# Chapter 2

# A Theory of Current Account Determination

In this chapter, we build a model of an open economy, that is, of an economy that trades in goods and financial assets with the rest of the world. We then use that model to study the determinants of the trade balance and the current account. In particular, we will find out how the external accounts respond to a variety of economic shocks, such as changes in income and the world interest rate, and how those responses depend on whether the shocks are of a permanent or temporary nature.

## 2.1 A two-period economy

### 2.1.1 Households

Consider an economy in which agents live for two periods, 1 and 2, and are endowed with $Q_1$ units of goods in period 1 and $Q_2$ units in period 2. Goods are assumed to be perishable in the sense that they cannot be stored from one period to the next. In addition, households are endowed with $B_0^*$ units of a bond. In period 1, the household receives interest on its bond holdings in the amount of $r_0 B_0^*$, where $r_0$ denotes the interest rate on bond holdings between periods 0 and 1. In period 1, the household's income is given by the sum of interest on its bond holdings and its endowment of goods, $r_0 B_0^* + Q_1$. The household can allocate its income to two alternative uses: purchases of consumption goods, which we denote by $C_1$, and purchases of bonds, $B_1^* - B_0^*$, where $B_1^*$ denotes bond holdings at the end of period 1.

Thus, in period 1 the household faces the following budget constraint:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1. \tag{2.1}$$

Similarly, in period 2 the representative household faces a constraint stating that consumption expenditure plus bond purchases must equal income:

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2, \tag{2.2}$$

where $C_2$ denotes consumption in period 2, $r_1$ denotes the interest rate on assets held between periods 1 and 2, and $B_2^*$ denotes bond holdings at the end of period 2. As explained in the previous chapter, by the no-Ponzi-game constraint households are not allowed to leave any debt at the end of period 2, that is, $B_2^*$ must be greater than or equal to zero. Also, because the world is assumed to last for only 2 periods, agents will choose not to hold any positive amount assets at the end of period 2. Thus, asset holdings at the end of period 2 must be exactly equal to 0:

$$B_2^* = 0. \tag{2.3}$$

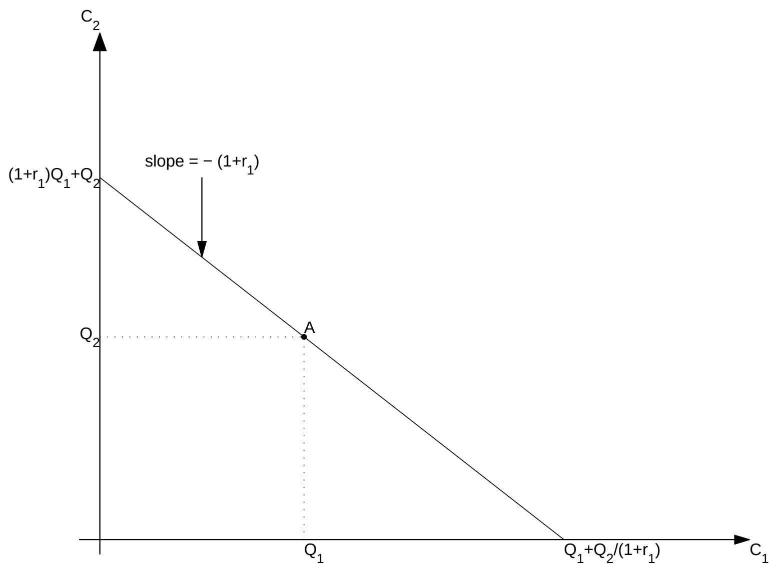
Combining the budget constraints (2.1) and (2.2) and the terminal condition (2.3) to eliminate $B_1^*$ and $B_2^*$, gives rise to the following lifetime budget constraint of the household:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}. \tag{2.4}$$

This intertemporal budget constraint requires that the present discounted value of consumption (the left-hand side) be equal to the initial stock of wealth plus the present discounted value of the endowment stream (the right-hand side). The household chooses consumption in periods 1 and 2, $C_1$ and $C_2$, taking as given all other variables appearing in (2.4), $r_0$, $r_1$, $B_0^*$, $Q_1$, and $Q_2$.

