

Transistor amplifiers may be connected to the outputs of the TWD. They only need to be adjusted to have equal gains over the bandwidth for whatever input VSWR: reflected waves will be absorbed in the TWD. The greater the number of ways, the smaller will be the resulting input-reflection coefficient. Combining the powers from individual amplifiers will be effected by a similar TWD connected to the outputs of each amplifier: the total electrical path will then be the same whatever the path (as is the dividing/combining structure of Fig. 6(b)). This results in a compact all-planar MIC power amplifier.

Another interesting feature may be expected from such a traveling-wave amplifier when the output load turns out to be highly mismatched. Power, in this case, will be reflected back from the output and divided before reaching the outputs of the different amplifiers. Clearly, the phase of the reflection coefficients seen from the successive amplifiers will be different, therefore the amplitude and phase of their output signals will be modified compared to the previously matched condition. Currents will then circulate through the bridging resistors and this will limit the effect of the mismatched load.

Due to its compact topology and to its matching properties, the traveling-wave amplifier concept may be useful for a monolithic design. A GaAs semi-insulating substrate of 100 μm would lead to much smaller transversal dimensions.

Furthermore, a $\pi/2$ phase lag between the power splitters is not necessary for these TWD's, especially of types *B* or *D*, for the input reflected waves to be absorbed within the resistors. This effect will be essentially governed

by the total longitudinal phase difference for whatever number of amplifiers used and so can be easily optimized.

ACKNOWLEDGMENT

The authors would like to thank G. Kantorowicz for his advice and helpful discussions. They also express their gratitude to Ms. A. Dottin for her measurements and technical assistance.

REFERENCES

- [1] K. Honjo, Y. Takayama, and A. Higashisaka, "Broad-band internal matching of microwave power GaAs MESFET's," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, no. 1, Jan. 1979.
- [2] J. Srue, and Y. Takayama, "Ku and K-band internally matched high power GaAs FET amplifiers," *Electron. Lett.*, Aug. 30, 1979.
- [3] N. Nagai, K. Ono, and E. Maekawa, "On some five-way hybrid power dividers," in *1978 Int. Microwave Symp. dig.*, p. 326.
- [4] K. B. Niclas, "A 400 mW 11-18 GHz GaAs MESFET amplifier using planar power combining techniques," in *Proc. ISSCC*, p. 122, 1979.
- [5] M. Cohn, B. D. Geller, and J. M. Schellenberg, "A 10 watt broadband FET combiner/amplifier," in *IEEE MTT Int. Microwave Symp. Proc.*, p. 292, 1979.
- [6] R. A. Pucel, "Power combiner performance of GaAs MESFET," *Microwave J.*, p. 51, Mar. 1980.
- [7] K. Kurokawa, "Design theory of balanced transistor amplifiers," *Bell Syst. Tech. J.*, Oct. 1965.
- [8] D. P. Hornbuckle and L. J. Kuhlman, "Broad-band medium power amplification in the 2-12.4 GHz range with GaAs MESFET's," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, no. 6, June 1976.
- [9] S. B. Cohn, "A class of broadband three-port tem-mode hybrids," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, no. 2, Feb. 1968.
- [10] L. Young, "Tables for cascaded homogeneous quarter wave transformers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-7, p. 233-237, Apr. 1959.

A Least Squares Solution for Use in the Six-Port Measurement Technique

GLENN F. ENGEN, SENIOR MEMBER, IEEE

Abstract—Although based on the use of simple amplitude detectors, it is possible to obtain complex values of reflection coefficient, via the six-port technique, from the intersection of three circles in the complex plane. In a typical case, the circle centers are determined primarily by the six-port design and are nominally constant, while the radii are proportional to the square root of the ratio of the output of three of the detectors to a

fourth one. As a practical matter, however, these circles will not intersect in a point because of noise or other errors in the detectors.

This paper develops a procedure for choosing Γ in this context. Moreover, the question of what may be inferred about the system performance from the extent of this intersection failure is briefly considered.

I. INTRODUCTION

THE SIX-PORT reflectometer, which uses four simple amplitude detectors, provides an alternative to the four-port reflectometer and complex ratio detector in im-

Manuscript received May 19, 1980.

The author is with the U.S. Department of Commerce, National Bureau of Standards, Boulder, CO 80383.

plementing an automatic network analyzer [1], [2]. At any specific frequency the six-port circuit may be approximately represented by a coupler which samples the incident wave amplitude, and three nondirective probes.¹ As explained in [1], it is convenient to visualize the six-port operation by means of a diagram in the complex plane. Here three circles are constructed whose intersection gives value of the reflection coefficient being measured. The circle centers are primarily determined by the six-port parameters and nominally independent of the reflection coefficient being measured. The circle radii, on the other hand, are proportional to the ratios of the probe responses to the incident wave amplitude as provided by the coupler. In this way, phase information is obtained from the use of detectors which, by themselves, respond only to magnitude.

