HKCEE 1991
Mathematics II

91 1. $(a^{2a})(3a^{4a})$

A. $3a^{6a}$
B. $(3a)^{6a}$
C. $3a^{8a}$
D. $4a^{6a}$
E. $(3^{4a})(a^{6a})$

91 2. $\dfrac{1}{1-x^2} - \dfrac{1}{(1+x)^2} =$

A. $\dfrac{2}{(1-x^2)(1+x^2)}$
B. $\dfrac{2x^2}{(1-x^2)(1+x^2)}$
C. $\dfrac{2x^2}{(1-x^2)(1+x)^2}$
D. $\dfrac{2}{(1-x)(1+x)^2}$
E. $\dfrac{2x}{(1-x)(1+x)^2}$

91 3. Which one of the following is a factor of $x^3 - 4x^2 + x + 6$?

A. $(x + 1)(x - 2)$
B. $(x + 1)(x + 2)$
C. $(x - 1)(x + 2)$
D. $(x - 1)(x - 3)$
E. $(x - 1)(x + 3)$

91 4. If $y = \sqrt{\dfrac{1+mx}{1-mx}}$, then $x =$

A. $\dfrac{m(y-1)}{y+1}$.
B. $\dfrac{y-1}{m(y+1)}$.
C. $\dfrac{(1-y^2)}{m(1+y^2)}$.
D. $\dfrac{m(y^2-1)}{(y^2+1)}$.
E. $\dfrac{(y^2-1)}{m(y^2+1)}$.

91 5. $\dfrac{\dfrac{1}{x^2}+\dfrac{1}{y^2}}{\dfrac{1}{x}+\dfrac{1}{y}} =$

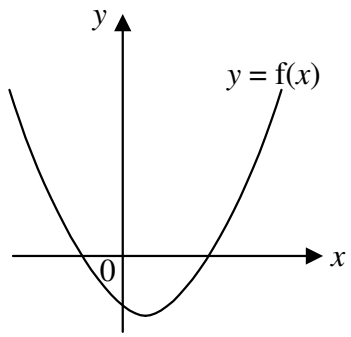
A. $\dfrac{1}{x^2} + \dfrac{1}{y^2}$
B. $\dfrac{1}{x^2} + \dfrac{1}{xy} + \dfrac{1}{y^2}$
C. $\dfrac{1}{x^2} + \dfrac{2}{xy} + \dfrac{1}{y^2}$
D. $\dfrac{1}{x^2} - \dfrac{2}{xy} + \dfrac{1}{y^2}$
E. $\dfrac{1}{x^2} - \dfrac{1}{xy} + \dfrac{1}{y^2}$

91 6. The L.C.M. of x, $2x^2$, $3x^3$, $4x^4$, $5x^5$ is

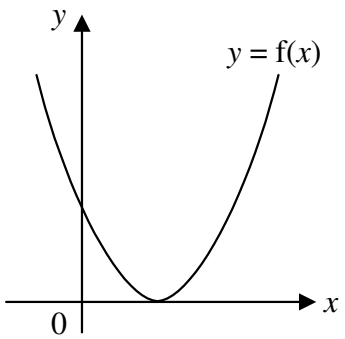
A. x.
B. $5x^5$.
C. $60x^5$.
D. $120x^5$.
E. $120x^{15}$.

91 7. In which of the following cases the equation f(x) = 0 **cannot** be solved by the method of bisection?

A.



B.



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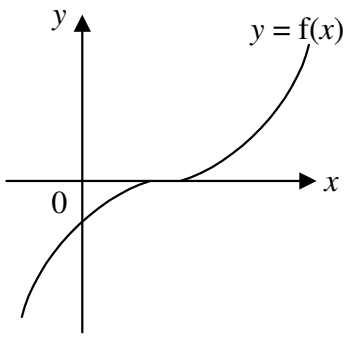

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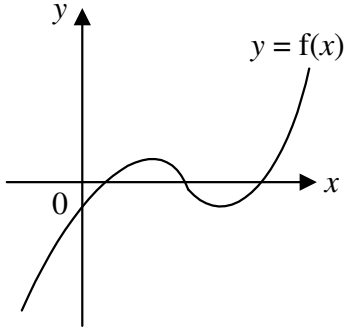
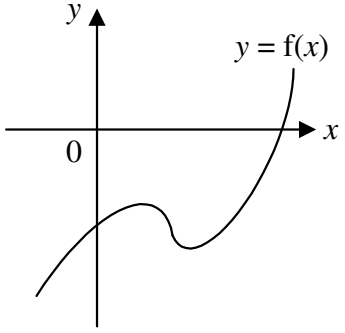


E.



