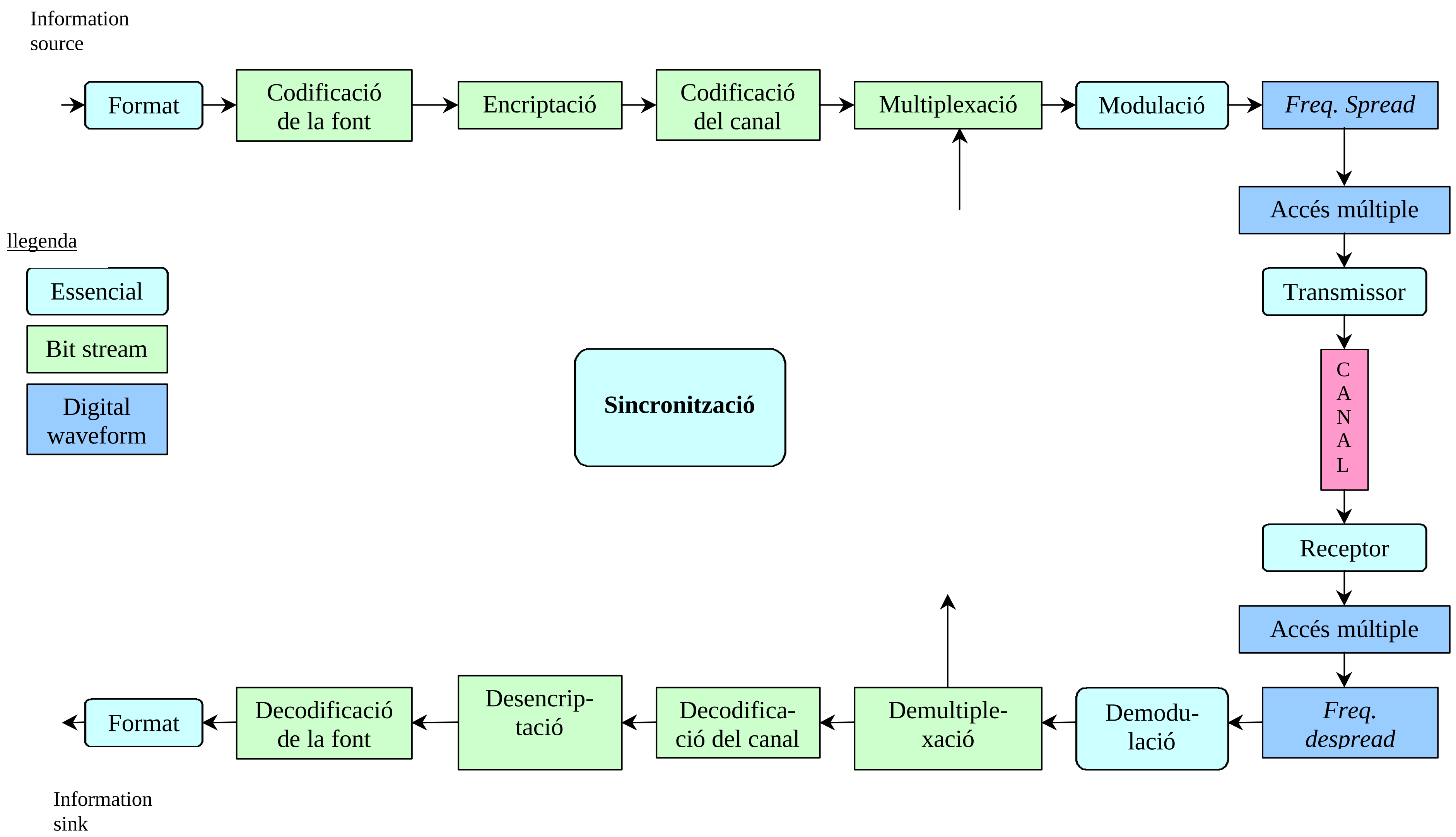
Esquema general d'un sistema de transmissió digital