In practice, because of measurement error, the three circles will not intersect in a point. This leads to the question, "How does one assign a value to Γ in this context?" It is also self-evident that the extent by which these circles fail to intersect is, in some sense, a measure of the system performance. It is the purpose of this paper to investigate these topics.

II. BACKGROUND

While possessing a widespread general utility, the preceding picture is only approximate in that in addition to the radii, the circle centers also have, in general, some functional dependence upon the detector readings. It has been shown, however, [3], [4] that one can write

$$\Gamma = \frac{\alpha w + \beta}{\gamma w + 1} \quad (1)$$

where α , β , and γ are parameters of the six-port and its detectors, while w satisfies the equations

$$|w|^2 = \frac{P_3}{P_4} \quad (2)$$

$$|w - w_1|^2 = \zeta \frac{P_5}{P_4} \quad (3)$$

$$|w - w_2|^2 = \rho \frac{P_6}{P_4} \quad (4)$$

where w_1 , w_2 , ζ , and ρ are also six-port parameters, and $P_3 \cdots P_6$ are the readings of the four amplitude or power detectors.

Inspection of (2)–(4) indicates that w is given by the intersection of three circles, centered at 0, w_1 , w_2 , and of radii $\sqrt{P_3/P_4}$, $\sqrt{\zeta P_5/P_4}$, $\sqrt{\rho P_6/P_4}$.

In actual practice, it is thus w rather than Γ which is determined from the intersection of three circles. Following this, Γ is obtained from w by use of (1).

In order to solve (2)–(4), a change of variables is useful.

¹In practice, other designs exist which avoid the frequency sensitivity inherent in this simple model.

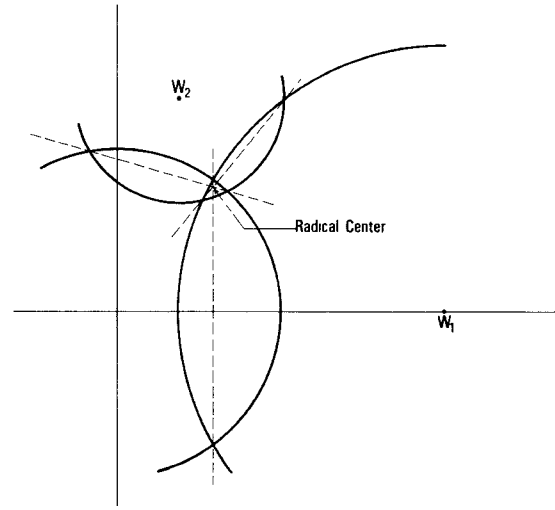


Fig. 1. The common chords of three circles intersect in a point known as the radical center.

Let

$$w = x + jy \quad (5)$$

thus,

$$x^2 + y^2 = r_0^2 \quad (6)$$

$$(x - x_1)^2 + y^2 = r_1^2 \quad (7)$$

$$(x - x_2)^2 + (y - y_2)^2 = r_2^2 \quad (8)$$

where $r_0^2 = P_3/P_4$, etc., and w_1 has been assumed to be real [4]. Subtracting (6) from (7) yields a linear equation in x which represents the common chord of these two circles. In a similar manner the common chords of the remaining circles are represented by the linear equations in x and y which result from subtracting (6) from (8), and (7) from (8). As shown in Fig. 1, these three lines intersect in a point, known as the radical center of the circles, whose coordinates are

$$x = \frac{r_0^2 - r_1^2 + x_1^2}{2x_1} \quad (9)$$

$$y = \frac{r_0^2 - r_2^2 + x_2^2 + y_2^2 - 2xx_2}{2y_2} \quad (10)$$

The foregoing represents a straightforward and useful solution to the problem when measurement accuracy can be traded for computational simplicity. It also provides a useful "initial estimate" for the alternative procedure which is next considered. This alternative procedure leads to a maximum likelihood estimate of w via a least squares procedure.

