

Interaction and modelling of entries to and exists from drug use

A draft of a draft!
Comments welcome!

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SIRUS
Oslo, April, 2001

Social interaction, treatment and modelling changes in the number of drug users

General introduction

Potential drug users learn both about the costs and benefits of using drugs and about the probabilities of experiencing these costs and benefits from existing users. The question in this paper is how could model this and what kind of insights we can gain from such a model.

To answer this question I will first present a simple aggregated model of entries and exits from drug use. As it turns out this model has some rather surprising properties – for instance that more money invested in treatment for “serious addicts” has absolutely no effect on the number of drug users. One might wonder whether this really is a “valuable insight” or whether the “no effect” result is simply an artefact of the model. To discuss this I will, first, examine whether there are plausible microfoundations behind the aggregated model. The next step is to see whether the result holds even if we relax some of the unrealistic assumptions. To do so, I will use a computer program that initially can be set to use assumptions like the mathematical model in this paper. The difference between a computer simulation and a mathematical model, however, is that whereas it is often very difficult to include too many realistic assumptions and many different types of social interaction in a formal mathematical model, this is far easier in a computer program. Hence, the program exemplifies both a way to test the importance of the unrealistic assumptions (jointly, not only isolated) and an independent perhaps sometimes more useful modelling approach.

Existing literature

A.M. Jones (1994, p. 101) writes that “The economic literature on social interaction and consumer behaviour is rather sparse.” This may be unsurprising considering the theoretical and empirical difficulty encountered when we try to model and test social interaction as described in two review articles; P. Kirman (199?) and C. Manski (1999). More specifically, about formal modelling of interaction in the field of addiction research, the most recent review (?) is Skog (1982). A recent contribution by an economist is Moene (1999) who presents a formal model of the consequences of the desire to conform when consuming drugs or alcohol. A classic contribution on interaction in economics is Schelling (1978).

The model

I will first try to create a simple model of how observational learning could influence the number of drug users. I will then explore the properties of this model and present some possible extensions – for instance about how to include interaction in preferences and not only beliefs.

Some definitions

Assume the population (N) at any time can be divided into three distinct groups: Seemingly happy drug users – Yuppies (Y); Seemingly unhappy drug users – Junkies (J) and those who do not use drugs in any form – Abstainers (A). We then have:

$$(1) N_t = A_t + Y_t + J_t$$

To make things easier later on, I will introduce the following definitions:

$$(2) \quad a_t = \frac{A_t}{N_t} \quad y_t = \frac{Y_t}{N_t} \quad j_t = \frac{J_t}{N_t} \quad d_t = j_t + y_t \quad a_t = 1 - d_t$$

In words, a_t is the share of the population at time t that does not use drugs, d_t is the share that use drugs (which includes both the happy and the unhappy users) and so on. These are all between zero and one and, by definition, $a_t + y_t + j_t = 1$.

The aggregate model

The three key equations of the model are:

$$(3) \quad y_t = y_{t-1} + \mathbf{b}_1 a_{t-1} (1 - p_t) - \mathbf{b}_2 y_{t-1}$$

$$(4) \quad j_t = j_{t-1} + \mathbf{d}_1 y_{t-1} - \mathbf{d}_2 j_{t-1}$$

$$(5) \quad p_t = \frac{j_{t-1}}{d_{t-1}}$$

Essentially the two first equations say that to find this year's share of yuppies or junkies in the population we simply start from last year's share and add those who begin and subtract those who quit. The last equation represents one way of estimating the probability of becoming a junkie. It is simply last year's share of junkies out of all drug users.

Slightly more detailed the story is as follows: Every year some abstainers start to use drugs (become yuppies). I shall assume that this depends linearly on the probability of becoming a junkie; The higher the probability of becoming a junkie the smaller the share of the abstainers who will begin to use drugs (\mathbf{b}_1 is the share of abstainers willing to try drugs when the probability of becoming a junkie is zero i.e. the model allows for a group of people who refuse regardless of the probability of becoming a junkie). \mathbf{b}_2 represents the share of yuppies who every year leave the group for whatever reason (There are many ways to leave the group: they could just quit drugs, they could die, they could be treated, or they could become junkies).

Similarly, there is an inflow of junkies since I assume that every year a share of the yuppies become junkies (\mathbf{d}_1 is the proportion of yuppies who every year turn into junkies). Moreover, every year some junkies exit from the group – they either die or are treated (become abstainers) (\mathbf{d}_2 is the share of junkies who exit the group every year).

