Electrical Resonance
(R-L-C series circuit)

APPARATUS

1. R-L-C Circuit board
2. Signal generator
3. Oscilloscope Tektronix TDS1002 with two sets of leads (see "Introduction to the Oscilloscope" on page 12)

INTRODUCTION

When a sinusoidally varying force of constant amplitude is applied to a mechanical system the response often becomes very large at specific values of the frequency. The phase difference between the driving force and the response also varies, from close to 90° when far from resonance to 0 when resonance occurs.

Resonance occurs in electrical circuits as well, where it is used to select or "tune" to specific frequencies. We will study the characteristics of sinusoidal signals and the behavior of a series R-L-C circuit.

First we review the mathematical description of the behavior of the R-L-C circuit when connected to a sinusoidal signal. Since current is the same at all points in the series R-L-C circuit, we write:

$$I = I_0 \, sin \, \omega t \quad \text{where} \quad \omega = 2 \, \pi \, f$$

The voltages across each component are then:

$$V_R = I_0 \, R \, sin \, \omega t \tag{1}$$

$$V_L = I_0 \, \omega \, L \, sin(\omega t + \pi/2)$$

and

$$V_C = I_0 \left(\frac{1}{\omega C} \right) \, sin(\omega t - \pi/2)$$

The sum of these voltages must equal the supply voltage:

$$V_s = V_0 \, sin(\omega t + \phi)$$

where

$$V_0 = I_0 \, Z \quad \text{and} \quad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \tag{2}$$

Z is called the impedance and is a minimum when

$$\omega L - \frac{1}{\omega C} = 0 \tag{3}$$

42

Under this condition, called resonance, the phase shift between the supply voltage and the current is also zero. This phase shift is given by the phase angle ϕ:

$$tan\,\phi = \frac{\omega L - \frac{1}{\omega C}}{R} \qquad (4)$$

Equation 3 can be used to determine the **resonance frequency** f_0. Engineers use the half-power frequencies f_1 and f_2 (see Fig. 1) as a measure of the width of the resonance, these are the frequencies, where $I_0(f_1) = I_0(f_2) = I_0(f_0)/\sqrt{2}$
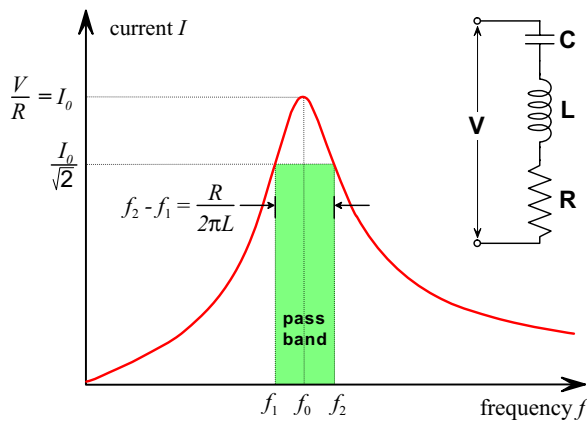and consequently, $\phi = 45°$.

Figure 1: Resonance in the R-L-C series circuit.
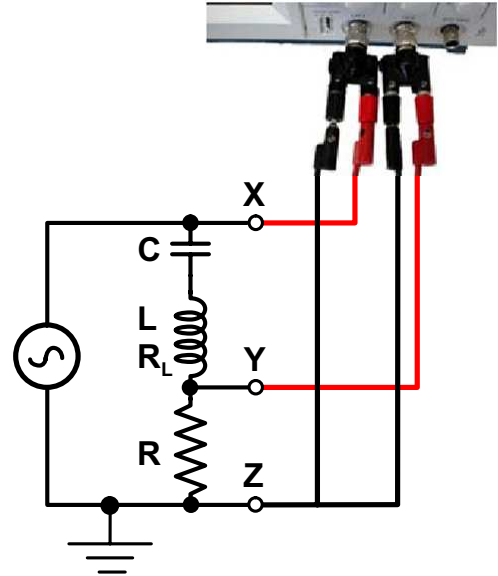

Figure 2: Wiring diagram.

