Chapter 8 More about Polynomials

Chapter 8A

1.

$$(5x^4+3x^3-8x^2+5)-(-6x^4+4x^3-x-1)$$
$$=5x^4+3x^3-8x^2+5+6x^4-4x^3+x+1$$
$$=\underline{\underline{11x^4-x^3-8x^2+x+6}}$$

2.

$$3P(x)-2Q(x)=3(x^2+2)-2(-3x+5)$$
$$=3x^2+6+6x-10$$
$$=\underline{\underline{3x^2+6x-4}}$$

3.

$$(4x^2-5x-1)\times(2+x)$$
$$=(4x^2-5x-1)(2)+(4x^2-5x-1)(x)$$
$$=8x^2-10x-2+4x^3-5x^2-x$$
$$=4x^3+3x^2-11x-2$$

$\therefore$ The coefficient of $x^2=\underline{\underline{3}}$

4.

$$(2x^3+x^2-x+1)(3x-2)$$
$$=(2x^3+x^2-x+1)(3x)-(2x^3+x^2-x+1)(2)$$
$$=6x^4+3x^3-3x^2+3x-4x^3-2x^2+2x-2$$
$$=\underline{\underline{-2+5x-5x^2-x^3+6x^4}}$$

5.

$$\begin{array}{r|l} & 2x^3+x^2-2x-1 \\ \hline 2x-1 & 4x^4+0x^3-5x^2+0x-3 \\ & \underline{4x^4-2x^3} \\ & \quad 2x^3-5x^2 \\ & \quad \underline{2x^3-x^2} \\ & \qquad -4x^2+0x \\ & \qquad \underline{-4x^2+2x} \\ & \qquad\quad -2x-3 \\ & \qquad\quad \underline{-2x+1} \\ & \qquad\qquad -4 \end{array}$$

$\therefore$ The quotient $=\underline{\underline{2x^3+x^2-2x-1}}$

The remainder $=\underline{\underline{-4}}$

6.

$$(2x-5)(3x+4)$$
$$=6x^2+8x-15x-20$$
$$=6x^2-7x-20$$

$\therefore\ 6x^2-7x-20\equiv 6x^2+Ax+B$

By equating coefficients of like powers of x, we have

$A=\underline{\underline{-7}}$, $B=\underline{\underline{-20}}$

7. Let $f(x)=9x^4+3x^3-x^2+12x-5$.

By the Remainder Theorem,

the remainder

$$=f\left(-\frac{2}{3}\right)$$
$$=9\left(-\frac{2}{3}\right)^4+3\left(-\frac{2}{3}\right)^3-\left(-\frac{2}{3}\right)^2+12\left(-\frac{2}{3}\right)-5$$
$$=\underline{\underline{\frac{-113}{9}}}$$

8. Let $f(x)=x^4+x^3-8x-8$.

$\because\ f(2)=2^4+2^3-8(2)-8$
$=0$

$\therefore$ $\underline{x-2 \text{ is a factor of } x^4+x^3-8x-8.}$

9.

$$\begin{array}{r|l} & x^2-2x+1 \\ \hline x+2 & x^3+0x^2-3x+2 \\ & \underline{x^3+2x^2} \\ & \quad -2x^2-3x \\ & \quad \underline{-2x^2-4x} \\ & \qquad\quad x+2 \\ & \qquad\quad \underline{x+2} \end{array}$$

$\therefore\ P(x)=(x+2)(x^2-2x+1)$

$=\underline{\underline{(x+2)(x-1)^2}}$

10. Let $f(x)=2x^3+9x^2+10x+3$.

$$f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+9\left(-\frac{1}{2}\right)^2+10\left(-\frac{1}{2}\right)+3$$
$$=0$$

$\therefore$ $2x+1$ is a factor of $f(x)$.

$$f(-3)=2(-3)^3+9(-3)^2+10(-3)+3$$
$$=0$$

$\therefore$ $x+3$ is a factor of $f(x)$.