Finally, the probability of becoming a junkie is given by the ratio of last year's junkies to last year's drug users – see (5). The more junkies there are out of those who have used drugs, the higher the probability of ending up as a junkie. Note that this is not necessarily the best possible – or “rational” - estimate. A rational person who knew the model above and believed it to be correct, could argue as follows: Assume I start to use drugs this year. What could happen to me next year and what are the probabilities? I could just quit drugs, I could die, I could become a junkie. The probability of becoming a junkie next year is \mathbf{d}_1 . The same is true in year 2, year 3 and so on (given that you still is a yuppie). It is, however, very difficult (?or?) to work out the true probability of becoming a junkie given that the model only

operates with an aggregated “exit” parameter. I have not used a separate parameter for death, treatment, and “just quitting.” Hence, if I start to use drugs at $t=0$, then the probability of becoming a junkie at $t=2$ must consider first the probability that I still am in the yuppie group and second the probability that I am among those yuppies that become junkies. The last is d_1 . The first is: $1 - b_2$. We must then work out the probability of becoming junkie at all future times (Could be a problem here with assuming infinite lives unless there is convergence – before 1?). (also problem: switching in and out) (I will work on this, but right now all I want to do is to examine the probability as it is specified. Alternative and more “rational” estimates will be investigated later).

We have some information about the size of these parameters. Mathematically, a logical consequence of the set up is that the inflow of junkies from yuppies cannot be larger than the share of yuppies which disappear from the group (i.e. $d_1 < b_2$). Also by definition all the numbers are between zero and one. If we allow “outside information” we could also make plausible guesses as to the general size of the other parameters. d_2 is at least larger than the death rate (which is around 3%). Moreover, one might believe that resources spend on treatment should affect d_2 (One could try to find out how much 1 million more spend on treatment will change d_2 ?). We know something about how many people who use drugs from questionnaires (often about 20% of youth report having tried cannabis, but only 3% of youths – 15-20 – report that they are regular users - yuppies). We also could also estimate the number of junkies in Norway to about 10 000 if we are willing to equate “junkies” with “injecting” drug abusers. Using this (and more) we can be more sure about where to start when we use numerical simulations to examine the properties of the system; and it may also be easier to find analytic solutions; and finally, we know something about in which region it is useful to look. A “strange” result with plausible parameters is much more interesting than a strange result using implausible parameters (e.g. in a computer simulation).

The properties of the model

What kind of questions would we want the answers to when faced with the model above? First of all I would want to know which variables (and how) influence the equilibrium share of junkies, the share of yuppies and the share of drug users. Of course, this presupposes the existence of an equilibrium and we have to investigate this too. Moreover, I want to know whether the equilibrium is unique or whether the system creates multiple equilibria. Finally, one might ask questions about the stability of the equilibria, the speed of convergence towards equilibria and whether the equilibria are optimal in some specified sense.

Define equilibrium as a situation where the number of junkies and yuppies is stable from year to year. This implies that every year the inflow must be equal to the outflow. Hence in equilibrium:

$$(6) \quad b_1 a(1 - p) = b_2 y$$

$$(7) \quad d_1 y = d_2 j$$

Substitute for a and p in (6) and we have:

$$(8a) \quad \mathbf{b}_1(1-d)\left(1-\frac{j}{d}\right) = \mathbf{b}_2y$$

$$(8b) \quad \mathbf{b}_1(d-j-d^2+jd) = \mathbf{b}_2yd$$

Substitute $d = j + y$ and simplify

$$(8c) \quad \mathbf{b}_1(y+j-j-(y+j)^2+j(y+j)) = \mathbf{b}_2y(y+j)$$

$$(8d) \quad \mathbf{b}_1(y+j-j-y^2-2yj-j^2+jy+j^2) = \mathbf{b}_2(y^2+yj)$$

$$(8e) \quad \mathbf{b}_1(y-y^2-2yj) = \mathbf{b}_2(y^2+yj)$$

All the elements contain y so this can be simplified by dividing each side by y . Note that division by y is not as innocent as it may look. First of all it excludes at least one possible equilibrium ($y=0, j=0$). Anyway, we find at least one possible equilibrium by dividing by y to get:

$$(8f) \quad \mathbf{b}_1(1-y-j) = \mathbf{b}_2(y+j)$$

Know from (7) $y = \frac{\mathbf{d}_2}{\mathbf{d}_1}j$

Substitute this and solve for j :

$$(9) \quad j^* = \frac{\mathbf{b}_1\mathbf{d}_1}{(\mathbf{b}_1 + \mathbf{b}_2)(\mathbf{d}_1 + \mathbf{d}_2)}$$