Figure 2.1 displays the pairs $(C_1, C_2)$ that satisfy the household's intertemporal budget constraint (2.4). For simplicity, we assume for the remainder of this section that the household's initial asset position is zero, that is, we assume that $B_0^* = 0$. Then, clearly, the basket $C_1 = Q_1$ and $C_2 = Q_2$ (point A in the figure) is feasible in the sense that it satisfies the intertemporal budget constraint (2.4). If the household wants to increase consumption in one period, it must sacrifice some consumption in the other period. In particular, for each additional unit of consumption in period 1, the household has to give up $1 + r_1$ units of consumption in period 2. This

Figure 2.1: The intertemporal budget constraint



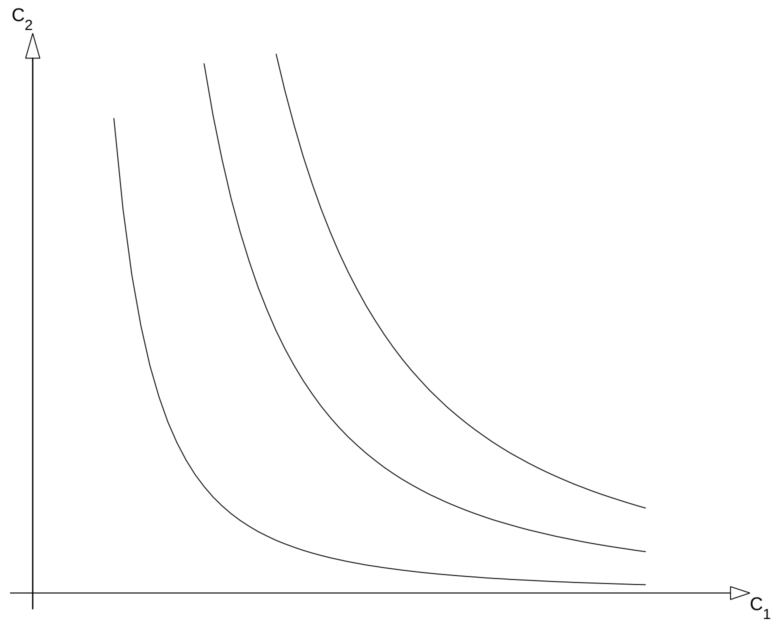

means that the slope of the budget constraint is $-(1+r_1)$. Note that points on the budget constraint located southeast of point A correspond to borrowing (or dissaving) in period 1 because $S_1 = Q_1 - C_1 < 0$; note also that because we are assuming that $B_0^* = 0$, the fact that $S_1 < 0$ implies, by the relation $S_1 = B_1^* - B_0^*$, that the household's asset position at the end of period 1, $B_1^*$, is negative. This in turn implies that a point on the budget constraint located southeast of the endowment point $A$ is also associated with positive saving in period 2 because $S_2 = B_2^* - B_1^* = -B_1^* > 0$. On the other hand, points on the budget constraint located northwest of A are associated with positive saving in period 1 and dissaving in period 2. If the household chooses to allocate its entire lifetime income to consumption in period 1, then $C_1 = Q_1 + Q_2/(1+r_1)$ and $C_2 = 0$. This point corresponds to the intersection of the budget constraint with the horizontal axis. If the household chooses to allocate all its lifetime income to consumption in period 2, then $C_2 = (1+r_1)Q_1 + Q_2$ and $C_1 = 0$; this basket is located at the intersection of the budget constraint with the vertical axis.