i.e. $\underline{(2x+1)(x+3) \text{ is a factor of } 2x^3+9x^2+10x+3.}$

Chapter 8B

1. (a)

$$(2x-1)^2+(x+2)(x^2+ax+3)$$
$$=4x^2-4x+1+x^3+2x^2+ax^2+2ax+3x+6$$
$$=\underline{\underline{x^3+(6+a)x^2+(2a-1)x+7}}$$

(b) (i)

Coefficient of $x=3$

$$2a-1=3$$
$$a=\underline{\underline{2}}$$

(ii)

$$\text{Coefficient of } x^2=6+a$$
$$=6+2$$
$$=\underline{\underline{8}}$$

2. (a) $3x^2+5x-2=\underline{\underline{(3x-1)(x+2)}}$

(b) (i)

$$f(x)=(2x-1)(3x^2+5x-2)-7x-14$$
$$=6x^3+10x^2-4x-3x^2-5x+2-7x-14$$
$$=6x^3+7x^2-16x-12$$

$$\begin{array}{r|l} & 6x+7 \\ \hline x^2+1 & 6x^3+7x^2-16x-12 \\ & 6x^3 \qquad\quad +6x \\ \hline & \qquad 7x^2-22x-12 \\ & \qquad 7x^2 \qquad\quad +7 \\ \hline & \qquad\qquad -22x-19 \end{array}$$

$\therefore$ The quotient $=\underline{\underline{6x+7}}$

The remainder $=\underline{\underline{-22x-19}}$

(ii)

$$f(x)=(2x-1)(3x^2+5x-2)-7x-14$$
$$=(2x-1)(3x-1)(x+2)-7x-14$$
$$=(2x-1)(3x-1)(x+2)-7(x+2)$$
$$=(x+2)[(2x-1)(3x-1)-7]$$
$$=(x+2)(6x^2-5x+1-7)$$
$$=(x+2)(6x^2-5x-6)$$
$$=\underline{\underline{(x+2)(2x-3)(3x+2)}}$$

3. (a)

$$\text{The remainder}=g(-1)$$
$$=2(-1)^{49}+1$$
$$=2(-1)+1$$
$$=\underline{\underline{-1}}$$

(b)

Let $f(x)=g(x+1)$.

$\therefore$ $f(x)=2(x+1)^{49}+1$

$\therefore$ The required remainder $=f(-2)$

$$=2(-2+1)^{49}+1$$
$$=2(-1)+1$$
$$=\underline{\underline{-1}}$$

4. (a)

$$f(3)=8(3)^3-26(3)^2+3(3)+9$$
$$=0$$

$\therefore$ $\underline{x-3 \text{ is a factor of } f(x).}$

(b)

$$\begin{array}{r} 8x^2 - 2x - 3 \\ x-3 \overline{\smash{)}\, 8x^3 - 26x^2 + 3x + 9} \\ \underline{8x^3 - 24x^2} \\ -2x^2 + 3x \\ \underline{-2x^2 + 6x} \\ -3x + 9 \\ \underline{-3x + 9} \end{array}$$

$\therefore \quad f(x) = (x-3)(8x^2 - 2x - 3)$

$= \underline{\underline{(x-3)(2x+1)(4x-3)}}$

(c)

$f(x) = 0$

$(x-3)(2x+1)(4x-3) = 0$

$\therefore \quad x - 3 = 0 \quad or \quad 2x + 1 = 0 \quad or \quad 4x - 3 = 0$

$x = \underline{\underline{3}} \quad or \quad \underline{\underline{-\frac{1}{2}}} \quad or \quad \underline{\underline{\frac{3}{4}}}$

5. (a)