Which gives y :

$$(10) \quad y^* = \frac{\mathbf{b}_1\mathbf{d}_2}{(\mathbf{b}_1 + \mathbf{b}_2)(\mathbf{d}_1 + \mathbf{d}_2)}$$

And, interestingly (since $d = j + y$)

$$(11) \quad d^* = \frac{\mathbf{b}_1}{\mathbf{b}_1 + \mathbf{b}_2}$$

After this I know the following:

(a) There are at least two equilibria ($y=0, j=0$) and the one given above (importance of knowing this? Non-linearities and lock-in? Policy-implications? Or just: understanding/explanation?)

(b) I suspect (based on numerical simulation) that $y=0, j=0$ is unstable and that the other is stable. (The numerical simulation also suggested the existence of other equilibria except the ones I have found). (I will try to investigate this more formally using Liapunov)

(c) Surprisingly (at least initially): The equilibrium share of drug users (d^*) is not affected by \mathbf{d}_1 (the proportion of yuppies becoming junkies every year) or \mathbf{d}_2 (the share of junkies who quit every year). This is important because treatment (of junkies) can be viewed as a way of increasing \mathbf{d}_2 . One might hope that this would reduce the equilibrium share of drug users, but in the model this is not the case. In fact, the result is quite intuitive: Treatment initially reduces the number of junkies. This, in turn, implies that there are fewer junkies and that the probability of becoming a junkie is reduced (if calculated as specified in our model). Since the probability of becoming a junkie is reduced, more abstainers are willing to try drugs and there is an increase in the share of yuppies. Finally, the model predicts that the reduction of junkies (as a result of treatment) will exactly balance the increase of yuppies and the net effect on the number of users is zero.

Note two crucial elements here. First of all, the way people form expectations about the probability of becoming a junkie is crucial. More sophisticated individuals might realize that the reduction of junkies as a consequence of treatment should not reduce the probability of becoming a junkie. The sophisticated individual might also base his estimate on more information than last year's shares. More sophisticated modelling of the expectation formation mechanisms is therefore one possible extension of the model. (But this also demands a more detailed model in which the "exit reason" is given)

Second, the transfer from yuppies to junkies is very fast in this model. A large inflow of yuppies will result in a large inflow of junkies after only one year. One might try to build more sophisticated models in which the individual spends several years as a yuppie before he (sometimes) becomes a junkie. The speed and symmetry (in the sense that implicitly people in the model are yuppies and junkies for an equal length of time) of the flow of people is crucial (I conjecture) to the conclusion that treating a junkie will not affect the overall number of drug users. The reason is as follows: As long as people are yuppies they are "contagious" in the "positive" sense that they increase drug use (by making it look less harmful to abstainers). Junkies are also "contagious", but this time in a "negative" sense: they reduce the number of drug users by frightening the potential users. If you spend the same amount of time being positive as negative contagious the net effect is zero. If, however, the time period as a yuppie is longer than the period as a junkie the net effect is positive (you recruit users). It would also imply that treating a junkie would increase the equilibrium share of users (d).

Note also: The speed of the adjustment is responsible for making it difficult to capture waves (say a drug is used for some time and then dies out because people learn about its harmful side effects).

