaxial tests. The data is given to the module into a separate window in pairs as (σ,τ) or (σ_1,σ_3) for direct shear and triaxial test, respectively. The module utilises the least square method to obtain the most fitting results.

2.2.3 Determination of Unit Weight of the Material

The procedure allows elementary calculation of unit weight of the soil for an arbitrary combination of the porosity or void ratio in dependence of the degree of saturation. The module allows also computation for submerged soils.

3 Ultimate Bearing Capacity Supervision

Menu *qult* allows the verification of the ultimate bearing capacity of a rectangular base according to the equations that have been given by Terzaghi, Meyerhof in Hansen (Cernica; 1995), and Vesic

(Bowles; 1996). Verification according to all the authors is predefined, but each method can also be switched off (Figure 4).

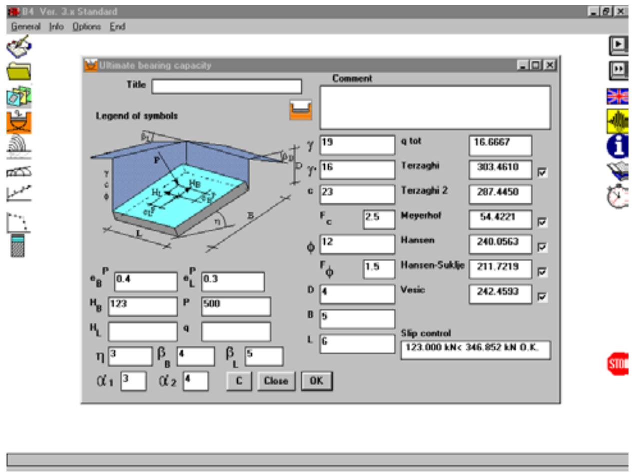

Figure 4: The input data form for the ultimate bearing capacity computation

The package yields six values for the ultimate bearing capacity - Terzaghi has namely given two solutions and two slightly different solutions also for the Hansen's theory exist. The first Terzaghi solution covers square bases and the second one strip bases. As both extreme cases are not very frequent in the engineering practice it can be considered that the actual solution lies between both boundary values. For the Hansen theory for rectangular base also two theories can be found (Šuklje, 1984 and Cernica, 1995). The two theories yield practically identical results except if the terrain around the base is sloped.

The package compares the load of the given base with all six computed values and in case that a computed value is smaller than the load of the base the computed value becomes a red background.

The module further performs the slip control of the base if the transverse load (concentrated load with eccentricity and uniform load) and inplane concentrated load are given.

The module also allows analysis on a multilayered soil. The analysis form requires the data about all layers (thickness, unit weight, cohesion and internal friction angle for each layer). The module afterwards calculates the average values of the cohesion and the angle of internal friction and implements these values in the analysis with a single layer (Figure 5).

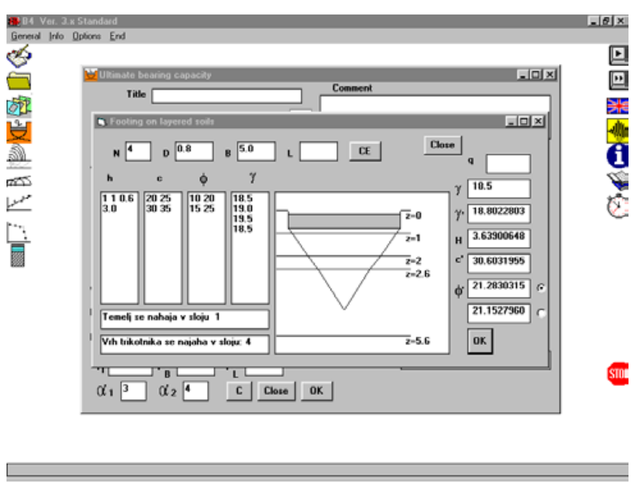

Figure 5: The input data form for the ultimate bearing capacity computation on a multilayered soil

This approach is very suitable as it allows that at some layer either cohesion or angle of internal friction is zero.