$(x+2)(x-3)(Ax+1) + B$

$= (x^2 - x - 6)(Ax+1) + B$

$= (x^2 - x - 6)(Ax) + (x^2 - x - 6)(1) + B$

$= Ax^3 - Ax^2 - 6Ax + x^2 - x - 6 + B$

$= Ax^3 + (1-A)x^2 - (1+6A)x - 6 + B$

$\therefore \quad Ax^3 + (1-A)x^2 - (1+6A)x - 6 + B$

$\equiv 2x^3 + Cx^2 + Dx - 3$

By equating coefficients of like powers of x, we have

$A = \underline{\underline{2}}$

$1 - A = C$

$C = 1 - 2$

$= \underline{\underline{-1}}$

$-(1+6A) = D$

$D = -[1 + 6(2)]$

$= \underline{\underline{-13}}$

$-6 + B = -3$

$B = \underline{\underline{3}}$

(b) (i) From **(a)**,

$(x+2)(x-3)(2x+1) - 3 \equiv 2x^3 - x^2 - 13x - 3$

$2x^3 - x^2 - 13x \equiv (x+2)(x-3)(2x+1) + 6$

$f(x) = x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$

$= \frac{1}{2}(2x^3 - x^2 - 13x) - 3$

$= \frac{1}{2}[(x+2)(x-3)(2x+1) + 6] - 3$

$= \underline{\underline{\frac{1}{2}(x+2)(x-3)(2x+1)}}$

(ii)

$f(x) = 0$

$\frac{1}{2}(x+2)(x-3)(2x+1) = 0$

$\therefore \quad x + 2 = 0 \quad or \quad x - 3 = 0 \quad or \quad 2x + 1 = 0$

$x = \underline{\underline{-2}} \quad or \quad \underline{\underline{3}} \quad or \quad \underline{\underline{-\frac{1}{2}}}$

Chapter 8C

1. $\because \quad f(x)$ is divisible by $x + 2$.

$\therefore \quad f(-2) = 0$

i.e. $(-2)^3 + k(-2)^2 - (-2) - 2 = 0$

$-8 + 4k + 2 - 2 = 0$

$4k = 8$

$\therefore \quad k = \underline{\underline{2}}$

2. When $f(x)$ is divided by $x - 1$, the remainder is $f(1)$ and when $f(x)$ is divided by $x + 1$, the remainder is $f(-1)$.

$\therefore \quad f(1) = f(-1)$

$(1)^3 - 2(1)^2 + k(1) - 1 = (-1)^3 - 2(-1)^2 + k(-1) - 1$

$1 - 2 + k - 1 = -1 - 2 - k - 1$

$2k = -2$

$\therefore \quad k = \underline{\underline{-1}}$

3. $\because$ $x+4$ is a factor of $f(x)$.

$\therefore$ $f(-4)=0$

i.e. $a(-4)^3+4(-4)^2-(-4)+b=0$

$-64a+64+4+b=0$

$-64a+b+68=0$ .... **(i)**

$\because$ $x+4$ is a factor of $g(x)$.

$\therefore$ $g(-4)=0$

i.e. $4(-4)^3+16(-4)^2-a(-4)+b=0$

$-256+256+4a+b=0$

$\therefore$ $4a+b=0$ ...... **(ii)**

(ii) − **(i)**, $68a-68=0$

$\therefore$ $a=\underline{\underline{1}}$

Substitute $a=1$ into **(ii)**, $4(1)+b=0$

$\therefore$ $b=\underline{\underline{-4}}$

4.

$\because$ $12x^3y^2=2^2\cdot 3\cdot x^3\cdot y^2$,

$6x^2y=2\cdot 3\cdot x^2\cdot y$

$\therefore$ H.C.F. $=2\cdot 3\cdot x^2\cdot y=\underline{\underline{6x^2y}}$

L.C.M. $=2^2\cdot 3\cdot x^3\cdot y^2=\underline{\underline{12x^3y^2}}$

5.

$\because$ $4(x+1)^2(x-1)^3=2^2(x+1)^2(x-1)^3$,

$6(x+1)(x-1)(x-3)=2\cdot 3(x+1)(x-1)(x-3)$

$\therefore$ H.C.F. $=\underline{\underline{2(x+1)(x-1)}}$

L.C.M. $=2^2\cdot 3(x+1)^2(x-1)^3(x-3)$

$=\underline{\underline{12(x+1)^2(x-1)^3(x-3)}}$

6.