Which consumption bundle on the budget constraint the household will choose depends on its preferences. We will assume that households like both $C_1$ and $C_2$ and that their preferences can be described by the utility function

$$U(C_1, C_2), \tag{2.5}$$

where the function $U$ is strictly increasing in both arguments. Figure 2.2 displays household's indifference curves. You should be familiar with the

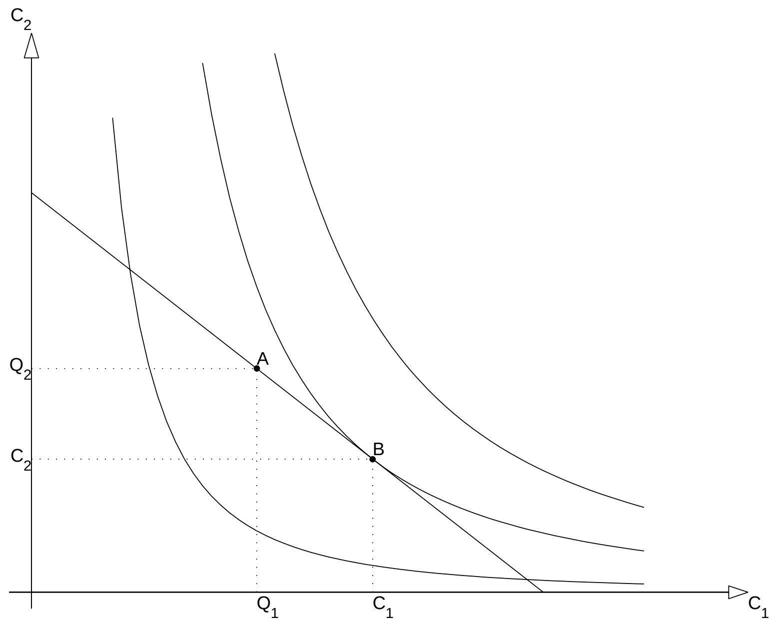
Figure 2.2: Indifference curves




concept of indifference curves from introductory Microeconomics. All consumption baskets on a given indifference curve provide the same level of utility. Because consumption in both periods are goods, that is, items for which more is preferred to less, as one moves northeast in figure 2.3, utility increases. Note that the indifference curves are convex toward the origin, so that at low levels of $C_1$ the indifference curves are steeper than at high levels of $C_1$. Intuitively, the convexity of the indifference curves means that at low levels of consumption in period 1, the household is willing to give up relatively many units of period 2 consumption for an additional unit of period 1 consumption. On the other hand, if period 1 consumption is high, then the household will not be willing to sacrifice much period 2 consumption for an additional unit of period 1 consumption. The negative of the slope of an indifference curve is known as the marginal rate of substitution of $C_2$ for $C_1$. Therefore, the assumption of convexity means that along a given indifference curve, the marginal rate of substitution decreases with $C_1$.

Households choose $C_1$ and $C_2$ so as to maximize the utility function (2.5) subject to the lifetime budget constraint (2.4). Figure 2.3 displays the life-

Figure 2.3: Equilibrium in the endowment economy




time budget constraint together with the household's indifference curves. At the feasible basket that maximizes the household's utility, the indifference curve is tangent to the budget constraint (point B). Formally, the tangency between the budget constraint and the indifference curve is given by the following first-order condition of the household's maximization problem:

$$U_1(C_1, C_2) = (1 + r_1)U_2(C_1, C_2), \tag{2.6}$$

where $U_1(C_1, C_2)$ and $U_2(C_1, C_2)$ denote the marginal utilities of consumption in periods 1 and 2, respectively. The marginal utility of consumption in period 1 indicates the increase in utility resulting from the consumption of an additional unit of $C_1$ holding constant $C_2$. Similarly, the marginal utility of period 2 consumption represents the increase in utility associated with a unit increase in $C_2$ holding constant $C_1$. Technically, the marginal utilities of $C_1$ and $C_2$ are defined as the partial derivatives of $U(C_1, C_2)$ with respect to $C_1$ and $C_2$, respectively.[1] The ratio $\frac{U_1(C_1,C_2)}{U_2(C_1,C_2)}$ represents the negative of the slope of the indifference curve at the basket $(C_1, C_2)$, or the marginal rate of substitution of $C_2$ for $C_1$.[2]

[1]That is, $U_1(C_1, C_2) = \frac{\partial U(C_1,C_2)}{\partial C_1}$ and $U_2(C_1, C_2) = \frac{\partial U(C_1,C_2)}{\partial C_2}$.