PROCEDURE

Part 1: Setting up the circuit

1a. Examine the circuit board. Copy the data from the card on the bottom of the board including the circled board number. (If you need to come back at a later time, you will need to use the same board.)
NOTE: In this experiment there are two resistances, one (R) used to sense the current, and the resistance of the wire that forms the inductor(R_L). Equations 2 and 3 refer to the sum of the resistances, $(R + R_L)$ while we will apply equation 1 separately to the current sensing resistor R.

1b. Identify each component on the circuit board. Note the labels on the diagram.

1c. Review the notes on the oscilloscope. Two cables have been provided for the oscilloscope. Connect the cable with the plugs to CH1, and the plugs to points X and Z, being careful about ground connections (black plug to the black jack - point X). CH1 will display the AC voltage supplied to the circuit, V_0.

1d. Connect the cable with the clips to CH2, and the clips to the solder lugs on either side of the resistor (points X and Y), with the ground side (black wire) next to point X. CH2 will display the voltage across the resistor, V_R.

1e. Plug the Cable from the circuit board into the signal generator, again observing the grounding (GND tab on the side of the plug). Set the generator frequency to 5.0 kHz. **NOTE:** Setting the frequency generally requires setting two controls, a dial control and a range multiplier. The dial setting may be 0.5, 5.0, or 50.0, depending on the particular generator used. The multiplier will vary accordingly.

$$5.0 \text{ kHz} = 0.5 \times 10^4 \text{ Hz} = 5.0 \times 10^3 \text{ Hz} = 50.0 \times 10^2 \text{ Hz}.$$

1f. Set the oscilloscope to display CH1, both VOLTS/DIV dials to 200 mV, SEC/DIV to 50 μs, Trigger source to CH1, Trigger Mode to AUTO.

1g. Now turn on power for both signal generator and oscilloscope. If you do not see a trace try varying the vertical POSITION control above the CH1 dial, or the horizontal position control above the SEC/DIV dial.

If the screen shows multiple traces, vary the "TRIGGER LEVEL" knob slowly until it locks the trace onto a single sine curve.

Locate the "Amplitude" or "Fine" control on the signal generator and use it to set the amplitude of the trace to about 0.5 Volt, i.e. 2.5 divisions at 200 mV/div.

Part 2: Measuring frequency

Change the frequency of the generator to 6.0 kHz, and adjust the SEC/DIV so as to display a little more than one complete cycle. Measure the period of the signal, i.e. the distance between corresponding points on the curve times the SEC/DIV setting. You can use the horizontal and vertical POSITION knobs to position the trace under the most closely divided scales. Calculate the frequency from your measurement of the period. What is the percentage difference from the value set on the generator? This is an indication of the accuracy of the equipment.

Part 3: Measuring magnitude

There are three measures of the magnitude of the AC signal that are used. amplitude, rms (root mean square) and peak-to-peak (p-p). Center the sine wave vertically on the screen and adjust the output of the generator until the amplitude is exactly 0.5 V.

The voltage reading from the bottom to top distance is the p-p value. It is the fastest measuring technique when switching back and forth between signals.

On your data sheet: Sketch one cycle and indicate the distances that represent the amplitude and the p-p values, and record these values.
What is the relation between these two values?
Calculate the rms (effective) value of the signal you observed.

Part 4: Finding resonance

4a. Switch both traces on using the CH1 MENU and the CH2 MENU buttons. Adjust VOLTS/DIV and vertical shifts so that both generator and resistor signals are visible, and do not overlap. Slowly vary the frequency up to about 20 kHz, then scan down to about 2 kHz.

Note several changes: the phase between the two signals, the size of the resistor signal on CH2 (proportional to the current) and the variation in generator output on CH1.

4b. Adjust the frequency until the phases of the signals match. (You may want to overlap the signals to do this.) Note the generator frequency.(Record it as the resonance frequency by phase.) At resonance, the current and voltage in a series R-L-C circuit are in phase (see equation 4 with $\phi = 0$).