$\because$ $9x^2-9=9(x^2-1)$

$=3^2(x-1)(x+1)$

$12x^2-30x+18=6(2x^2-5x+3)$

$=3\cdot 2(x-1)(2x-3)$

$\therefore$ H.C.F. $=\underline{\underline{3(x-1)}}$

L.C.M. $=2\cdot 3^2(x-1)(x+1)(2x-3)$

$=\underline{\underline{18(x-1)(x+1)(2x-3)}}$

7.

$$\frac{3}{x+1}-\frac{3x}{(x+1)^2}$$

$$=\frac{3(x+1)-3x}{(x+1)^2}$$

$$=\frac{3x+3-3x}{(x+1)^2}$$

$$=\underline{\underline{\frac{3}{(x+1)^2}}}$$

8.

$$\frac{m^2-1}{m^2+2m-3}\div\frac{m^2+2m+1}{m+3}$$

$$=\frac{(m+1)(m-1)}{(m+3)(m-1)}\times\frac{m+3}{(m+1)^2}$$

$$=\underline{\underline{\frac{1}{m+1}}}$$

9. The restrictions are $x\neq\pm 2,\ \frac{1}{2}$.

$$\frac{3}{x-2}+\frac{1}{x+2}=\frac{1}{2x-1}$$

$$\frac{3(x+2)+(x-2)}{(x-2)(x+2)}=\frac{1}{2x-1}$$

$$\frac{4x+4}{x^2-4}=\frac{1}{2x-1}$$

$$(4x+4)(2x-1)=x^2-4$$

$$8x^2+4x-4=x^2-4$$

$$7x^2+4x=0$$

$$x(7x+4)=0$$

$$x=\underline{\underline{0}}\ \text{or}\ \underline{\underline{-\frac{4}{7}}}$$

Chapter 8D

1. (a)

$\because \quad f(x)=(x^2-5x+4)(ax+b)$

$\therefore \quad f(2)=[2^2-5(2)+4](2a+b)$

$\because \quad f(2)=-8$

$\therefore \quad -2(2a+b)=-8$

$2a+b=4$ .... **(i)**

Also, $f(-1)=[(-1)^2-5(-1)+4](-a+b)$

$\because \quad f(-1)=10$

$\therefore \quad 10(-a+b)=10$

$-a+b=1$ .... **(ii)**

(i) − (ii) : $3a=3$

$a=\underline{\underline{1}}$

Substitute $a=1$ into **(i)**, we have

$2(1)+b=4$

$b=\underline{\underline{2}}$

(b) From **(a)**,

$f(x)=(x^2-5x+4)(x+2)$

$=(x-1)(x-4)(x+2)$

$\because \quad f(x)\cdot g(x)=\text{H.C.F.}\times\text{L.C.M.}$

$\therefore \quad g(x)=\dfrac{[(x-1)(x+2)][(x-1)^2(x+2)(x-4)(x-3)]}{(x-1)(x-4)(x+2)}$

$=\underline{\underline{(x-1)^2(x+2)(x-3)}}$

2. (a)

$x^2-5x-6=\underline{\underline{(x-6)(x+1)}}$

$x^2+3x+2=\underline{\underline{(x+1)(x+2)}}$

(b)

H.C.F. $=\underline{\underline{x+1}}$

L.C.M. $=\underline{\underline{(x+1)(x-6)(x+2)}}$

(c)

$\dfrac{3}{x^2-5x-6}+\dfrac{2}{x^2+3x+2}$

$=\dfrac{3}{(x-6)(x+1)}+\dfrac{2}{(x+1)(x+2)}$

$=\dfrac{3(x+2)+2(x-6)}{(x-6)(x+1)(x+2)}$

$=\dfrac{3x+6+2x-12}{(x-6)(x+1)(x+2)}$

$=\underline{\underline{\dfrac{5x-6}{(x-6)(x+1)(x+2)}}}$

3. (a) $\because \quad f(x)$ is divisible by $x+2$.

$\therefore \quad f(-2)=0$

i.e. $9(-2)^3+18(-2)^2+a(-2)+b=0$

$-72+72-2a+b=0$

$-2a+b=0$ .... **(i)**

$\because \quad g(x)$ is divisible by $x+2$.