91 8. Solve the following equations :

$x - 1 = y + 2 = x + y - 5$

A. $x = 1, y = -2$
B. $x = 1, y = 4$
C. $x = 4, y = 1$
D. $x = 7, y = -2$
E. $x = 7, y = 4$

91 9. Let y vary partly as $\frac{1}{x}$ and partly as x. When $x = 1$, $y = 5$ and when $x = 4$, $y = \frac{25}{2}$. Find y when $x = 2$.

A. $\frac{5}{2}$
B. 4
C. $\frac{25}{4}$
D. 7
E. $\frac{17}{2}$

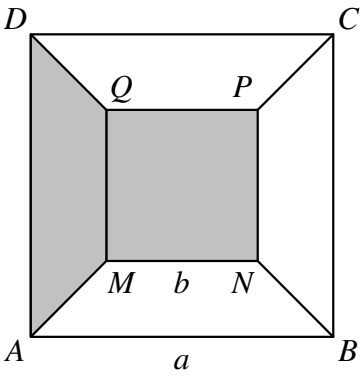
91 10. If $\frac{1}{a} : \frac{1}{b} = 2 : 3$ and $a : c = 4 : 1$, then $a : b : c =$

A. $12 : 8 : 3$.
B. $8 : 3 : 2$.
C. $4 : 6 : 1$.
D. $2 : 3 : 8$.
E. $2 : 3 : 4$.

91 11. A blanket loses 10% of its length and 8% of its width after washing. The percentage loss in area is

A. 18.8% .
B. 18% .
C. 17.2% .
D. 9% .
E. 8% .

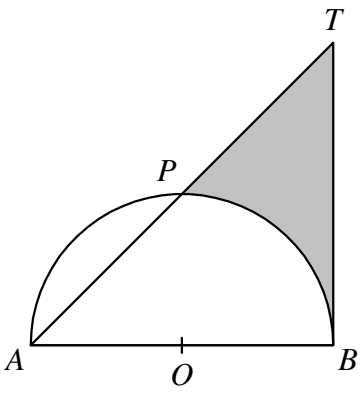
91
12.



In the figure, $ABCD$ is a square of side a and $MNPQ$ is a square of side b. The four trapeziums are identical. The area of the shaded region is

A. $\frac{3b^2 + a^2}{4}$.
B. $\frac{3b^2 - a^2}{2}$.
C. $\frac{5b^2 + a^2}{4}$.
D. $\frac{5b^2 - a^2}{4}$.
E. $\frac{(a-b)^2}{4} + b^2$.

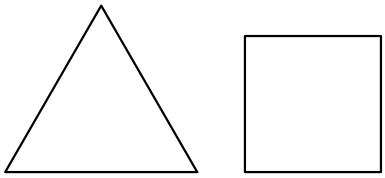
91
13.



In the figure, TB touches the semi-circle at B. TA cuts the semi-circle at P such that $TP = PA$. If the radius of the semi-circle is 2, find the area of the shaded region.

A. $12 - \pi$
B. $8 - \pi$
C. $6 - \pi$
D. $4 - \pi$
E. $2(4 - \pi)$

91
14.



An equilateral triangle and a square have equal perimeters.

$\frac{\text{Area of the triangle}}{\text{Area of the square}} =$

A. $\frac{9\sqrt{3}}{16}$.
B. $\frac{\sqrt{3}}{4}$.
C. $\frac{\sqrt{3}}{3}$.
D. $\frac{4\sqrt{3}}{9}$.
E. 1.

91
15.
A man borrows \$10 000 from a bank at 12% per annum compounded monthly. He repays the bank \$2000 at the end of each month. How much does he still owe the bank just after the second repayment?