4 Estimation of Vertical Stresses and Settlements

This module doubtlessly represents the most characteristic and innovative component of the package as it embodies several new original equations for various types of bases. For almost all of these equations the final forms have been obtained by symbolic integration, and only for circular loads the integration is performed numerically. All these equations allow the point of the consideration to be chosen completely arbitrarily.

4.1 Data input

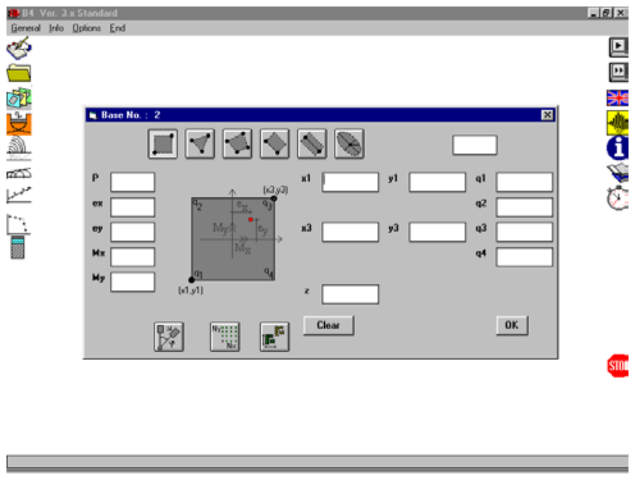

Figure 6: The menus for description of the base type

The program reads the input data about the soil half-space and the surface load from an input file that is a standard ASCII file. The data can be written into this file in two ways: using either interactive pre-processor or by manually writing the pure code into a file.

4.1.1 The pre-processor

For the preparation of the input data the program offers a graphically supported user-friendly pre-processor that allows the user to model the problem.

The pre-processor was designed in the approach that allows the user to prepare the input data using several forms that in a sequence appear on the screen. After defining the number of bases the user selects the appropriate menu for the description of the base under consideration (Figure 6).

Consequently, a new form designed to description of the selected base appears and allows the user to describe the position of the base and its load by filling in the numerical values into prepared cells. Once the base is being defined it is possible to use it for the description of new bases: a previously defined base can be namely rotated around a given point, an option that is very adequate for description of semi arches; and the base can be further multiplied to obtain a row, a column or an array of bases with identical dimensions. It is further possible to paste a pattern of previously defined bases to a new location.

4.1.2 Manual input of the data

All the input data can also be written in the input file directly using one of the standard editor programs as for example *Notepad*. The contents of the input file can also be supervised and as well modified during the implementation of the package with the program *Notepad* that is a standard part of the system Windows. This option is very convenient in the case when the obtained analysis results show that the input data has been modelled incorrectly, and the input data can thus be corrected immediately. The program automatically invokes the editor *Notepad* although all other editors can be used if the text output file has the extension *dat*.

4.2 Data required for Vertical Stresses and Settlements Computation

The entire geometry of the problem is defined with two independent types of data: load data and soil data.

4.2.1 Description of the Load Data

Load data can be further divided into:

Number of the bases and their positions

Version 3.0 of the program B4 allows the analysis of various types of loads. The origin of coordinate system can be chosen completely arbitrary and the boundaries of the bases can be oriented arbitrary against the coordinate axes.
If a rectangular base is parallel to the axes the data needed for the description of the base is reduced to minimum. In this case the location of the base in a chosen coordinate system (except of the depth z) is completely defined with only four values. The package in this case requires only the coordinates of the two diagonal points - the left bottom and right upper point.
If a rectangular base is not parallel to the axes eight values are required and all four nodal coordinates that must be given in a clockwise direction are required. To describe a rectangular base not parallel to the coordinate axes exactly (all opposite boundaries are strictly parallel to each other) two special options are available in the pre-processor.
The remaining types of bases (triangular, quadrilateral and circular/elliptic) require different types of data and are also completely supported by adequate forms in the pre-processor.