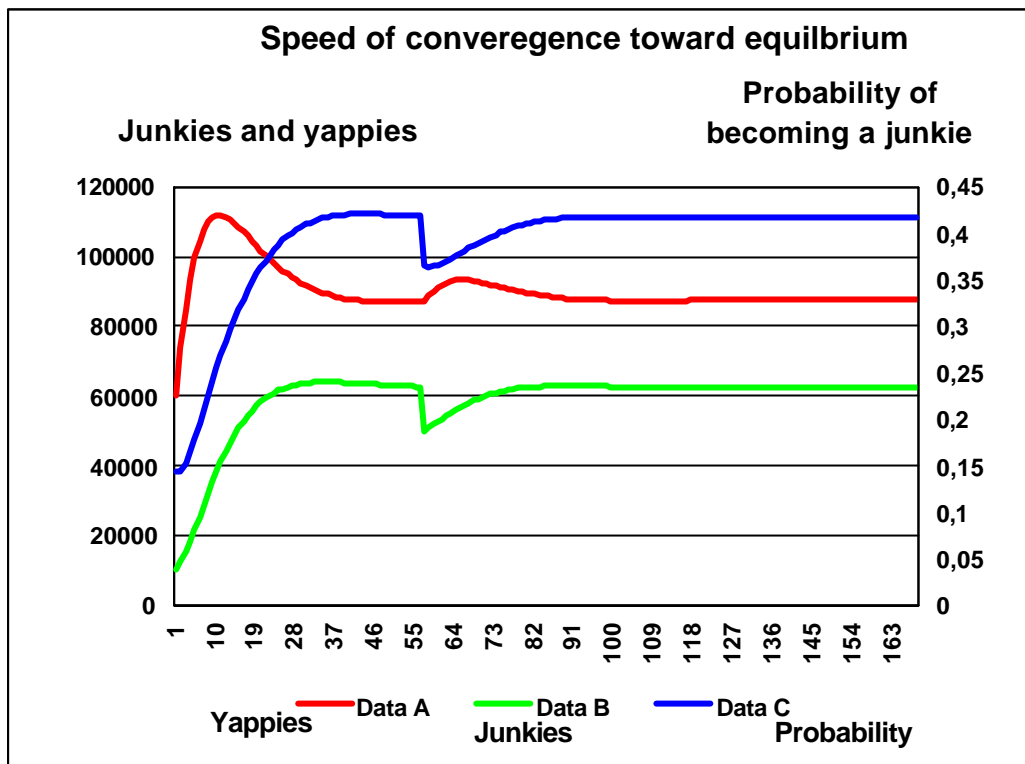
All of this points to the need for a more sophisticated model since there is no reason to assume that people are junkies and yuppies for an exactly equal time period. This could be done either by using a computer model (in which j_t easily can depend on y_{t-8}), or introducing some kind of mathematically tractable lags (Almond/Koyeck?)

(d) Not only do I know which variables enter into the equilibrium solution. I also know how the variables enter. For instance, the number of users is simply the share of abstainers willing to use drugs when $p = 1$ divided the sum of the same variable and the share of yuppies who disappear from the group every year. It is more difficult to interpret j and y verbally (except just to state it verbally), but we at least we know in a mathematical sense how the variables relate to each other in equilibrium.

(e) I have so far not said anything about the speed of convergence towards equilibrium. Analytically it is difficult (and it may be impossible) to solve the system since you have to solve a non-linear difference equation. There are well known methods for solving linear difference equations, but non-linear are often close to impossible to solve. As a substitute, however, I ran some simulations to examine the speed of convergence. The figure below shows the result from one such simulation in which the starting point was set as follows:

60000Y0 (yuppies)
10000J0 (junkies)
0,142857Initial probability of becoming a junkie
70000Total drug users
30000(b1)vanskelig tolkning? generasjonsstørrelse?
0,2(b2)andel yuppies som slutter
0,05(d1)andel som blir junkies hvert år
0,07(d2)andel av junkies som forsvinner hvert år

I wanted to examine how fast the system converged upon equilibrium so I introduced a “shock” after about 50 periods. The number of junkies was then about 62 000 and I just assumed an exogenous shock that reduced their numbers to 50 000. It turned out that the system returned to (the same) (almost) equilibrium after about 25 years.



(f) I can try to estimate the effect of changing the parameters. For instance, more treatment could be viewed as a way of increasing d_2 . We could then use the analytic solution to say something about how much an increase in treatment (say of 0.1) will change the equilibrium levels of junkies and yuppies. (And we might try to calculate the monetary cost of increasing d_2 by 0.1). We could then say things like “10 000 000 more spent on treatment (of junkies) will result in a (long run) reduction in the number of junkies by x% and an increase in the share of yuppies by y%” It would here be interesting to compare, for instance, the effect of treating yuppies (increasing b_2) vs. treating junkies (increasing d_2). Of course, the faith we have in the conclusions depends on whether we believe the model is realistic. This, in turn, depends partly on the realism of the assumptions. (Why “partly”? We may believe the results of a model even if the assumptions are unrealistic when we know that the assumptions are not important in generating the results i.e. we could relax the assumption and still get the same result)

So, Try to explore these question:

- (1) Realism of model and
- (2) Do we get the same results when relaxing assumptions.
(Have already noted some potential important assumptions.)