[2]To see that (2.6) states that at the optimum the indifference curve is tangent to the

The intuition behind condition (2.6) is as follows. Suppse that the consumer sacrifices one unit of consumption in period 1 and saves it. Then his utility in period 1 falls by $U_1(C_1, C_2)$. In period 2, he receives $(1 + r_1)$ units of consumption each of which gives him $U_2(C_1, C_2)$ units of utility, so that his utility in period 2 increases by $(1 + r_1)U_2(C_1, C_2)$. If the left-hand side of (2.6) is greater than the right-hand side, the consumer can increase his lifetime utility by saving less (and hence consuming more) in period 1. Conversely, if the left-hand side of (2.6) is less than the right-hand side, the consumer will be better off saving more (and consuming less) in period 1. At the optimal allocation, the left- and right-hand sides of (2.6) must be equal to each other, so that in the margin the consumer is indifferent between consuming in period 1 and consuming in period 2.[3]

### 2.1.2 Equilibrium

We assume that all households in the economy are identical. Thus, by studying the behavior of an individual household, we are also learning about the behavior of the country as a whole. For this reason, we will not distinguish between the behavior of an individual household and that of the country as a whole. To keep things simple, we further assume that there is no investment in physical capital. (In chapter **??**, we will extend the model by allowing for production and capital accumulation.) Finally, we will assume that the country has free access to international capital markets. This means that the domestic interest rate, $r_1$, must be equal to the world interest rate, which we will denote by $r^*$, that is,

$$r_1 = r^*.$$

If this condition is satisfied we will say that *interest rate parity* holds. The country is assumed to be sufficiently small so that its savings decisions do not affect the world interest rate. Because all households are identical, at any point in time all domestic residents will make identical saving decisions.

budget constraint, divide the left and right hand sides of that equation by $-U_2(C_1, C_2)$ to obtain

$$-\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = -(1 + r_1)$$

and recall that $-(1 + r_1)$ is the slope of the budget constraint.

[3]One way of obtaining (2.6) is to solve for $C_2$ in (2.4) and to plug the result in the utility function (2.5) to get rid of $C_2$. The resulting expression is $U(C_1, (1+r_0)(1+r_1)B_0^* + (1 + r_1)Q_1 + Q_2 - (1 + r_1)C_1)$ and depends only on $C_1$ and other parameters that the household takes as given. Taking the derivative of this expression with respect to $C_1$ and setting it equal to zero—which is a necessary condition for a maximum—yields (2.6).

This implies that domestic households will never borrow or lend from each other and that all borrowing or lending takes the form of purchases or sales of foreign assets. Thus, we can interpret $B_t^*$ ($t = 0, 1, 2$) as the country's net foreign asset position in period $t$.

An equilibrium then is a consumption bundle $(C_1, C_2)$ and an interest rate $r_1$ that satisfy the household's intertemporal budget constraint, the household's first-order condition for utility maximization, and interest rate parity, that is,

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

$$U_1(C_1, C_2) = (1+r_1)U_2(C_1, C_2)$$

and

$$r_1 = r^*,$$

given the exogenous variables $\{r_0, B_0^*, Q_1, Q_2, r^*\}$.

At this point, we will pause to revisit the basic balance-of-payments accounting in our two-period model. We first show that the lifetime budget constraint of the household can be expressed in terms of current and expected future trade balances. Begin by rearranging terms in the intertemporal budget constraint (2.4) to express it in the form

$$(1+r_0)B_0^* = -(Q_1 - C_1) - \frac{(Q_2 - C_2)}{1+r_1}.$$

In our simple economy, the trade balance in period 1 equals the difference between the endowment of goods in period 1, $Q_1$, and consumption of goods in period 1, $C_1$, that is, $TB_1 = Q_1 - C_1$. Similarly, the trade balance in period 2 is given by $TB_2 = Q_2 - C_2$. Using these expressions for $TB_1$ and $TB_2$ and recalling that in equilibrium $r_1 = r^*$, we can write the lifetime budget constraint as:

$$(1+r_0)B_0^* = -TB_1 - \frac{TB_2}{1+r^*}. \tag{2.7}$$

This expression, which should be familiar from chapter 1, states that a country's present discounted value of trade deficits must equal its initial net foreign asset position including net investment income. If the country starts out as a debtor of the rest of the world ($B_0^* < 0$), then it must run a trade surplus in at least one period in order to repay its debt ($TB_1 > 0$ or $TB_2 > 0$ or both). Conversely, if at the beginning of period 1 the country is a net creditor ($B_0^* > 0$), then it can use its initial wealth to finance current or future trade deficits. In particular, it need not run a trade surplus in either period. In the special case in which the country starts with a zero stock of foreign wealth ($B_0^* = 0$), a trade deficit in one period must be offset by a trade surplus in the other period.