Base depth

As each base (regardless of its shape) can be on different depth the base depth data must be given before the load data is described. With the pre-processor this data can be omitted as the pre-processor automatically implements the value 0.

The load in the corner nodes

Two basic load cases are distinguished: a uniform load and a nonuniform load. If the load is uniform the value of the load can be given either in the first corner node only or in all nodes if the load is nonuniform. In the first case the ordering of the corner nodes is insignificant. However, when a nonuniform load has to be described over a base the prescribed node order must be taken into consideration. The first value of the load is considered to take place in the first corner node indicated in the figure that appears on the menu and remaining load values are assigned to corner nodes in a clockwise direction. In case of triangular bases the nonuniform load is automatically linear, but in the case of rectangular bases the nonuniform load is also linear if the condition

$q_1+q_3=q_2+q_4$ is satisfied (q_1, q_2, q_3 and q_4 represent the load values in the first, second, third and fourth corner, respectively). If this condition is not satisfied the non-linear load is being described over the base. The program allows also the input of an eccentric concentrated vertical load acting on a rectangular base that is converted into a linear pressure over the base.
All base types can have nonuniform distribution of the load except the circular and quadrilateral bases where the load must be strictly uniform. The linear load over a quadrilateral can be modelled by two triangles, each with a linear load.

4.2.2 Description of the Soil Stratum

The first data for the description of the soil is number of points with given stratographies and additional points where the analysis of stresses and settlements will be performed. The soil stratum is described with points where stratographies - the information about underlying layers are known. These points (required is at least one point with a given stratography) are defined with coordinates x and y. The additional points are those points where the exact stratography is not known but it can be assumed that in such point engineering interesting (for example maximal settlement) results will occur. Such points can also be defined with actual coordinates and with the reference to a previously given stratography for which is being assumed that will adequately represent the soil stratum at the considered point.
Each point requires the data about the underlying soil layers – first, the number of all underlying layers, and afterwards the modulus of elasticity and Poisson's coefficient for each layer. This is the minimum data required for each layer but for the purposes of analysis also other data can be given (to perform the consolidation analysis the coefficients of permeability are required, for example).

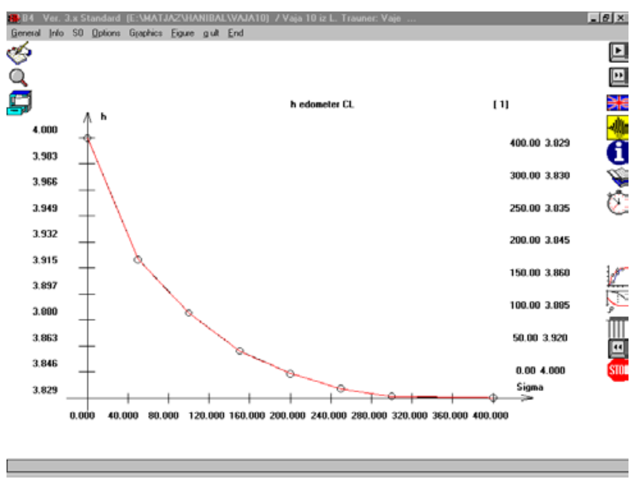

Figure 7: Computational curve obtained from the laboratory obtained data

The margins of each layer are defined with the corresponding initial depth and ending depth. For the first layer the initial depth i.e. z distance from the origin of the coordinate system, and ending depth (initial depth plus thickness of the layer) are required. For all following layers only ending depth are required as initial depth is assumed to be equal to the ending depth of the upperlying layer. Two additional information are also required: modulus of elasticity and Poisson's ratio. If laboratory obtained curves for a layer's material are available it is possible to include them into the analysis by replacing the value of the modulus of elasticity with the number describing the test data set. For such a material the program computes the modulus of elasticity according to the laboratory test type (h oedometer, e oedometer or triaxial test) and the parameters provided by the user. As the laboratory obtained data is given as a finite set of discrete pairs of points the missing data of the curve among given discrete points can be obtained using either piecewise linear interpolation or