- (1) Depends heavily on “microfoundations” so discuss this.
- (2) Then explore whether the assumptions can be relaxed. How: Use computer simulation of more complex assumptions.

But note here: The list (a) to (f) represents “new” knowledge gained by mathematical modelling (and partly unavailable to other techniques/verbal reasoning?). Of course, whether

this is useful knowledge is another question (again must be realistic to have policy lessons. is it? examine microfoundations and other assumptions)

Microfoundations

The aim of this section is to examine possible microfoundations to the aggregate functions explored above.

The general idea is that each year every abstainer decides whether to begin to use drugs or not. (An alternative interpretation is that every year a new generation – say all those who turn 20 – make up their minds. Why? Seems unrealistic to assume that everybody – children and pensioners – decide this every year). Assume they do so by weighting the costs and benefits of taking drugs and the probabilities of experiencing these costs and benefits. A very general formulation of this would be that at time “*t*” person “*i*” decides to use drugs (D) if the expected utility of drugs exceeds the expected benefits of some other action he might take. We assume that once a person has decided to use drugs there are only two relevant end-states to be considered: Either he will end up as a junkie or he will end up as a yuppie. Life as a yuppie is considered to be attractive; life as a junkie is less attractive. The expected utility of taking drugs is then:

$$EU_{it}(D) = p_{it}U_{it}(J) + (1 - p_{it})U_{it}(Y)$$

where p_{it} is the individual’s probability of ending up as a junkie and $U_{it}(\cdot)$ represents the person’s estimate of the total (discounted) utility of your life (!) depending on whether you end up as a junkie or yuppie.

Given the formulation above there are only two ways in which social interaction can be important: Either your estimate of the probability of becoming a junkie is influenced by other people in the population or your estimate of the utilities is influenced by other people. (A third possibility might be whether D is available at all – something that often depends on “others.”). In this paper I only try to model one particular way in which a person’s probability of becoming a junkie is influenced by observational learning. To make things simpler then, I shall make the following assumptions:

$$\begin{aligned} p_{it} &= p_t & \forall i \\ U_{it}(\cdot) &= U(\cdot) & \forall it \end{aligned}$$

i.e. that every individual uses the same probability of becoming a junkie and that the utility of ending up as a junkie or as a yuppie is the same for every individual at all times. [I also exclude – for now – interaction and learning about the size of the costs and benefits; the utilities). This implies that at abstainers at time *t* base their decision as to whether or not to begin to use drugs on:

$$EU_{it}(D) = p_t U(J) + (1 - p_t) U(Y)$$

The formulation so far is, perhaps, alien enough to deserve some comments. The equations above are deliberately stripped of some features that are often considered to be important in the decision to use drugs. For instance, Ainslie (199?) argues that discounting based on inconsistent time-preferences is potentially important. I have simply assumed this problem

away by ignoring explicit modelling of time-preferences. All I need is that the individual has some idea of the utility associated with life as a junkie or life as a yuppie (as end-states). This is not because I believe discounting is unimportant, but because the *focus of this paper* is on interaction and not on discounting. Similarly, Becker and Murphy (19xx) argue that people have different utilities associated with drug use and that this is important in a story of entries and exits from drug use (some people have more to lose; some people have miserable lives anyway and so on). I assume that everybody has the same utility attached to the alternatives. Once again this is not to deny the importance of heterogeneous utility estimates, but it is impossible to model everything at once and my focus is – initially – not on different utility estimates. (Orphanides + Z. model is more similar in structure)

There are, however, some simplifications that cannot be defended with the phrases “this is not my topic.” First and foremost my microfoundation seems to imply that all abstainers either start to use drugs or do not start. I need some kind of heterogeneity that allows some abstainers to begin to use drugs, while others do not. Here are some suggestions:

1. Different p 's since different people meet different people

If we relax the assumption that everybody has the same probability estimate, we could create a model in which some people (randomly) have high estimates while others get low estimates of the probability of becoming a junkie.

Interpret p as the result of people YOU have met (i.e. not the true population ratio). Some people will by coincidence meet more yuppies than others; others will – by coincidence – meet more junkies. We have:

$$p_{it} = \frac{j_{i,t-1}}{d_{i,t-1}}$$

Mathematically it would not be too difficult to find out the proportion of all abstainers that would get a p lower than a certain limit given that the true p is j/d . E.g. binomial distribution, so has variance $np(1-p)$ and expectation (np) where n is the number of “meetings” we assume every year. If need: could use normal approximation to find answer. All the people with a p lower than some limit p_0 would start to use drugs.

