

Example on Bisection Method 分半方法的例題

- (a) Show that the equation $x^2 + 4x - 3 = 0$ has a root between -4 & -5.
 証明在 -4 和 -5 之間方程 $x^2 + 4x - 3 = 0$ 最少有一實根。
- (b) Using the method of bisection, find the value of root correct to 3 decimal places.
 用分半方法求方程 $x^2 + 4x - 3 = 0$ 最的實根，準確至小數點後三個位。

Solution 題解

(a) Let 設 $f(x) = x^2 + 4x - 3$
 $f(-4) = (-4)^2 + 4(-4) - 3 = -3$
 $f(-5) = (-5)^2 + 4(-5) - 3 = 2$

Since $f(-4) < 0$ & $f(-5) > 0$, there must be a root between -4 & -5.
 (因 $f(-4) < 0$ 及 $f(-5) > 0$, 所以在 -4 和 -5 之間最少有一實根。)

- (b) Using the method of bisection, we have the following table:
 (用分半方法，得出下表：)

Interval (區間)			f(x)		
a	$m = \frac{a+b}{2}$	b	f(a)	f(m)	f(b)
-5	-4.5	-4	+	-	-
-5	-4.75	-4.5	+	+	-
-4.75	-4.625	-4.5	+	-	-
-4.75	-4.6875	-4.625	+	+	-
-4.6875	-4.6562	-4.625	+	+	-
-4.6562	-4.6406	-4.625	+	-	-
-4.6562	-4.6484	-4.6406	+	+	-
-4.6484	-4.6445	-4.6406	+	-	-
-4.6484	-4.6465	-4.6445	+	+	-
-4.6465	-4.6455	-4.6445	+	-	-
-4.6465	-4.6460	-4.6455	+	+	-
-4.6460	-4.6457	-4.6455	+	+	-
-4.6457	-4.6456	-4.6455	+	-	-
-4.6457	-4.6457	-4.6456	+	-	-
-4.6457		-4.6457			

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∴ Root 根 = -4.646, correct to 3 decimal places (準確至小數點後三個位).

【 Root 根 = -4.6, correct to 1 decimal places (準確至小數點後一個位).

Root 根 = -4.65, correct to 2 decimal places (準確至小數點後二個位).】

Alternative Method for (b)

Let x_0 be the root.

Using the method of bisection, we have the following table:

Bracketing Interval (B. I.)	Mid-value	Sign of $f(x)$
$-5 < x_0 < -4$	-4.5	-
$-5 < x_0 < -4.5$	-4.75	+
$-4.75 < x_0 < -4.5$	-4.625	-
$-4.75 < x_0 < -4.625$	-4.6875	+
$-4.6875 < x_0 < -4.625$	-4.6562	+
$-4.6562 < x_0 < -4.625$	-4.6406	-
$-4.6562 < x_0 < -4.6406$	-4.6484	+
$-4.6484 < x_0 < -4.6406$	-4.6445	-
$-4.6484 < x_0 < -4.6445$	-4.6465	+
$-4.6465 < x_0 < -4.6445$	-4.6455	-
$-4.6465 < x_0 < -4.6455$	-4.6460	+
$-4.6460 < x_0 < -4.6455$	-4.6457	+
$-4.6457 < x_0 < -4.6455$	-4.6456	-
$-4.6457 < x_0 < -4.6456$	-4.6457	-
$-4.6457 < x_0 < -4.6457$		

$\therefore x_0 = -4.646$, correct to 3 decimal places.

【 $x_0 = -4.6$, correct to 1 decimal places.
 $x_0 = -4.65$, correct to 2 decimal places.】

Supplementary Exercise on Bisection Method 分半方法的補充練習

Let 設 $f(x) = x^3 - 12x + 4$

- (a) Calculate the values of $f(3)$ and $f(4)$.

計算 $f(3)$ 和 $f(4)$ 的值。

- (b) Hence, use the method of bisection, find the root of $x^3 - 12x + 4 = 0$, if $3 < x < 4$, correct to 2 decimal places.

由此，用分半方法求區間 $3 < x < 4$ 內方程 $x^3 - 12x + 4 = 0$ 的實根，準確至小數點後兩個位。

Solution 題解

(a) $f(3) = ()^3 - 12() + 4 =$

$f(4) =$

- (b) 【Remark: Since $f(3) < 0$ & $f(4) > 0$, there must be a root between -4 & -5 .

註：因 $f(3) < 0$ 及 $f(4) > 0$ ，所以在 3 和 4 之間最少有一實根。】

Using the method of bisection, we have the following table:

(用分半方法，得出下表：)

Interval (區間)			f(x)		
a	$m = \frac{a+b}{2}$	b	f(a)	f(m)	f(b)
3	3.5	4	-		+

$\therefore$ Root 根 = , correct to 2 decimal places (準確至小數點後兩個位).

1995 Solution 題解

(iii) Let 設 $f(h) = h^3 - 192h + 672$

$$f(0) = (0)^3 - 192(0) + 672 = 672$$

$$f(6) = (6)^3 - 192(6) + 672 = -264$$

Since $f(0) > 0$ & $f(6) < 0$, there must be a root between 0 & 6.

(因 $f(0) > 0$ 及 $f(6) < 0$, 所以在 0 和 6 之間有一實根。)

Using the method of bisection, we have the following table:

(用分半方法, 得出下表:)

Interval (區間)			f(x)		
a	$m = \frac{a+b}{2}$	b	f(a)	f(m)	f(b)
0	3	6	+	+	-
3	4.5	6	+	-	-
3	3.75	4.5	+	+	-
3.75	4.125	4.5	+	-	-
3.75	3.9375	4.125	+	-	-
3.75	3.84375	3.9375	+	-	-

$\therefore h =$, correct to 1 decimal places (準確至小數點後一個位).

1995 Solution 題解

(iii) Let 設 $f(h) = h^3 - 192h + 672$

$$f(0) = (0)^3 - 192(0) + 672 = 672$$

$$f(6) = (6)^3 - 192(6) + 672 = -264$$

Since $f(0) > 0$ & $f(6) < 0$, there must be a root between 0 & 6.

(因 $f(0) > 0$ 及 $f(6) < 0$, 所以在 0 和 6 之間有一實根。)

Using the method of bisection, we have the following table:

(用分半方法, 得出下表:)

Interval (區間)			f(x)		
a	$m = \frac{a+b}{2}$	b	f(a)	f(m)	f(b)
0	3	6	+	+	-
3	4.5	6	+	-	-
3	3.75	4.5	+	+	-
3.75	4.125	4.5	+	-	-
3.75	3.9375	4.125	+	-	-
3.75	3.84375	3.9375	+	-	-

$\therefore h =$, correct to 1 decimal places (準確至小數點後一個位).