A. \$6181
B. \$6200
C. \$6201
D. \$8304
E. \$8400

91
16.
$\left[\frac{1}{\cos\theta} + \tan\theta\right](1 - \sin\theta) =$

A. $\sin\theta$
B. $\cos\theta$
C. $\cos^2\theta$
D. $1 + \sin\theta$
E. $\sin\theta\tan\theta$

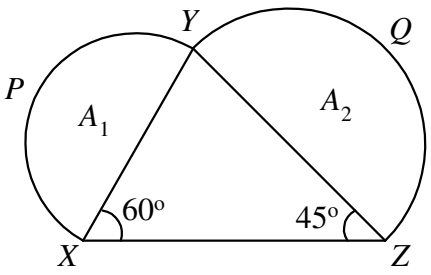
91
17. $\dfrac{\sin(\theta - 90^\circ)}{\tan(\theta + 180^\circ)} =$

A. $\cos\ \theta$
B. $-\cos\ \theta$
C. $\dfrac{\cos^2\theta}{\sin\theta}$
D. $-\dfrac{\cos^2\theta}{\sin\theta}$
E. $\dfrac{1}{\sin\theta}$

91
18. For $0 \le \theta < 2\pi$, how many roots does the equation $\tan\ \theta + 2\sin\ \theta = 0$ have?

A. 1
B. 2
C. 3
D. 4
E. 5

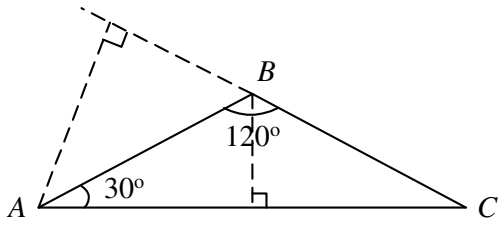
91
19.



In the figure, XPY and YQZ are semi-circles with areas A_1 and A_2 respectively. $\angle YXZ = 60^\circ$ and $\angle YZX = 45^\circ$. The ratio $A_1 : A_2 =$

A. $\sqrt{2} : \sqrt{3}$.
B. $\sqrt{2} : 3$.
C. $2 : 3$.
D. $2 : \sqrt{3}$.
E. $\sqrt{3} : \sqrt{2}$.

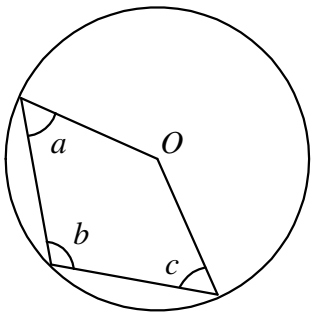
91
20.



In the figure, $\angle A = 30^\circ$ and $\angle B = 120^\circ$. The ratio of the altitudes of the triangle ABC from A and from B is

A. $2 : 1$.
B. $\sqrt{3} : 1$.
C. $\sqrt{2} : 1$.
D. $1 : \sqrt{2}$
E. $1 : \sqrt{3}$

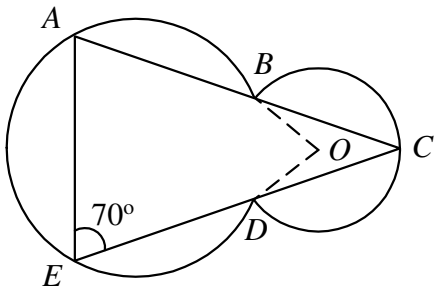
91
21.



In the figure, O is the centre of the circle. Find $a + c$.

A. b
B. $2b$
C. $180^\circ - b$
D. $360^\circ - b$
E. $360^\circ - 2b$

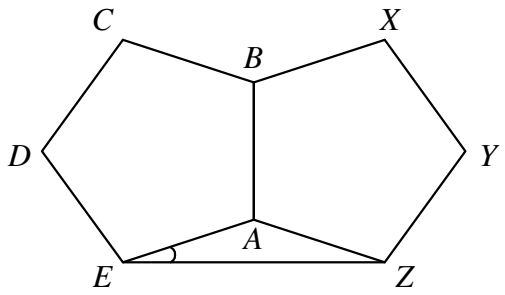
91
22.



In the figure, O is the centre of the circle BCD. ABC and EDC are straight lines. $BC = DC$ and $\angle AED = 70^o$. Find $\angle BOD$.

A. 40^o
B. 70^o
C. 80^o
D. 90^o
E. 140^o

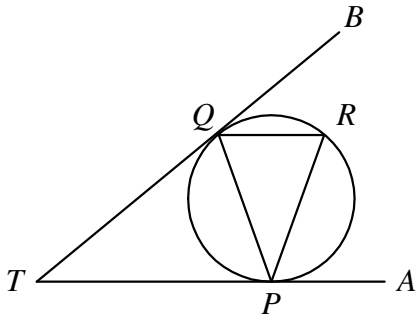
91
23.