We could then find an equilibrium (not corner solutions) even if people have the same utility functions and so on; In a group of similar people some will meet more yuppies and hence begin to use drugs (not because they are “different” people but because they by accident meet more yuppies than the true population level). One could here make some speculative predictions about how the number of friends would affect drug use (if people in general have few friends, many people will have probability estimates that are way off the true population shares. Alternative interpretation: People with few friends become drug users more often than people with many friends? Assuming true level is on average deterring.

The aggregate function of entries would, however, no longer be a linear function of p (conjecture).

It would be quite easy to include speculations about several mechanisms that affect p :

- overconfidence (decrease p)
- a mechanism that works like this: the more users there are the less the stigma is and the more “visible” happy users become (p decrease)

- a mechanisms to the effect that junkies are more visible (walk around openly) while many yuppies hide their use (since it is illegal + stigma) = p increase

2. Third parameter: Moral cost of drug

We could introduce a third parameter. Say, for instance, the population is heterogeneous with respect to some “moral cost” associated with the use of drugs. We would then have:

$$EU_{it}(D) = p_i U(J) + (1 - p_i) U(Y) - m_i$$

Alternatively there could be a variable associated with the alternative (not taking drugs) that varied across the population. Whether this would produce a linear increase of yuppies in p depends on the distribution of m. If it is uniform, we have a linear cumulative distribution. If it is, say, normally distributed, then the cumulative function is more like an S and we will have multiple equilibria.

3. Different risk attitudes

My initial idea was to have a population with different risk attitudes. That is, some people would take drugs when the probability of becoming a junkie was 0.1, others would not. This is possible, but it turned out to be more difficult that I thought to formulate this rigorously enough to justify the claim that my aggregated functions had good microfoundations. This is something which I will work on. The problem is that risk-attitudes is usually incorporated in the utility function (e.g. when measured by Pratt-Arrow: a measure of the degree to which the indifference curve is bent or straight) so then I have to allow heterogeneous utility estimates. This is OK, but it turned out to be slightly more difficult to rigorously justify the link between the utility functions and my linear aggregates: What assumptions must be made about the distribution and nature of individual utility functions in order to make my aggregated function linear in p? On the other hand, there is not need to insist on linearity and I could try to experiment with non-linear effects of changes in p (on the share of junkies and yuppies).

Initially I will just use (2), but since this is almost like cheating, I will also try to do some work on (1). I will – at least right now - forget about (3) although there might be some interesting results waiting to be discovered there?

I also want to make the link between micro and macro even “tighter.” All the macro equations should be formally derived from micro assumptions.

Computer program

I am close to finishing the computer program. The idea is as follows:

- Start with a population of – say – 1000 people
- Every person is given some characteristics
 - is he a junkie, a yuppie or an abstainer
 - his subjective “fear” limit (He will not use drugs when his p is above that)
 - his subjective cost estimate
 - his subjective “benefit” estimate
 - his “moral cost” of using drug
 - many other variables could be included: gender; age; start of drug career; discounting/impatience; strength of “autonomy”; children; number of friends ...

- Initially the program tells the computer how many people that should have each characteristic (how many junkies, yuppies, abstainers there are and how the probabilities and utility estimates are)

- **Interaction**

Then there is interaction. How? I simply say that each person should meet n other person (randomly selected). The characteristics of the people he will meet then affect that person (Should there be interaction both ways or just one affected by the other? Both ways). n could be a constant for all – or we could let n vary – some have many “meetings” while others have few. Could also let people to a greater extent only interact with people who are “similar” in terms of characteristics?). Interaction changes the probabilities and cost estimates and maybe some of the other characteristics. If you meet many happy users, your fear of using drugs decreases and maybe also the benefit estimate increases.

(So far interaction is only “local” – could have more “global” interaction like a person estimate of stigma is affected by overall number of users eg. as reported in newspaper)

- **Entries and exits**

After interaction all the abstainers decide whether to use drugs or not by calculating

$$EU_{it}(D) = p_t U(J) + (1 - p_t) U(Y) - m_i$$

and comparing this to their “threshold.” Based on this some change their status from abstainers to yuppies. Some yuppies also become junkies. And some quit, die, are treated in both groups and exit for that reason.