Lagrange interpolation. If a large amount of the input data is obtained from the laboratory test it is preferable to use the linear interpolation as it produces more stable and reliable results (Figure 7), and at the other hand the Lagrange interpolation produces non-reliable curves.
Regardless to the curve type the modulus of elasticity can be determined using either the tangent or secant method. The secant modulus computes the modulus of elasticity between the initial and final state of stresses, i.e. uses actual increase of stresses, while the tangent modulus determines the modulus of elasticity according to the final state of stresses. The predefined options are the piecewise linear interpolation and the secant method for determination of modulus of elasticity. When using the laboratory obtained curves it is important to forward to the program also the information about unit weight for all soil layers materials as with this data it is possible to obtain stress state prior to the applied load.
As each layer is characterised by the starting depth, the stratography data end with the final depth.
If the stratography is identical in several points of interest this information does not need to be repeated for each consecutive point that has identical stratography. The information about equal stratographies is shared among several points using reference points. Such points are normally defined in regions where maximum values are assumed and do not improve the quality of the analysis as they can also be selected later.

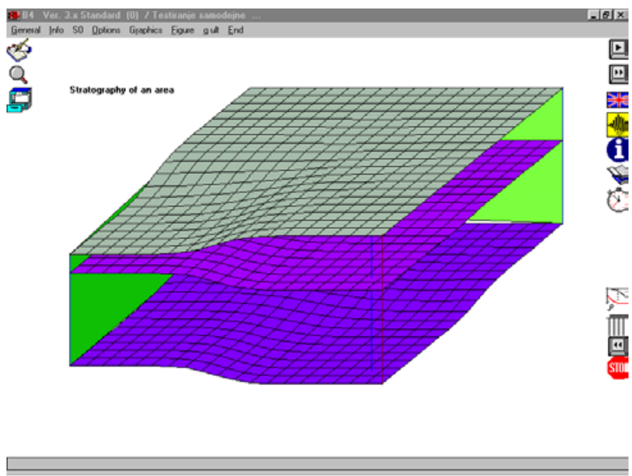

Figure 8: Interpolated soil stratum of two layers

Although the stratographies can be defined in discrete points only it is possible to predict the stratography also outside these points. The package namely offers several options for numerical interpolations of the stratographies that differ in layer's thicknesses only (i.e. number of underlying layers, their Poisson's ratios and modulus of elasticity must be identical for all points). Several types of interpolation are available. For interpolation beneath the road embankments the piecewise linear interpolation is usually reliable enough as the points with known stratographies are given more or less beneath the embankment. For cases when known stratographies are given in a wider region two additional interpolation types are available. In the first algorithm new stratography is obtained on the basis of four closets given stratographies by transferring them into a bi-unit square where all interpolations are performed. The second algorithm considers all user predefined points to create a new stratography. The weight coefficient for each predefined stratography are for each new point computed as a function of the distance from the computed new point and predefined stratography. Using a simple user selected parameter it is possible to influence the results and thus obtain more realistic results (Figure 8).

4.3 Output of the data

All numerical results are stored beside all input data in the output file that is also a standard ASCII file with an extension *lis*. Similar to the input file also the output file can be accessed from the program during the analysis using program *Notepad*. Graphical output is due to the large amount of required memory not stored automatically but each figure can be stored optionally (with a double click on it) into a separated output file in the Windows Bitmap (*bmp*) standard. Output files (numerical results and graphics) can be later retrieved into a text processor and adequately formatted.
It is also possible to create the analysis reports using the word processor directly without pasting the results from the output file with extension *lis*. The package namely supports the word processor WinWord - versions 6, 7 (95), 8 (97) and higher. If this option is selected the package establishes a link to WinWord and navigates it to create the final report. Such a report is much more distinguishing than the standard output of the data as it consists of all main implemented equations accompanied with obtained results.