III. MATHEMATICAL FORMULATION

In what follows, it will be assumed that true values for the six-port parameters, α , β , γ , w_1 , w_2 , ζ , and ρ are known.² Any given set of power detector readings, $P_3 \cdots P_6$

²In principle, it would be possible to generalize the approach such that improved estimates of w_1 , w_2 , ζ , and ρ are obtained as well, and this may be desirable as a future extension of the theory.

however, will differ from their true or correct values due to detector noise or other error. Thus it is not possible, in general, to choose a value for w such that (2)–(4) are simultaneously satisfied. Instead, whatever value for w is ultimately chosen explicitly reflects, via (2)–(4), some assignment of measurement error to the ratios P_3/P_4 , P_5/P_4 , and P_6/P_4 . Thus, although the choice of w is the ultimate objective, as an intermediate question it is appropriate to ask: Given the observed set $P_3 \cdots P_6$, and the constraint which is implicit in (2)–(4), what is the most likely set of true values? This in turn, leads directly to the most probable value of w .

Let $P_{3t} \cdots P_{6t}$ represent an estimate of the true values of $P_3 \cdots P_6$. If the power meter error is assumed to be Gaussian, it can be shown that a maximum likelihood estimate for $P_{3t} \cdots P_{6t}$ is that one which minimizes F' where

$$F' = \sum_{i=3}^6 \left(\frac{P_i - P_{it}}{\sigma_i} \right)^2 \quad (11)$$

where σ_i is the standard deviation in the observed P_i , and where, as already noted, the set $P_{3t} \cdots P_{6t}$ is subject to the implicit constraint³ contained in (2)–(4) which is such that the choice of three of the P_{it} determines the fourth. In order to impose this constraint it is convenient to retain P_{4t} as one of the independent variables and then eliminate P_{3t} , P_{5t} , and P_{6t} from (11) by means of (2)–(4) such that the real and imaginary parts of w become the remaining independent parameters.

Let

$$\epsilon_4 = P_4 - P_{4t} \quad (12)$$

while by use of (2)–(4)

$$\epsilon_3 = P_3 - P_{3t} = P_3 - P_{4t}|w|^2 \quad (13)$$

$$\epsilon_5 = P_5 - P_{5t} = P_5 - P_{4t}|w - w_1|^2/\zeta \quad (14)$$

$$\epsilon_6 = P_6 - P_{6t} = P_6 - P_{4t}|w - w_2|^2/\rho. \quad (15)$$

These results may be substituted in (11) to obtain

$$F' = \sum_{i=3}^6 \left(\frac{\epsilon_i}{\sigma_i} \right)^2 \quad (16)$$

where the ϵ_i are functions of w and P_{4t} . At this point it is merely necessary to determine the values of w and P_{4t} which make F' a minimum. The solution to this problem is given in the Appendix. Once the values of w and P_{4t} have been obtained via this procedure, the most likely set for $P_{3t} \cdots P_{6t}$ may be readily computed by further use of (2)–(4), but this last step is not required if one is only interested in w .

IV. INTERPRETATION OF F'

Error expressions associated with most measuring instruments are typically given by the sum of two terms, the first independent of the value being measured while the

second is proportional to it. If one assumes the latter term dominates the former, then a convenient choice for σ_i in (16) is merely P_i . By inspection F' is now the sum of the squares of the fractional errors in the power meter readings. This, however, assumes that x , y , and P_{4t} are all known.

Conversely, if F_m is the minimum value of F , as found in the Appendix, then E , where

$$E = \frac{\sqrt{F_m}}{2P_4} \quad (17)$$

is a lower limit to the average fractional error for a particular measurement. On a statistical basis it appears reasonable to anticipate that for a large collection of measurements, the average value of E would be proportional to the fractional error. Confirmation of this, plus a determination of the proportionality constant, however, must await either the results of a more complete theoretical analysis, or more likely, the outcome of a computer simulation study.

In actual practice, it is desirable to include a constant term in σ_i to keep it from becoming too small if $P_i \rightarrow 0$. In this case, instead of distributing the fractional error more or less equally among the power meters, the foregoing technique will assign most of it to the meter which is measuring the small value. While this represents the appropriate action in this context, it also means that the associated value of E will now primarily represent the error in this particular reading, rather than the performance of the power meters as a whole.

V. SUMMARY

This technique, as outlined above, provides both improved accuracy (statistically), and (via (17)) a continuous monitor of system performance. In actual practice the latter feature may prove of more value than the former. Although there is more to be done in developing an interpretation of this residual, it has already proven its worth in flagging performance deterioration which in one case was traced to excessive harmonic content in the signal source, and in another case, to inadequate RF bypassing in the power detectors (thermistor mounts) used with the six-port.

As already noted, the selection of the function to be minimized is appropriate if the power meter error is assumed to have a Gaussian distribution. In the existing implementations there is probably a sizable component of quantization error as well and the impact of this upon the solution is unknown at this time. In any case, however, the foregoing merely represents the current status of this problem, not its ultimate solution.