Part 5: Lissajous figure

This oscilloscope allows another way to observe resonance. If we set the Mode to XY (enter the DISPLAY menu and choose Format: XY) so that CH1 is depicted on the horizontal axis and CH2 on the vertical axis, we can observe Lissajous figures, which will be tilted ellipses for the sinusoidal signals present on both axes. You may have to adjust vertical and horizontal POSITION and the VOLTS/DIV dials to resize and center the pattern. Again, scan through the range of frequencies and note the variation in the display. Sketch and label the patterns at 2 kHz and 20 kHz. When the two signals are in phase, the pattern should become a straight line. Use this property of the Lissajous figure to find the resonance frequency again.

Part 6: Impedance

Leaving the frequency set at this last value, set V_0 to 0.5 Volts (1 Volt peak-to-peak on CH1) and measure V_R on CH2. You can switch between oscilloscope channels to observe each signal separately. Remember that V_R is proportional to the current I_0. (Both V_R and V_0 can be left as p-p values, since the ratio is the same as for the amplitudes.) Calculate I_0 using the appropriate value of resistance, and calculate the magnitude of the impedance using equation 2.

Part 7: Taking a resonance curve

We will measure pairs of values of V_0 and V_R at different frequencies, so as to be able to plot a resonance curve as in Figure 2. Note that, as you saw in Part IV, the generator output changes with frequency, so that both values have to be measured for each frequency. The resonance curve is a plot of I_0/V_0 vs. frequency. Tabulate your data using the column headings shown below. Peak-to-peak (p-p) values are recommended, since we will take a ratio. (Note that I_0 involves only the resistance of the resistor, not the whole circuit.)

f (kHz)	V_0 (V)	V_R (V)	I_0 (A)	I_0/V_0 (1/Ω)

Measure both V_0 and V_R at frequencies from 2.0 kHz to about 9.0 kHz. Note that in the central region, 5.0 kHz ... 7.0 kHz, steps of 0.2 kHz or 0.25 kHz (depending on the scale divisions of your signal generator) are required to follow the rapidly changing impedance. Larger steps can be taken outside of the central region.

Part 8: Measuring phase shift

If you still have time, set the frequency to 2 kHz. Make sure that the time base is in the calibrated position and set so that the screen shows 2 or 3 cycles of the signal on CH1. Measure the period of this signal. Now switch both channels on and measure the time between corresponding points on the two signals. Calculate the phase shift from the ratio of the time difference between the signals to the time of one period. Convert this to an angle in degrees.

ANALYSIS

1. Calculate the resonance frequency f_0 of the R-L-C series circuit, using the values on the bottom of the circuit board. Compare with the values measured in parts 4b. and 5. Compare the measured value of the impedance at resonance with the theoretical value.

2. Use your data to calculate the magnitude of the impedance Z of the circuit at one frequency in your data below 5 kHz. Compare with the values calculated from the circuit values. (Show your calculation.)

3. Plot I_0/V_0 against frequency. Include the points from part 6. Draw a SMOOTH curve that comes CLOSE to your data points. (See Figure 2 for a typical resonance curve.) This type of resonance curve is characteristic of many physical systems from atoms to radio receivers.

   The sharpness of the resonance is measured by the half-power points. These occur at 0.7071 (i.e. $1/\sqrt{2}$) times the peak value. Locate them on your graph. Label them f_1 and f_2, and record these values. Engineers use a Quality Factor Q to characterize the "sharpness" of a resonance, where
   $$Q = \frac{f_0}{f_2 - f_1}$$
   Calculate Q from your data. From equation 2, one can calculate a theoretical value:
   $$Q = \frac{2\pi \, f_0 \, L}{R_{total}}$$
   How does this value compare with your calculated value?

4. If you did part 8, then compare with the theoretical value.

Questions (to be answered in your report):

1. Explain why the current I_0 in the circuit is a maximum at the resonance frequency f_0 .

2. Show that if the phase difference between the voltages V_R and V_0 equals $90°$, the Lissajous figure would be an ellipse.

46