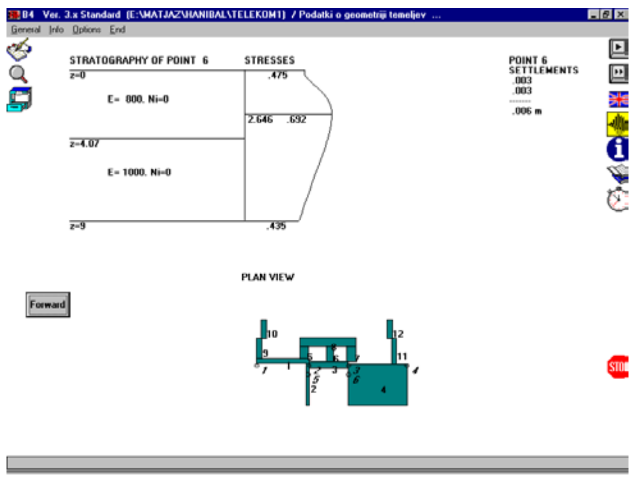

Figure 9: Typical screen during an analysis

To create the output file (with the extension *doc*) corresponding templates were created but these can be further modified according to the user needs (first page and headers with the logo of the user, for example). For some cases it is possible to distinguish between short and full reports (in the ultimate bearing capacity analysis it is possible to chose the report with main results only or the complete report where also the minor results are included).

5 Analysis

During the analysis of each point the most important results are simultaneously presented symbolically and graphically (Figure 9).
In the left upper corner a figure divided into two parts is presented. The left part of the figure displays the stratography and settlements of the point under consideration, and on the right part of the same picture the vertical stresses distribution is presented with the location and the value of maximum stress is indicated. In the right upper corner the deformations of all underlying layers of considered computational point appear on the screen during the calculation. Their sum representing the complete settlements of the point is also displayed. All these results are automatically stored also in the output file. The figures are not stored automatically but optionally. In the bottom part of the screen all bases with computational points are presented in the plan view.

6 Graphical representation of the obtained results

When analysing stresses and settlements it is often required to study their behaviour not only in selected predefined points but also either in given planes or given regions. The package thus offers menus that allow a comparison of all the results in various forms of graphical representation.

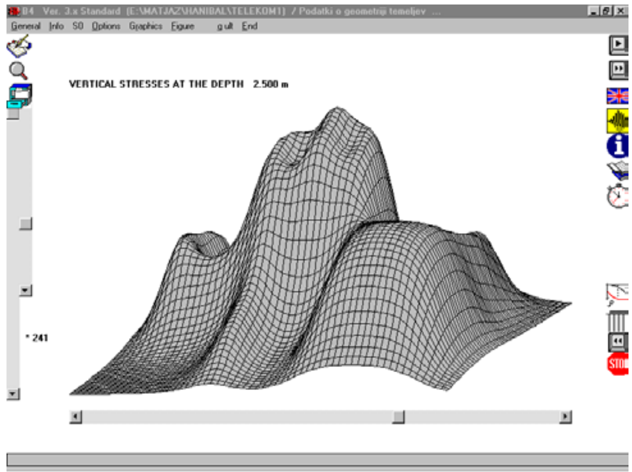

Figure 10: Distribution of stresses in a selected depth for a defined rectangular area