$\therefore \quad g(-2)=0$

i.e. $3(-2)^3+10(-2)^2-a(-2)+b=0$

$-24+40+2a+b=0$

$2a+b+16=0$ .... **(ii)**

(i) + (ii), $2b+16=0$

$b=\underline{\underline{-8}}$

(ii) − (i), $4a+16=0$

$a=\underline{\underline{-4}}$

(b)

$$\begin{array}{r} 9x^2 \qquad\qquad -4 \\ x+2\,\overline{)\,9x^3+18x^2-4x-8} \\ \underline{9x^3+18x^2\qquad\qquad} \\ -4x-8 \\ \underline{-4x-8} \end{array}$$

$\therefore \quad f(x)=(x+2)(9x^2-4)$

$=\underline{\underline{(x+2)(3x+2)(3x-2)}}$

$$\begin{array}{r|l} & 3x^2 + 4x - 4 \\ x+2 & 3x^3 + 10x^2 + 4x - 8 \\ & 3x^3 + 6x^2 \\ & \quad 4x^2 + 4x \\ & \quad 4x^2 + 8x \\ & \qquad -4x - 8 \\ & \qquad -4x - 8 \end{array}$$

$\therefore \quad g(x) = (x+2)(3x^2 + 4x - 4)$

$= \underline{\underline{(x+2)(3x-2)(x+2)}}$

(c)

$f(x) = g(x)$

i.e. $(x+2)(3x+2)(3x-2) = (x+2)(3x-2)(x+2)$

$(x+2)(3x+2)(3x-2) - (x+2)(3x-2)(x+2) = 0$

$(x+2)(3x-2)[(3x+2)-(x+2)] = 0$

$(x+2)(3x-2)(2x) = 0$

$x+2=0$ or $3x-2=0$ or $2x=0$

$\therefore \quad x = \underline{\underline{-2}}$ or $\underline{\underline{\frac{2}{3}}}$ or $\underline{\underline{0}}$

4. (a)

$g(1) = 2(1)^3 + 3(1)^2 - 8(1) + 3$

$= 0$

$\therefore$ $\underline{x-1 \text{ is a factor of } g(x).}$

$h(1) = 2(1)^3 + (1)^2 - 5(1) + 2$

$= 0$

$\therefore$ $\underline{x-1 \text{ is a factor of } h(x).}$

(b)

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ x-1 & 2x^3 + 3x^2 - 8x + 3 \\ & 2x^3 - 2x^2 \\ & \quad 5x^2 - 8x \\ & \quad 5x^2 - 5x \\ & \qquad -3x + 3 \\ & \qquad -3x + 3 \end{array}$$

$\therefore \quad g(x) = (x-1)(2x^2 + 5x - 3)$

$= \underline{\underline{(x-1)(2x-1)(x+3)}}$

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ x-1 & 2x^3 + x^2 - 5x + 2 \\ & 2x^3 - 2x^2 \\ & \quad 3x^2 - 5x \\ & \quad 3x^2 - 3x \\ & \qquad -2x + 2 \\ & \qquad -2x + 2 \end{array}$$

$\therefore \quad h(x) = (x-1)(2x^2 + 3x - 2)$

$= \underline{\underline{(x-1)(2x-1)(x+2)}}$

(c)

$$\frac{g(x)}{h(x)} - \frac{h(x)}{g(x)}$$

$$= \frac{(x-1)(2x-1)(x+3)}{(x-1)(2x-1)(x+2)} - \frac{(x-1)(2x-1)(x+2)}{(x-1)(2x-1)(x+3)}$$

$$= \frac{x+3}{x+2} - \frac{x+2}{x+3}$$

$$= \frac{(x+3)^2 - (x+2)^2}{(x+2)(x+3)}$$

$$= \frac{(x^2+6x+9) - (x^2+4x+4)}{(x+2)(x+3)}$$

$$= \underline{\underline{\frac{2x+5}{(x+2)(x+3)}}}$$

Chapter 8E

1. A	**2.** A
3. D	**4.** A
5. C	**6.** E
7. D	**8.** B
9. B	**10.** B
11. D	**12.** A
13. B	**14.** B

(End of Ch.8 Sol'n)