El sistema de transmisión de datos

Pulse Amplitude Modulation (PAM)

$$x_T(t) = \sum_{n=-\infty}^{+\infty} a_n p(t - nT)$$

M posibles símbolos $\quad \text{b bits} = \log_2 M \quad \text{M} = 2^{\text{b}}$

$$r = \frac{1}{T} \ [\text{símbolos/seg}]$$

$$r_b = rb\,[bits/seg]$$

$$R_{X_T X_T}(t+t,t) = E\{x_T(t+t)x_T^*(t)\} : \text{cicloestacionario}$$

$$\overline{R}_{X_T X_T}(t) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} R_{aa}(n) R_{pp}(t - mT)$$

$$S_{X_T X_T}(f) = F\{\overline{R}_{X_T X_T}(t)\} = \sum_{n=-\infty}^{+\infty} R_{aa}(n) e^{-2pfnT} \frac{|P(f)|^2}{T}$$

$$m_a = E\{a_n\}$$

$$a_n \equiv A_n + m_a$$

$$S_{x_T x_{Tt}}(f) = \left[\sum_{n=-\infty}^{+\infty} R_{AA}(n) e^{-2pfnT}\right] \frac{|P(f)|^2}{T} + \frac{|m_a|^2}{T^2} \sum_{n=-\infty}^{+\infty} |P(\frac{k}{T})|^2 \, d(f - \frac{k}{T})$$

(a) a[n] estadísticamente independientes

$$S_{x_T x_T}(f) = rs_a^2 |P(f)|^2 + r^2 |m_a|^2 \sum_{n=-\infty}^{+\infty} |P(kr)|^2 \, d(f - kr)$$

(b) a[n] estadísticamente independientes y media nula

$$S_{x_T x_T}(f) = rs_a^2 |P(f)|^2$$

$$x_R(t)_{|t=t_k=kT} = \sum_{n=-\infty}^{+\infty} p_r(kT - nT) + w(t) = \underbrace{a_k p_r(0)}_{\text{señal}} + \underbrace{\sum_{n=-\infty, n\neq k}^{+\infty} p_r((k-n)T)}_{ISI} + \underbrace{w(t)}_{ruido}$$

$$p_r(t) \equiv p_T(t) * h_T(t) * h_C(t) * h_R(t)$$

$$w(t) \equiv n(t) * h_R(t)$$

Condición no ISI. Teorema de Nyquist

$$p(kT) = \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases} \quad \Leftrightarrow \quad r\sum_{k=-\infty}^{+\infty} P(f - kr) = cte$$

Teoría de la decisión

Suponemos que no hay ISI, y que trabajamos, pues, con pulsos ideales.

$$y_k = y(kT) = a_k + w(kT)$$

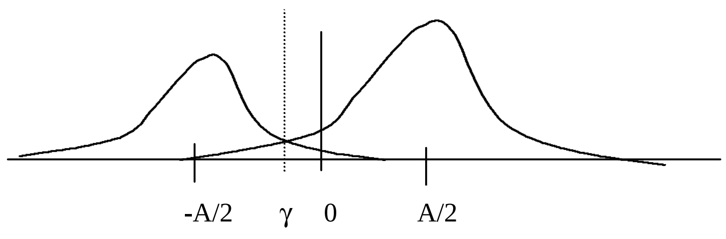
$$P_{error} = P_e = P(e|'1')P('1') + P(e|'0')P('0')$$

$$P_{error} = BER \Leftarrow \text{los símbolos son binarios}$$

$$H_0 = '0', H_1 = '1' \quad \text{hipótesis}$$

$$P_e = P(e|H_0)P(H_0) + P(e|H_1)P(H_1)$$

$$P(e|H_1) = \int_{-\infty}^{+\infty} f_{y|H_1}(y|H_1)dy = \int_{-\infty}^{g} f_w(y + \frac{A}{2})dy$$