The lifetime budget constraint can also be written in terms of the current account. To do this, recall that the current account is equal to the sum of net investment income and the trade balance. Thus in period 1 the current account is given by $CA_1 = r_0 B_0^* + TB_1$ and the current account in period 2 is given by $CA_2 = r^* B_1^* + TB_2$. Using these two definitions to eliminate $TB_1$ and $TB_2$ from equation (2.7) yields

$$(1 + r_0)B_0^* = -(CA_1 - r_0 B_0^*) - \frac{(CA_2 - r^* B_1^*)}{1 + r^*}.$$

Using the definition $CA_1 = B_1^* - B_0^*$ to eliminate $B_1^*$, we obtain, after collecting terms,

$$B_0^* = -CA_1 - CA_2.$$

This alternative way of writing the lifetime budget constraint makes it clear that if the country is an initial debtor, then it must run a current account surplus in at least one period ($CA_1 > 0$ or $CA_2 > 0$). On the other hand, if the country starts out as a net creditor to the rest of the world, then it can run current and/or future current account deficits. Finally, if the country begins with no foreign debt or assets ($B_0^* = 0$), a current account deficit in one period must be offset by a current account surplus in the other period.

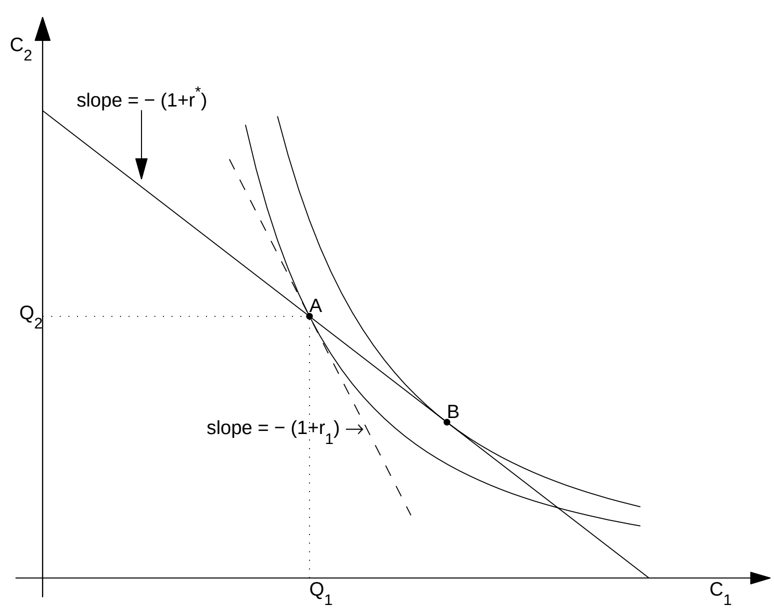
Let's now go back to the equilibrium in the small open economy shown in figure 2.3. At the equilibrium allocation, point B, the country runs a trade deficit in period 1 because $Q_1 - C_1$ is negative. Also, recalling our maintained assumption that foreign asset holdings in period 0 are nil, the current account in period 1 equals the trade balance in that period ($CA_1 = r_0 B_0^* + TB_1 = TB_1$). Thus, the current account is in deficit in period 1. The current account deficit in period 1 implies that the country starts period 2 as a net debtor to the rest of the world. As a result, in period 2 the country must generate a trade surplus to repay the debt plus interest, that is, $TB_2 = Q_2 - C_2 > 0$.