I am almost finished with a version of the program, but it is too early to say anything about the results. At least I can numerically investigate some of the properties of the model and examine the effects of changing the parameters more easily. I can also more easily create more complex models and simulation. (Initially the model can be set to use values that make it equal the mathematical model – then I can introduce more complex assumption). I can also examine the question of how long time on average a person is a yuppie and a junkie in the simple model.

Idea: To focus more on the exit decision. So far most of the attention has been focused on abstainers deciding to be yuppies and the factors that enter that decision. What about the decision to “just quit” drugs? Also influenced by p ? Could model this too.

Conclusion

Too early to say!

Appendix 1: Modelling the absolute numbers and not the share of junkies, yuppies

We have:

$$Y_t = Y_{t-1} + \mathbf{b}_1(1 - p_t) - \mathbf{b}_2 Y_{t-1}$$

$$J_t = J_{t-1} + \mathbf{d}_1 Y_{t-1} - \mathbf{d}_2 J_{t-1}$$

$$p_t = \frac{J_{t-1}}{D_{t-1}}$$

The main difference from the system in the paper is that this time I use variables that are measured in absolute numbers. I have also changed the first equation. I might, of course, use big A (as I used little A in the first model), but I want to try this in order to see whether the interpretation of the model becomes easier.

The solution becomes even simpler than the first model. In equilibrium:

$$\mathbf{b}_1 \left(1 - \frac{J}{J+Y}\right) = \mathbf{b}_2 Y$$

$$\mathbf{d}_1 Y = \mathbf{d}_2 J$$

From the first equation:

$$(J+Y)\mathbf{b}_1 \left(1 - \frac{J}{J+Y}\right) = \mathbf{b}_2 Y(J+Y)$$

$$\mathbf{b}_1(J+Y-J) = \mathbf{b}_2 YJ + \mathbf{b}_2 Y^2$$

$$\mathbf{b}_1(J+Y-J) = \mathbf{b}_2 YJ + \mathbf{b}_2 Y^2$$

$$\mathbf{b}_1 Y = \mathbf{b}_2 YJ + \mathbf{b}_2 Y^2$$

$$\mathbf{b}_1 = \mathbf{b}_2 J + \mathbf{b}_2 Y$$

Know: $Y = \frac{\mathbf{d}_2}{\mathbf{d}_1} J$

$$\mathbf{b}_1 = \mathbf{b}_2 J + \mathbf{b}_2 J \frac{\mathbf{d}_2}{\mathbf{d}_1}$$

$$\mathbf{b}_1 \mathbf{d}_1 = \mathbf{b}_2 \mathbf{d}_1 J + \mathbf{b}_2 J \mathbf{d}_2$$

$$J^* = \frac{\mathbf{b}_1 \mathbf{d}_1}{\mathbf{b}_1 (\mathbf{d}_1 + \mathbf{d}_2)}$$

which implies that,

$$Y^* = \frac{\mathbf{b}_1 \mathbf{d}_2}{\mathbf{b}_2 (\mathbf{d}_1 + \mathbf{d}_2)}$$

$$D^* = \frac{\mathbf{b}_1}{\mathbf{b}_2}$$

(Compared to the solution in “shares” this is an even more “elegant” solution – at least in the sense that it reduces to a very short expression)

How should the parameters be interpreted in this model?

\mathbf{b}_2 , \mathbf{d}_1 , \mathbf{d}_2 have the same interpretation

\mathbf{b}_2 : The share of yuppies who leave the group every year

\mathbf{d}_1 : The share of yuppies which every year become junkies

\mathbf{d}_2 : The share of junkies who leave the group every year

\mathbf{b}_1 however has a different interpretation. It all depends on how you view the model. One possible interpretation could be that we assume that every year only people who turn 20 year old make a choice as to whether or not to use drugs. \mathbf{b}_1 could then be the number of people in that group (say about 30 000 individuals). This is related to the interpretation of “share of people willing to use drugs.” Both try to capture the size of potential users. However, the problem is that \mathbf{b}_1 is also intended (?) to adjust the effect of changes in the probability of becoming a junkie. There is no reason to assume (?) that a 20% increase in the probability of becoming a junkie necessarily will lead to a 20% increase in the number starting to use drugs (which would be the case without \mathbf{b}_1). In short, there remains some work on the interpretation here and the intuition of the model. (We might also have to worry more about population growth when we use the model with absolute numbers.)