$$g_{\text{óptima}} \Rightarrow \frac{\partial}{\partial g} P_e(g) = 0$$

$$f_{y|H_0}(y \mid H_0)P(H_0) = f_{y|H_1}(y \mid H_1)P(H_1)|_{y=g}$$

$$f_w(g + \frac{A}{2})P(H_0) = f_w(g - \frac{A}{2})P(H_1)$$

$$\text{si } f_W(w) \approx Gaussiana(0, s_w^2),\ f_W(w) = \frac{1}{\sqrt{2p s_w^2}} e^{-\frac{1}{2}\frac{w^2}{s_w^2}}$$

$$\Rightarrow Q(x) \equiv \int_x^{+\infty} \frac{1}{\sqrt{2p}} e^{-\frac{1}{2}w^2} dw \Rightarrow g_{\text{óptima}} = -\frac{s_w^2}{A} \ln \frac{P(H_1)}{P(H_2)}$$

Si a[n] es una v.a, entonces

$$f_a(a) = P(H_0)d(a + \frac{A}{2}) + P(H_1)d(a - \frac{A}{2})$$

si $g = x + y$, con x e y v.as, $f_g(g) = f_x(g) * f_y(y)$ si x, y son indep.

$$f_y(y) = f_a(y) * f_w(y)$$

$$P(e \mid H_0) = Q(\frac{g + A/2}{s_w}) \qquad P(e \mid H_1) = Q(\frac{g - A/2}{s_w})$$

$$\text{si } g = 0 \Rightarrow P_e = Q(\frac{A}{2s_w})$$

Teoría de la detección óptima

$$H_R^{\text{óptimo}}(f) = l \frac{P^*(f)}{S_{nn}(f)} e^{-2pft_o}$$

$$\text{Si n(t)} \approx \text{AWGN}, S_{nn}(f) = \frac{N_0}{2} \Rightarrow H_R^{\text{óptimo}}(f) = l p^*(td - t)$$

s'anomena Filtre Adaptat, i és el detector óptim amb soroll gaussiá

$$h_r^{\text{óptima}}(t) = ap^*(td - t)$$

$$a = \frac{1}{E_p} = \frac{1}{\int_{-\infty}^{+\infty} |p(t)|^2 \, dt} \Rightarrow h_r^{\text{óptima}}(t) = \frac{1}{E_p} p^*(td - t)$$

$$S_T(t) = E\{|x(t)|^2\} = E\{|\sum a_n p(t - nT)|\}$$

$$S_T = \frac{1}{T}\int_T S_T(t)dt = r s_a^2 E_p \overset{\text{binari polar}}{=} r\left[\frac{1}{2}(\frac{A}{2})^2 + \frac{1}{2}(-\frac{A}{2})^2\right]E_p = r\frac{A^2}{4}E_p = S_T$$

$$E_b = S_T T = \frac{A^2}{2} = \sqrt{\frac{E_b}{2E_p}} \Rightarrow BER = P_e = Q(\frac{A}{2s_w}) = Q(\sqrt{\frac{E_b}{2}})$$

Sistemas multinivel

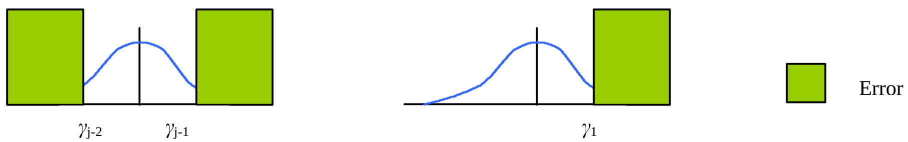
$$a_k = \{\pm\frac{A}{2}, \pm 3\frac{A}{2}, 5\pm\frac{A}{2}, \ldots, \pm(M-1)\frac{A}{2}\} \qquad M \text{ símbolos equiprobables } p(H_i) = \frac{1}{M}$$

$$h_r(t) = \frac{1}{E_p} p^*(td - t)$$

$$f_{y|H_i}(y \mid H_i) = f_w(y - a_i)$$

$$min\{P_e\} = \sum_{i=1}^{M} P(e \mid H_i)P(H_i) = \frac{1}{M}\sum_{i=1}^{M} P(e \mid H_i)$$