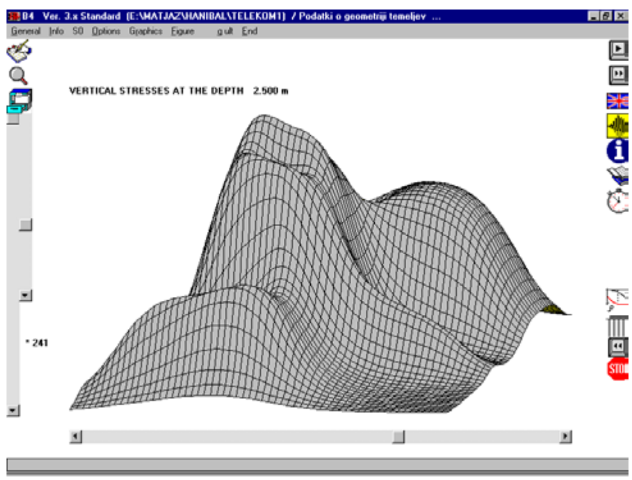

Figure 11: Equal distribution of stresses from a different angle of view

Probably the most effective one is a three-dimensional representation of the results over a selected rectangular area (Figure 10).
Three-dimensional representation offers a complete information about the distribution of the results with the implementation of view angle change (Figure 11).
With the change of the scale for plots the optimal graphical representation can be obtained. In this way stresses and settlements can be represented.
3D representations of the results can be also replaced by representation with isolines (Figure 12) or isosurfaces (Figure 13).

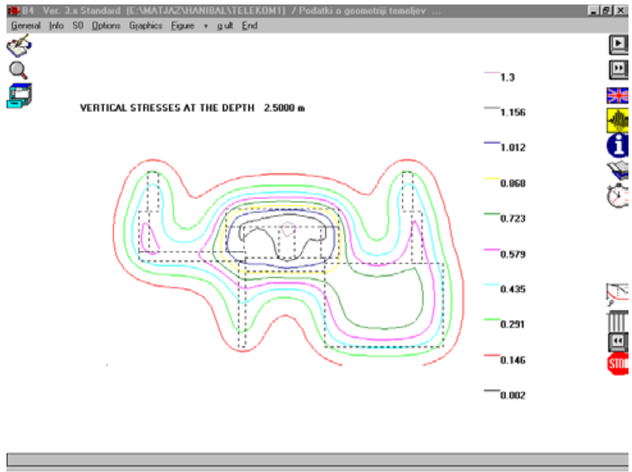

Figure 12: Graphical representation of the results using isolines

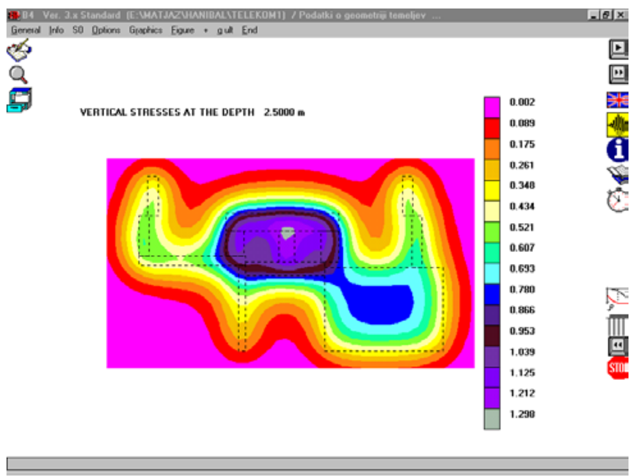

Figure 13: Graphical representation of the results using isosurfaces

The advantage of such representations over 3D representation is comprehensible distribution of the results without further modifications of the figures.
Another essential advantage of such a representation is that it is possible to select additional single points for verification of stresses and settlements directly from the figure using mouse only. Although such additional points can also be selected manually the implementation of the mouse allows more user friendly selection of desired points.

7 Computational Speed

To test the computational speed the task to compute stresses and settlements of a region described with 10000 points (grid of 100·100 points) loaded with two rectangular bases with linear nonuniform load can be used as an indicator of the actual computational speed. As both loads are rectangular the results will be obtained using two discretisations - triangular and rectangular. The given stratography consists of three layers. The tests were performed on a Pentium I PC 166 MHz with 64 MB of memory.