In the figure, $ABCDE$ and $ABXYZ$ are two identical regular pentagons. Find $\angle AEZ$.

A. 15^o
B. 18^o
C. 24^o
D. 30^o
E. 36^o

91
24.

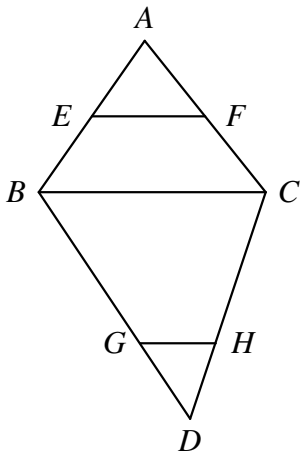


In the figure, TPA and TQB are tangents to the circle at P and Q respectively. If $PQ = PR$, which of the following **must** be true?

I. $\angle APR = \angle QRP$
II. $\angle QTP = \angle QPR$
III. $\angle QPR = \angle APR$

A. I only
B. II only
C. III only
D. I and II only
E. I and III only

91
25.



In the figure, E and F are the mid-points of AB and AC respectively. G and H divide DB and DC respectively in the ratio 1 : 3. If $EF = 12$, find GH.

A. 3
B. 4
C. 6
D. 8
E. 12

91
26. The circle $x^2 + y^2 + 4x + ky + 4 = 0$ passes through the point (1, 3). The radius of the circle is

A. $\sqrt{68}$.
B. $\sqrt{48}$.
C. $\sqrt{17}$.
D. 6 .
E. 3 .

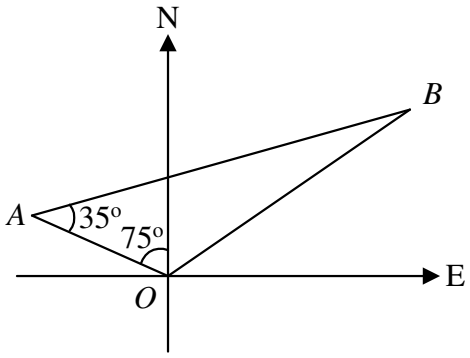
91
27. Let A and B be the points (4, −7) and (−6, 5) respectively. The equation of the line passing through the mid-point of AB and perpendicular to $3x - 4y + 14 = 0$ is

A. $3x - 4y - 1 = 0$.
B. $3x + 4y + 7 = 0$.
C. $4x - 3y + 1 = 0$.
D. $4x + 3y - 7 = 0$.
E. $4x + 3y + 7 = 0$.

91 28. $PQRS$ is a parallelogram with vertices $P = (0, 0)$, $Q = (a, b)$ and $S = (-b, a)$. Find R.

A. $(-a, -b)$
B. $(a, -b)$
C. $(a - b, a - b)$
D. $(a - b, a + b)$
E. $(a + b, a + b)$

91 29.



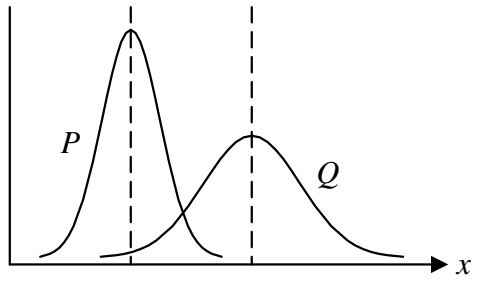
In the figure, A and B are the positions of two boats. The bearing of B from A is

A. N55°E .
B. N70°E .
C. N20°E .
D. S35°E .
E. S75°E .

91 30. The mean and standard deviation of a distribution of test scores are m and s respectively. If 4 marks are added to each score of the distribution, what are the mean and standard deviation of the new distribution?

	Mean	Standard Deviation
A.	$m + 4$	s
B.	$m + 4$	$s + 2$
C.	$m + 4$	$s + 4$
D.	m	$s + 2$
E.	m	$s + 4$

91 31.



The graph shows the frequency curves of two symmetric distributions P and Q.
Which of the following is /are true?

I. The mean of P < the mean of Q.
II. The mode of P > the mode of Q.
III. The inter-quartile range of P < the inter-quartile range of Q.