2 casos: centrales y extremos



Transmissió de dades

$$P_e = \overbrace{\frac{2}{M}P(e|H_1)}^{\text{extrems}} + \overbrace{\frac{1}{M}\sum_{i=2}^{M-1}P(e|H_i)}^{\text{centrals}} = \frac{2}{M}\int_{g_1}^{+\infty} f_W(y-a_1)dy + \frac{1}{M}\sum_{i=2}^{M-1}(1-\int_{g_{i-1}}^{g_i} f_W(y-a_i)dy$$

$$P_e(g_1, g_2, \ldots, g_{M-1}) \Rightarrow \frac{\partial}{\partial g_j}P_e = 0 \;\; \forall j \Leftrightarrow f_W(g_j - a_j) = f_W(g_j - a_{j-1})$$

$$g_j^{\text{óptima}} = \frac{a_{j+1}+a_j}{2} \Leftrightarrow \begin{cases} P\{H_i\} = \frac{1}{M} \\ f_W(w) = f_W(-w) \end{cases}$$

$$P_e = \frac{2M-2}{M}Q(\frac{A}{2s_w}) \qquad BER \cong \frac{P_e}{\log_2 M}$$

$$E_b \rightarrow \frac{E_b}{N_o} \rightarrow \text{Governa la qualitat del sistema}$$

$$E_b \equiv S_T \cdot T_b \equiv S_T\frac{T}{b}$$

$$T_b \equiv \frac{T}{b} = \frac{T}{\log_2 M}$$

$$S_T \begin{cases} R_{X_T X_T}(0) \\ \frac{1}{T}\int_T E\{|x(t)|^2\}dt \\ \text{estats independents} \end{cases}$$

$$s(t) = a_k p(t-kT)$$

$$E_s = \int_{-\infty}^{+\infty} E\{|S(t)|^2\}dt = E\{|a_k|^2\}\int_{-\infty}^{+\infty}\overbrace{E\{|p(t)|^2\}dt}^{E_p} = E\{|a_k|^2\}E_p$$

$$E_b = \frac{E_s}{b} = \frac{m^2-1}{12}A^2\frac{E_p}{M\log_2 M}$$

$$P_e = \frac{2M-2}{M}Q(\sqrt{\frac{6\log_2 M}{m^2-1}(\frac{E_b}{N_o})})$$

Polsos de Nyquist

$$p(t) = sinc(rt) = \frac{sin(prt)}{prt} \leftrightarrow P(f) = T\prod(Tf)$$

$$p_N(t) \equiv p_b()sinc(rt) \leftrightarrow P_N(f) = P_B(f) * \prod(Tf), P_B(f) = 0 \forall |f| \geq BW_B$$

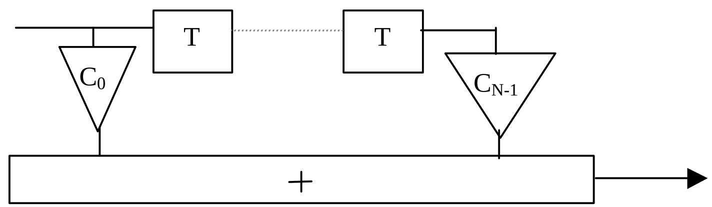
$$BW_N = \frac{r}{2} + BW_B \equiv \frac{r}{2}(1+b)$$

$$b \equiv \frac{BW_B}{r/2} : \text{factor de ROLL-OFF (\%)}$$

Ecualización

Igualación (compensación) del canal
Canal: forzadores de ceros, si el ruido afecta poco a la calidad del sistema
Canal + ruido: ecualización, forzadores de ceros.