VI. APPENDIX

The appendix will outline the mathematical details in finding the minimum of F' , (16). In the interests of computational convenience, some further changes in notation

³Apart from this constraint, there would be a trivial solution to (11): $P_{it} = P_i$, $i = 3 \cdots 6$.

will prove useful. Let

$$P_1 = \zeta P_5 \quad (18)$$

$$P_2 = \rho P_6 \quad (19)$$

$$d_1 = \zeta \sigma_5 \quad (20)$$

$$d_2 = \rho \sigma_6 \quad (21)$$

$$d_3 = \sigma_3 \quad (22)$$

$$d_4 = \sigma_4 \quad (23)$$

$$z = 1/P_{4t} \quad (24)$$

$$w_3 = 0 \quad (25)$$

$$f_i(x, y, z) = -z\epsilon_i/\sigma_i = \frac{|w-w_i|^2 - zP_i}{d_i}, \quad i=1 \dots 3 \quad (26)$$

$$f_4(z) = -z\epsilon_4/\sigma_4 = \frac{1 - zP_4}{d_4}. \quad (27)$$

(Note that $|w-w_i|^2$ is a compact notation for $(x-x_i)^2 + (y-y_i)^2$, etc.) With these changes, (16) now becomes

$$F' = \frac{F}{z^2} \quad (28)$$

where

$$F = \sum_{i=1}^4 f_i^2. \quad (29)$$

Since the difference in the end result is negligible, and a considerable simplification is obtained, the function which is minimized is F rather than F' .

A necessary (but by no means sufficient) condition for a minimum of F is

$$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = \frac{\partial F}{\partial z} = 0. \quad (30)$$

Differentiation of F leads to

$$\frac{\partial F}{\partial x} = 2G = 2 \sum_{i=1}^3 f_i \frac{(x-x_i)}{d_i} \quad (31)$$

$$\frac{\partial F}{\partial y} = 2H = 2 \sum_{i=1}^3 f_i \frac{(y-y_i)}{d_i} \quad (32)$$

$$\frac{\partial F}{\partial z} = -2 \sum_{i=1}^4 f_i \frac{P_i}{d_i}. \quad (33)$$

Setting $\partial F/\partial z = 0$ and solving for z yields

$$z = \frac{\sum_{i=1}^3 \frac{|w-w_i|^2 P_i}{d_i^2} + \frac{P_4}{d_4^2}}{\sum_{i=1}^4 \frac{P_i^2}{d_i^2}}. \quad (34)$$

This result may now be substituted in (31) and (32) to yield a pair of simultaneous equations to be solved for x and y . Unfortunately, however, these are cubic, and at this point one must proceed with numerical rather than algebraic methods.

The Newton method (in two dimensions) has proven useful in this context. This calls for the derivatives of G and H with respect to x and y . Rather than performing the actual substitution of (34) in (31) and (32), one can write

$$G = G[x, y, z(x, y)] = 0 \quad (35)$$

from which

$$\left. \frac{\partial G}{\partial x} \right|_{y=c} = \left. \frac{\partial G}{\partial x} \right|_{y=c} + \frac{\partial G}{\partial z} \frac{\partial z}{\partial x} \quad (36)$$

and similarly for the other derivatives. To continue

$$\left. \frac{\partial G}{\partial x} \right|_{y=c} = \sum_{i=1}^3 \frac{f_i}{d_i} + \frac{2(x-x_i)^2}{d_i^2} \quad (37)$$

$$\left. \frac{\partial G}{\partial y} \right|_{x=c} = \left. \frac{\partial H}{\partial x} \right|_{y=c} = 2 \sum_{i=1}^3 \frac{(x-x_i)(y-y_i)}{d_i^2} \quad (38)$$

$$\left. \frac{\partial H}{\partial y} \right|_{x=c} = \sum_{i=1}^3 \frac{f_i}{d_i} + \frac{2(y-y_i)^2}{d_i^2} \quad (39)$$

$$\frac{\partial G}{\partial z} = - \sum_{i=1}^3 \frac{(x-x_i)P_i}{d_i^2} \quad (40)$$

$$\frac{\partial H}{\partial z} = - \sum_{i=1}^3 \frac{(y-y_i)P_i}{d_i^2} \quad (41)$$

$$\frac{\partial z}{\partial x} = \frac{2 \sum_{i=1}^3 \frac{(x-x_i)P_i}{d_i^2}}{\sum_{i=1}^4 \frac{P_i^2}{d_i^2}} \quad (42)$$

$$\frac{\partial z}{\partial y} = \frac{\sum_{i=1}^3 \frac{(y-y_i)P_i}{d_i^2}}{\sum_{i=1}^4 \frac{P_i^2}{d_i^2}}. \quad (43)$$