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II and III

91 32. A fair die is thrown 3 times. The probability that "6" occurs exactly once is

A. $\frac{1}{3}$.
B. $\left(\frac{1}{6}\right)^3$.
C. $\frac{1}{3} \times \frac{1}{6}$.
D. $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$.
E. $3\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$.

91 33. If $(\sqrt{3} + 1)\sqrt{x} = 2$, then $x =$

A. $2 - \sqrt{3}$.

B. $\sqrt{3}-1$.
C. 1 .
D. $2(2-\sqrt{3})$.
E. $4-\sqrt{3}$.

91 34. If $\log x : \log y = m : n$, then $x =$

A. $\frac{my}{n}$.
B. $(m-n)y$.
C. $m-n+y$.
D. $y^{\frac{m}{n}}$.
E. $\frac{m\log y}{n}$.

91 35. If $f(x) = x - \frac{1}{x}$, then $f(x) - f\left(\frac{1}{x}\right) =$

A. 0 .
B. $2x$.
C. $-\frac{2}{x}$.
D. $2\left(x-\frac{1}{x}\right)$.
E. $2\left(\frac{1}{x}-x\right)$.

91 36. If $p(x^2 - x) + q(x^2 + x) = 4x^2 + 8x$, find p and q.

A. $p = 4, q = 8$
B. $p = -8, q = 4$
C. $p = -2, q = 6$
D. $p = 2, q = 6$
E. $p = 6, q = -2$

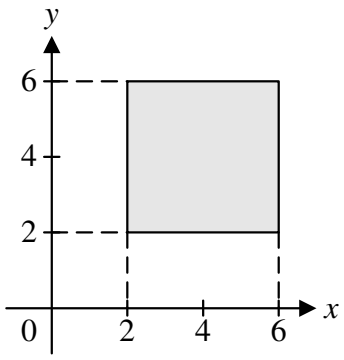
91 37. If $x < 0 < y$, then which one of the following **must** be positive?

A. $x + y$
B. $x - y$
C. $y - x$
D. xy
E. $\frac{y}{x}$

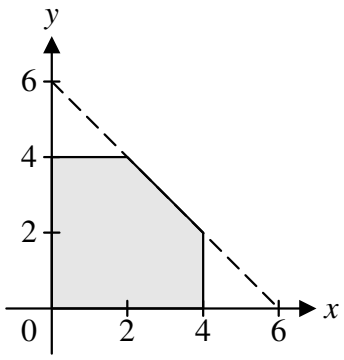
91 38. Which one of the following shaded regions represents the solution of

$$\begin{cases} 2 \le x + y \le 6 \\ 0 \le x \le 4 \\ 0 \le y \le 4 \end{cases} ?$$

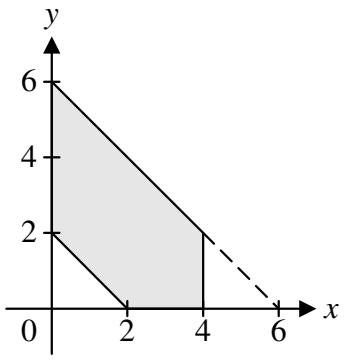
A.



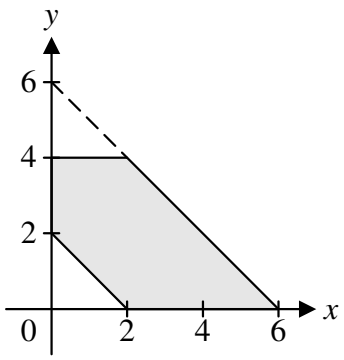
B.



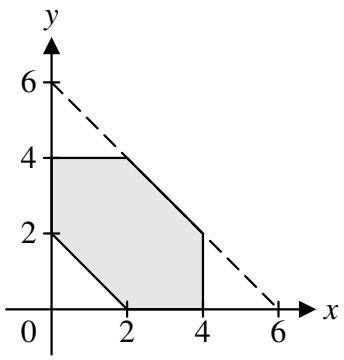
C.



D.



E.



91 39. If $(x - 2)(x - 3) = (a - 2)(a - 3)$, solve for x.