Forzador de ceros



$$\overbrace{\begin{vmatrix} P(1) & 0 & 0 \\ p(2) & p(1) & 0 \\ p(3) & p(2) & p(1) \end{vmatrix}}^{P} \overbrace{\begin{vmatrix} C_0 \\ C_1 \\ C_2 \end{vmatrix}}^{C} = \overbrace{\begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}}^{\bar{1}} \text{donde haya más señal}$$

$$P \cdot C = 1$$

$$C = P^{-1}1$$

$$C_{opt}^{ZF} = (P^T P)^{-1}P^T \cdot 1 \Leftarrow MECM$$

El uso de forzadores de cero incrementa la pot

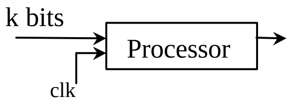
Waveform coding

Orthogonal signals

$$\Rightarrow \frac{1}{E}\int_0^T s_i(t)s_j(t)dt = \begin{cases}1 & i=j\\ 0 & otherwise\end{cases}$$

$$E_i = \int_0^T s_i^2(t)dt$$

M-ary signaling



Produces one of $M=2^k$ waveforms

Orthogonal codes

Data set	Orthogonal codeword set
0	0 0
1	0 1

$$H_K = \begin{bmatrix} H_{k-1} & H_{k-1}\\ H_{k-1} & \overline{H}_{k-1}\end{bmatrix}$$

$$z_{ij} = d[i-j]$$

$$P_E(k) \le (2^k-1)Q\left(\sqrt{\frac{kE_b}{N_0}}\right)$$

$$\frac{P_B(k)}{P_E(k)} = \frac{2^{k-1}}{2^{k-1}-1}$$

Biorthogonal codes

Biorthogonal signal set of M signals or codewords can be obtained from an orthogonal set of M/2 signals. It consists of a combination of orthogonal and antipodal signals.

$$B_k = \begin{bmatrix} H_{k-1}\\ \overline{H}_{k-1}\end{bmatrix}$$

$$z_{ij} = d[i-j] - d\left[|i-j| - \frac{M}{2}\right] = \begin{cases}1 & i=j\\ -1 & |i-j| = \frac{M}{2}\\ 0 & otherwise\end{cases}$$

$$P_E(k) \le (2^k-2)Q\left(\sqrt{\frac{kE_b}{N_0}}\right) + Q\left(\sqrt{\frac{2kE_b}{N_0}}\right)$$

$$P_B(k) \cong \frac{P_E(k)}{2}, k > 3$$

i.e.:

Data set	Orthogonal codeword set
0 0	0 0 0 0
0 1	0 1 0 1
1 0	0 0 1 1
1 1	0 1 1 0

$$H_2 = \begin{bmatrix} 0&0&0&0\\ 0&1&0&1\\ 0&0&1&1\\ 0&1&1&0\end{bmatrix}$$

Data set	Biorthogonal codeword set
000	0 0 0 0
001	0 1 0 1
010	0 0 1 1
011	0 1 1 0
100	1 1 1 1
101	1 0 1 0
110	1 1 0 0
111	1 0 0 1

$$H_2 = \begin{bmatrix} 0&0&0&0\\ 0&1&0&1\\ 0&0&1&1\\ 0&1&1&0\\ 1&1&1&1\\ 1&0&1&0\\ 1&1&0&0\\ 1&0&0&1\end{bmatrix}$$

Transorthogonal (Simples) Codes

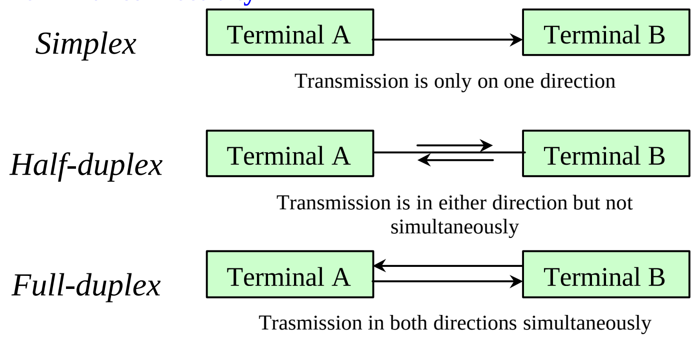
Results from deleteing the first digit of each orthogonal codeword.