Task	Triangular	Rectangular
Stresses	31 sec	31 sec
Settlements	96 sec	33 sec

Table 1: Comparison of computational speed for various discretisations of the problem

It is obvious from Table 1 that when computing stresses both discretisations practically require the same time. On the other hand, when computing settlements the rectangular discretisation is about three times faster than triangular discretisation. The reason for that is that when using triangular discretisation each triangular base is further discretisied with six right-angle triangles.
Regardless the discretisation the computational speed is good enough to produce all results in a very short time.

8 Consolidation

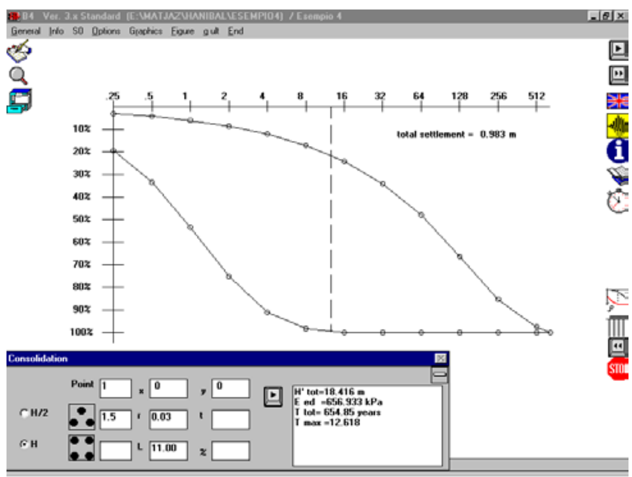

Figure 14: The results of the consolidation analysis

In order to perform the consolidation analysis the coefficients of the permeability (for horizontal and vertical direction or a unique coefficient) must be given for each layer. The analysis can be performed without or with vertical drains. In the case of vertical drains the user can select the topology of the vertical drains (triangular or rectangular), the distance among them, the radius of vertical drains and their length. It is possible to consider consolidation either in both vertical directions (up and down) or towards the surface only.
The package displays the following results: computational height of a supplementary virtual layer, the average oedometric modulus of such a layer and final times of the consolidation – for case without or with vertical drains. The module also plots the time history for the case without or with vertical drains using logarithmic scale (Figure 14) and the data used for the plot are stored as a table in the output file. It is further possible to determine the settlement for a given time or to determine the required time for a given rate of consolidation.

9 Soil Characteristics Improvement using Slopes

If the computed settlements are in an unacceptable range the soil characteristics can be improved using slopes. Package considers lime and gravel slopes. For the given data the program calculates the virtual soil characteristics for the region improved with slopes and afterwards computes corresponding settlements.

10 Documentation of the package

The package is supported with a 100 page manual, which covers all phases of the analysis: preparation of data, manipulation and computation and also interpretation of obtained results. The manual summarises also the basic theories implemented in the package and all package options are demonstrated with examples.

11 WWW on line information

Information about upgrades and new versions are also available on the Internet address: http://www.geocities.com/b4bymcs/B4/B4homeE.htm, where some files containing information are stored. A CD with longer demo containing ScreenCam movie file can be obtained by sending a request to B4@email.si.

12 Conclusions

The presented paper briefly presents the essential features of an advanced software package B4. The package can be characterised by two main advantages: simplicity and innovative equations that allow the computation of much wider problems than just a simple rectangular bases loaded with a uniform load.

The amount of information the package offers in real time is practically inaccessible if classical approaches are used. As the package runs on the popular and widespread Windows platforms (Windows 2000, ME, '98 & '95, Windows 3.1+) it is available as an excellent educational tool not only to students but also to a wide spectrum of civil engineers involved in geomechanical problems.

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