A. $x = 0$ or 5
B. $x = 2$ or 3
C. $x = a$ or 2
D. $x = a$ or 3
E. $x = a$ or $5 - a$

91 40. If the sum to n terms of an A.P. is $n^2 + 3n$, find the 7^{th} term of the A.P.

A. 16
B. 18
C. 54
D. 70
E. It cannot be found.

91 41. If x, y, z are in G.P, which of the following **must** be true?

I. $x + 3$, $y + 3$, $z + 3$ are in G.P.
II. $3x$, $3y$, $3z$ are in G.P.
III. x^2, y^2, z^2 are in G.P.

A. I only
B. II only
C. III only
D. I and II only
E. II and III only

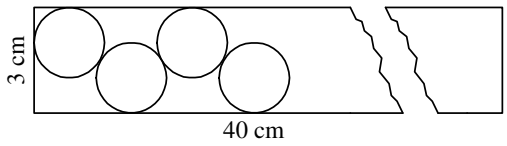
91 42. 3 kg of a solution contains 40% of alcohol by weight. How much alcohol should be added to obtain a solution containing 50% of alcohol by weight?

A. 0.3 kg
B. 0.6 kg
C. 0.75 kg
D. 1.5 kg
E. 3.75 kg

91 43. P sold an article to Q at a profit of 25%. Q sold it to R also at a profit of 25%. If Q gained \$500, how much did P gain?

A. \$250
B. \$320
C. \$333
D. \$400
E. \$500

91 44.

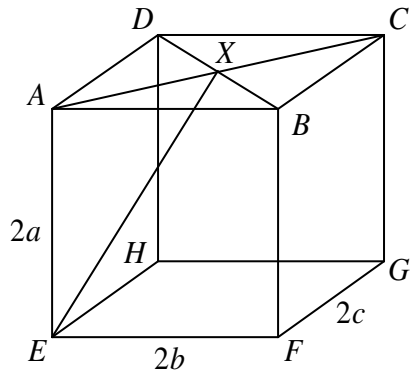


From a rectangular metal sheet of width 3 cm and length 40 cm, at most how many circles each of radius 1 cm can be cut?

A. 20
B. 21
C. 22
D. 23
E. 24

DIRECTIONS: Question 45 and 46 refer to the figure below, which shows a cuboid $ABCDEFGH$ with $AE = 2a$, $EF = 2b$ and $FG = 2c$. AC and BD intersect at X.

91
45.



$XE =$

A. $\sqrt{a^2+b^2+c^2}$
B. $\sqrt{a^2+b^2+(2c)^2}$
C. $\sqrt{a^2+(2b)^2+c^2}$
D. $\sqrt{(2a)^2+b^2+c^2}$
E. $2\sqrt{a^2+b^2+c^2}$

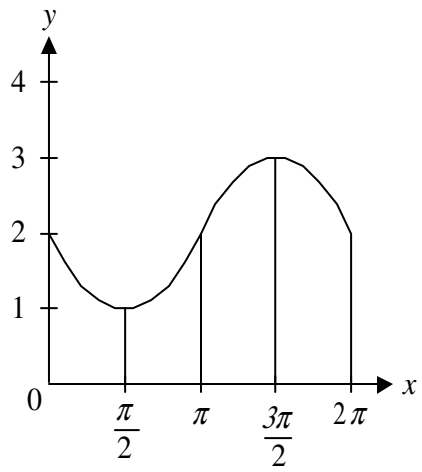
91
46. If the angle between XE and the plane EFGH is θ, then $\tan\theta =$

A. $\frac{a}{b}$.
B. $\frac{2a}{b}$.
C. $\frac{\sqrt{(2a)^2+c^2}}{b}$.
D. $\frac{a}{\sqrt{b^2+c^2}}$.
E. $\frac{2a}{\sqrt{b^2+c^2}}$.

91
47. $\cos\frac{\pi}{2}+\cos\pi+\cos\frac{3\pi}{2}+\cos 2\pi+\ldots+\cos 10\pi =$

A. 0
B. 1
C. −1
D. 10
E. −10

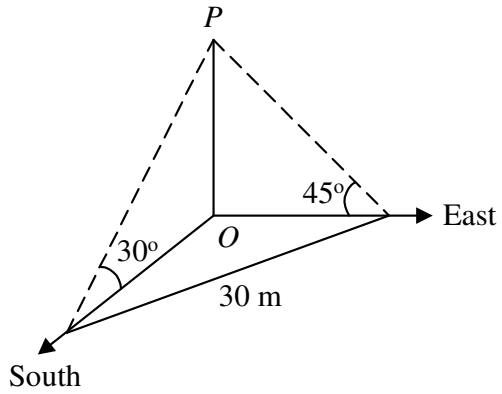
91
48.



