Antennas

Radiation Integrals and Auxiliary Potential Functions

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The vector potential $\vec{A}$ for an electric current source $\vec{J}$

$\nabla \cdot \vec{B} = 0 \Leftrightarrow \vec{B}$ is solenoidal $\quad \nabla \cdot \nabla \times A = 0$

$$\vec{B}_A = m\vec{H}_A = \nabla \times \vec{A}$$

$$\vec{H}_A = \frac{1}{m} \nabla \times \vec{A}$$

$$\vec{E}_A = -\nabla f_e - jw\vec{A}$$

$$\nabla \times H_A = \vec{J} + jwe\vec{E}_A$$

$$\nabla \cdot \vec{A} = -jwem f_e \Rightarrow f_e \equiv \frac{1}{jwem} \nabla \cdot \vec{A}$$

Lorentz condition: $\nabla^2 \vec{A} + k^2 \vec{A} = -m\vec{J}$

$$\vec{E}_A = -\nabla f_e - jw\vec{A} = -jw\vec{A} - j\frac{1}{wme} \nabla \left(\nabla \cdot \vec{A} \right)$$

The vector potential $\vec{F}$ for a magnetic current source $\vec{M}$

$$\vec{E}_F = -\frac{1}{e} \nabla \times \vec{F} \qquad f_m = -\frac{1}{jwem} \nabla \cdot \vec{F}$$

$$\vec{H}_F = -\nabla f_m - jw\vec{F} \qquad \nabla^2 \vec{F} + k^2 \vec{F} = -e\vec{M}$$

$$\nabla \times \vec{E}_F = -\vec{M} - jwm\vec{H}_F \qquad \vec{H}_F = -jw\vec{F} - \frac{j}{wme} \nabla (\nabla \cdot \vec{F})$$

Electric and magnetic fields for electric ($\vec{J}$) and magnetic ($\vec{M}$) current sources

1. Specify $\vec{J}$ and $\vec{M}$ (electric and magnetic current density).
2. Find $\vec{A}$ (due to $\vec{J}$) using:

$$\vec{A} = \frac{m}{4p} \iiint_V \vec{J} \frac{e^{-jkR}}{R} dv'$$

3. Find $\vec{F}$ (due to $\vec{M}$) using:

$$\vec{F} = \frac{e}{4p} \iiint_V \vec{M} \frac{e^{-jkR}}{R} dv'$$

4. The total fields are:

$$\vec{E} = \vec{E}_A + \vec{E}_F = -jw\vec{A} - j\frac{1}{wme} \nabla \left(\nabla \cdot \vec{A} \right) - \frac{1}{e} \nabla \times \vec{F}$$

$$\vec{H} = \vec{H}_A + \vec{H}_F = \frac{1}{m} \nabla \times \vec{A} - \frac{1}{jwm} \nabla \times \vec{E}_F$$

Far-field radiation

$$\vec{A} = \hat{a}_r A_r(r,q,f) + \hat{a}A_q(r,q,f) + \hat{a}A_f(r,q,f)$$

$$\vec{A} \cong \left(\hat{a}_r A_r'(r,q,f) + \hat{a}A_q'(r,q,f) + \hat{a}A_f'(r,q,f) \right) \frac{e^{-jkr}}{r}, r \to \infty$$

Far-field region:

$$\left. \begin{array}{l} E_r \cong 0 \\ E_q \cong -jwA_q \\ E_f \cong -jwA_f \end{array} \right\} \vec{E}_A \approx -jw\vec{A} \text{ (not for } E_r \text{)}$$

$$\left. \begin{array}{l} H_r \cong 0 \\ H_q \cong +j\frac{w}{h} A_f \\ H_f \cong -j\frac{w}{h} A_q \end{array} \right\} \vec{H}_A \approx \frac{\hat{a}_r}{h} \times \vec{E}_A = -j\frac{w}{h} \hat{a}_r \times \vec{A} \text{(not for } H_r \text{)}$$

$$\vec{H}_F \approx -jw\vec{F}$$

$$\vec{E}_F \approx jwh\hat{a}_r \times \vec{F}$$

References

[1] Balanis C. Antenna Theory: Analysis and Design. 2nd edition. Wiley. 1997.