After collecting the results an improved estimate of x, y is obtained from

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - J^{-1} \begin{bmatrix} G \\ H \end{bmatrix} \quad (44)$$

where x_0, y_0 are the current estimates of x, y , $G = G(x_0, y_0, z_0)$, $H = H(x_0, y_0, z_0)$, and

$$J = \begin{bmatrix} \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} \end{bmatrix}. \quad (45)$$

A convenient initial estimate for x and y is obtained from (9) and (10). Following this, two or three iterations ordinarily suffice for making both G and H vanish within the limits of the machine precision.

While both intuition and practical experience strongly suggest that this will ordinarily yield a minimum for F , this conclusion must be approached with a measure of

caution. If in Fig. 1 the radius of the circle centered at the origin is increased such that it is nominally tangent to the line between w_1 and w_2 , it then appears that two possible minima for F will exist, especially if σ_3 is large. These correspond approximately to the points of intersection of the circles centered at w_1 and w_2 . Alternatively, if the radius of the circle centered at the origin and σ_3 are both very small, and if the circles centered at w_1 and w_2 intersect at the origin, then the origin may actually correspond to a (local) maximum of F , while if one traverses the perimeter of the circle centered at the origin one will alternately encounter a series of four minima and four saddle points. (These correspond to the set of nine possible roots for a pair of simultaneous cubic equations.)

Fortunately, however, considerations of these types tend

to be somewhat remote from the projected applications. Further study of this topic is in progress and will be reported in due time.

REFERENCES

- [1] G. F. Engen, "The six-port reflectometer: an alternative network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1075-1079, Dec. 1977.
- [2] C. A. Hoer, "A network analyzer incorporating two six-port reflectometers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1070-1074, Dec. 1977.
- [3] G. F. Engen and C. A. Hoer, "Thru-reflect-line: an improved technique for calibrating the dual six-port automatic network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 987-993, Dec. 1979.
- [4] G. F. Engen, "Calibrating the six-port reflectometer by means of sliding terminations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 951-957, Dec. 1978.

Circular-Electric Mode Waveguide Couplers and Junctions for Use in Gyrotron Traveling-Wave Amplifiers

LARRY R. BARNETT, MEMBER, IEEE, J. MARK BAIRD, MEMBER, IEEE, ARNE W. FLIFLET, AND VICTOR L. GRANATSTEIN

Abstract—Recent gyrotron traveling-wave amplifier experiments in the TE_{01}^c mode have led to the developing of 2-port and 4-port devices potentially useful as input couplers, severers, and beam-RF separators for collector designs. The couplers are moderately wide-band, have high transmission efficiencies, and low reflection coefficients. In addition, they are relatively easy to construct. We present analytical and experimental results.

INTRODUCTION

RECENT EXPERIMENTS with a gyrotron traveling-wave amplifier operating in the TE_{01} circular waveguide mode [1] have spurred interest in developing circular-electric mode components. The recently reported amplifier experiments [1], which operate at 35 GHz, attained 30-dB gain in a single stage with 10-kW output. The useful small-signal bandwidth was on the order of 1

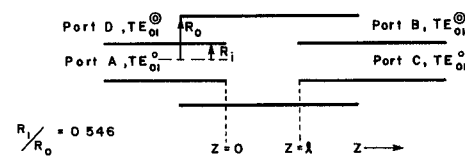


Fig. 1. Marcattili 4-port hybrid coupler.

GHz. The input coupler in this amplifier consisted of a TE_{01} coaxial mode to TE_{01} circular mode junction. The TE_{01} coaxial mode was produced from TE_{10} rectangular waveguide by a coaxial sector waveguide taper. A taper study is described in the companion paper [2]. The work has expanded beyond the original coupler development and has resulted in devices potentially useful in circular-electric mode amplifiers and oscillators or designs with circular-electric outputs [3], [4], [8].

BACKGROUND

The initial design for the 2-port input coupler was derived from the Marcattili 4-port circular hybrid junction [5], [6]. The 4-port hybrid is shown in Fig. 1. An input at

Manuscript received May 20, 1980; revised September 18, 1980. This work was supported by the Naval Electronics System Command, Washington, DC, under Task XF 54 581 007.

L. R. Barnett, J. M. Baird, and A. W. Fliflet are with B-K Dynamics, Inc., Rockville, MD, 20850.

V. L. Granatstein is with the Naval Research Laboratory, Washington, DC 20375.