$$z_{ij} = \begin{cases}1 & i=j\\ \frac{-1}{M-1} & i\ne j\end{cases}$$

represents minimum energy

Types of error control

Therminal connectivity



Transmissió de dades

Automatic Repeat Request (ARQ methods)

Stop-and-wait ARQ (half-duplex)	Transmitter waits for an ACK or a NAK.
Continuous ARQ with pullback (full-duplex)	When the transmitter recevices a NAK it also receives in which transmission block was the error and restarts transmission from that transmission block.
Continuous ARQ with selective repeat (full-duplex)	Only the corrupted message is repeated.

Forward Error Correction (FEC)

- **A reverse channel is not available or the delay with ARQ would be excessive.**
- **The retransmission strategy is not conveniently implemented.**
- **The expected number of errors, without corrections, would require excessive retransmissions.**

Structured sequences

Orthogonal coding techniques have inefficient use of bandwidth.

Channel Modes

Discrete Memory Less Channel (DMC)

$$P(j \mid i)\{1 \le i \le M, 1 \le j \le Q\}$$

Discrete input alphabet — i : modulator M-ary symbol

Discrete output alphabet — j : demodulator Q-ary output symbol

Set of conditional probabilities — $P(i \mid j)$: prob. of receiving j given that i was transmitted.

Input sequence $U = u_1,\ldots,u_m$

Output sequence $Z = z_1,\ldots,z_m$

$$P(Z \mid U) = \prod_{n=1}^{m} P(z_m \mid u_m)$$

Binary Symmetric channel (BSC) “Hard Decisions”

$$P(0 \mid 1) = P(1 \mid 0) = p$$

$$P(1 \mid 1) = P(0 \mid 0) = 1 - p$$

$$p = Q\left(\sqrt{\frac{2E_c}{N_0}}\right)$$

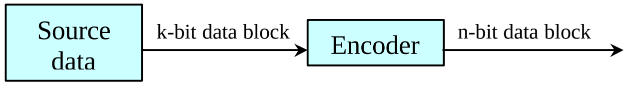
Gaussian Channel “Soft Decisions”

Discrete input

Continious output

$$p(z \mid u_k) = \frac{1}{s\sqrt{2p}} e^{-\frac{(z-u_k)^2}{2s^2}}$$

Code Rate and Redundancy



n-k bits: redundant bits, parity bits or check bits.

Fundamentals of Statistical Decision Theory

Bayes' Theorem

$$P(A|B)=\frac{P(B|A)P(A)}{P(B)}$$

$$P(s_i|z_j)=\frac{P(z_j|s_i)P(s_i)}{P(z_j)}=\frac{P(z_j|s_i)P(s_i)}{\sum_{p=1}^{M}P(z_j|s_p)P(s_p)}$$

$P(s_i)$: a priori probability

$P(z_j|s_i)$: likelihood

$P(s_i|z_j)$: a posteriori probability

Mixed form:

$$P(s_i|z)=\frac{P(z|s_i)P(s_i)}{P(z)}=\frac{P(z|s_i)P(s_i)}{\sum_{p=1}^{M}P(z|s_p)P(s_p)}$$

z: continious sample

Decision theory

$$z(T)=a_i(T)+n_0(T)$$
$$r(n)=a(n)+w(n)$$

MAP: $\arg_i max\{P(s_i|z)\}=\arg_i max\{P(z|s_i)P(s_i)\}$

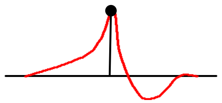
ML: $\arg_i max\{P(z|s_i)\}$

Binary

MAP: $P\{z|s_1\}P\{s_1\}>P\{z|s_2\}P\{s_2\}\Rightarrow H_{s_1}$

ML: $P\{z|s_1\}>P\{z|s_2\}\Rightarrow H_{s_1}$

MLSE (Maximum Likelihood Sequence Estimation)