The figure shows the graph of the function

A. $y = 2\cos x$.
B. $y = 2 - \sin x$.
C. $y = 2 + \sin x$.
D. $y = 2 - \cos x$.
E. $y = 2 + \cos x$.

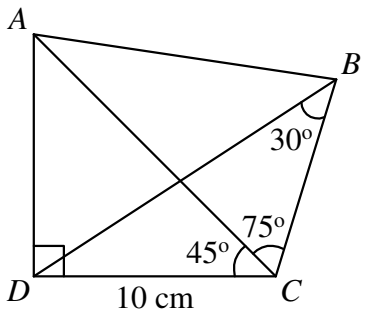
91
49.



In the figure, the height of the vertical pole PO is

A. 7.5 m .
B. 15 m .
C. $15\sqrt{2}$ m .
D. $15\sqrt{3}$ m .
E. 45 m .

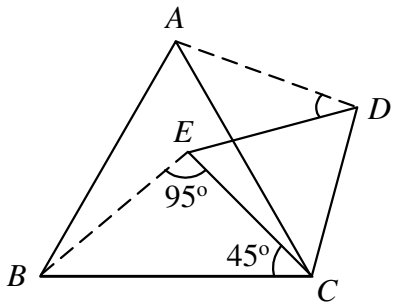
91
50.



In the figure, find the length of AB, correct to the nearest cm.

A. 14 cm
B. 15 cm
C. 16 cm
D. 17 cm
E. 18 cm

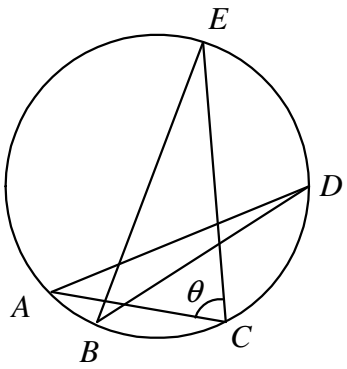
91
51.



In the figure, ABC and CDE are equilateral triangles. Find $\angle ADE$.

A. 15^o
B. 35^o
C. 40^o
D. 45^o
E. 50^o

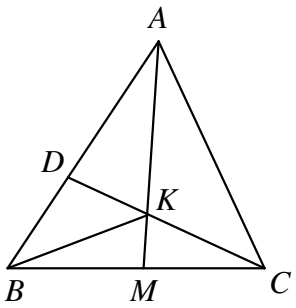
91
52.



In the figure, arc AB : arc BC : arc CD : arc DE : arc EA = 1 : 2 : 3 : 4 : 5. Find θ.

A. 30^o
B. 36^o
C. 60^o
D. 72^o
E. 120^o

91
53.

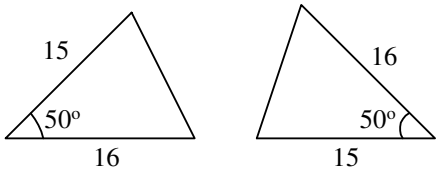


In the figure, M is the mid-point of BC and $AD = 2DB$. AM and CD intersect at K. Find $\dfrac{\text{area of } \Delta ADK}{\text{area of } \Delta AKC}$.

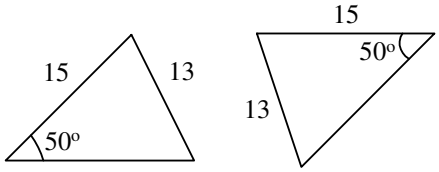
A. $\frac{1}{2}$
B. $\frac{2}{3}$
C. $\frac{3}{4}$
D. $\frac{4}{5}$
E. 1

91
54. In the figure, which of the pairs of triangles **must** be congruent?

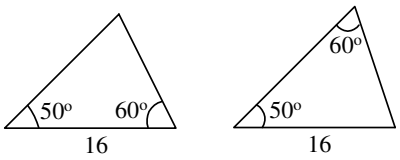
I.



II.



III.



A. I only
B. II only
C. I and III only
D. II and III only
E. I, II and III