M_1: samples before maximum

M_2: samples after maximum

L symbols $\Rightarrow y(\text{-}M_1)...y(L-1-M_2)$

$y_{received}(t)$

Gaussian Noise: $\hat{s}_i=\arg_{s_i}\min\{\sum_{k=-M_1}^{L-1-M_2}(y_r(k)-s_i(k))^2\}$

Laplacian Noise: $\hat{s}_i=\arg_{s_i}\min\{\sum_{k=-M_1}^{L-1-M_2}|y_r(k)-s_i(k)|\}$

Codificación de la fuente

$I(X_j)$: autoinformación

$I(X_j) = -\log_2 p_j$

X_j : *símbolo*

$P(X_j) = p_j$

$H(X)$: entropía de la fuente

$$H(X) = E\{I(X_j)\} = -\sum_{j=1}^{N} p_j \log_2 p_j$$

$$M_{tupla} \Rightarrow P(X_1,\dots,X_M) = \prod_{m=1}^{M} P(X_m)$$

$$H_M(X) = H(X)$$

$$H_M(X)_{\text{con memoria}} < H_M(X)_{\text{sin memoria}}$$

Límites de la transmisión de datos

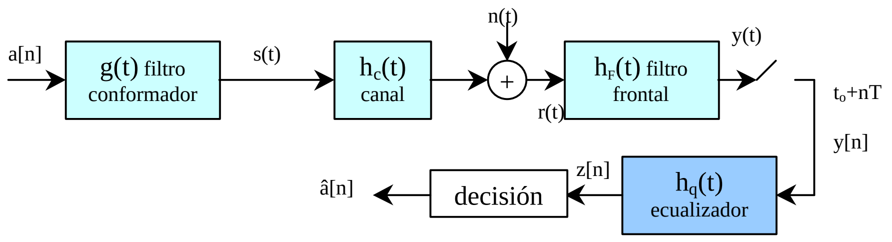
- Ancho de banda
- Potencia de señal
- Complejidad

1948, Shannon $C = W\log_2(1+SNR)$ bps : capacidad

$E = \dfrac{r_t}{W}$ $(bits/s/Hz)$: eficiencia espectral

$r_b = r_s \log_2 \underbrace{M}_{\text{nº de símbolos}}$ (bps)

PAM



$$s(t) = \sum_{n=-\infty}^{+\infty} a(n)g(t-nT)$$

$$r(t) = s(t)*h_c(t) + n(t) = \sum_{n=-\infty}^{+\infty} a(n)g(t-nT)*h_c(t) + n(t)$$

$$y(t) = s(t)*h_c(t)*h_f(t) + n(t)*h_f(t)$$

$$h(t) = n(t)*h_f(t)$$

$x(t) \equiv g(t)*h_c(t)*h_f(t)$: respuesta impulsional

$x_d(t) \equiv g(t)*h_f(t)$: respuesta impulsional de diseño (canal ideal)

$$y(t) = \sum_{n=-\infty}^{+\infty} a(n)x(t-nT) + h(t)$$

$$y(t_0+nT) = \sum_{m=-\infty}^{+\infty} a(m)x(t_0+nT-mT) + h(t_0+nT)$$

$$y(n) = \sum_{m=-\infty}^{+\infty} a(m)x(n-m) + h(n) = a(n)*x(n) + h(n)$$

$$z(n) = a(n)*h(n)$$

$$h(n) \equiv h_q(n)*x(n)$$

Filtro frontal y filtro conformador

$$h(t) = n(t)*h_f(t)$$

$$E\{h^2(\mathrm{t})\} = \frac{N_0}{2}\int_{-W}^{+W} |H_f(f)|^2\,df = \frac{N_0}{2}\int_{0}^{t} |h_f(t)|^2\,dt$$

$h_f(t) = g(t_0 - t)$: filtro adaptado

$$\Rightarrow SNR_{max} = \frac{E\{a^2(n)\}}{\dfrac{N_0}{2}}$$