## 2.2 Capital controls

Current account deficits are often viewed as something bad for a country. The idea behind this view is that by running a current account deficit the economy is living beyond its means and accumulating external debt. As a result, the argument goes, the country will face future economic hardship in the form of reduced consumption and investment spending when the foreign debt becomes due. A policy recommendation frequently offered to countries undergoing external imbalances is the imposition of capital controls. In their most severe form, capital controls consist in the prohibition of borrowing from the rest of the world. Milder versions take the form of taxes on international capital inflows.

We can use the model economy developed in this chapter to study the welfare consequences of prohibiting international borrowing. Suppose that the equilibrium under free capital mobility is as described in figure 2.3. That is, agents optimally choose to borrow from the rest of the world in period 1 in order to finance a level of consumption that exceeds their endowment. Assume now that the government prohibits international borrowing. The new equilibrium is depicted in figure 2.4. If agents cannot borrow from the

Figure 2.4: Equilibrium under capital controls



rest of the world in period 1, that is, $B_1^* \geq 0$, then in that period they can at most consume their endowment. Because under free capital mobility $C_1$ was greater than $Q_1$, the borrowing constraint will be binding, so that in the constrained equilibrium $B_1^* = 0$ and $C_1 = Q_1$. The fact that consumption equals the endowment implies that the trade balance in period 1 is zero ($TB_1 = 0$). Given our assumption that the initial net foreign asset position is zero ($B_0^*$), the current account in period zero is also nil ($CA_1 = 0$). Because the country starts period 2 with zero external debt ($B_1^* = 0$), it can use its entire period 2 endowment for consumption purposes ($C_2 = Q_2$).

Under capital controls the domestic interest rate $r_1$ is no longer equal to the world interest rate $r^*$. In fact, it must be higher than $r^*$ to discourage domestic residents from borrowing. Graphically, $1 + r_1$ is given by the negative of the slope of the indifference curve at point A, which is not only the endowment point but also the optimal consumption bundle under capital controls. Only at that interest rate are households willing to consume their endowment.

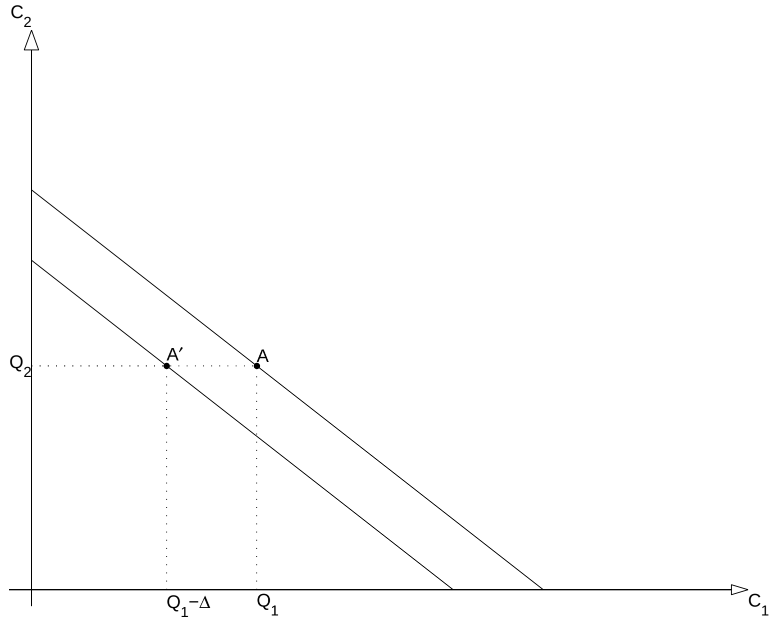
The indifference curve that passes through the endowment point A lies southwest of the indifference curve that passes through point B, the optimal consumption bundle under free capital mobility. Therefore, the level of utility, or welfare, is lower in the absence of free capital mobility. [*Question*: Suppose the equilibrium allocation under free capital mobility lay northwest of the endowment point A. Would it still be true that eliminating free international capital mobility is welfare decreasing?]

## 2.3 Current account adjustment to output, terms of trade, and world interest rate shocks

### 2.3.1 Temporary Output Shocks

Consider first the case of a temporary output shock. Specifically, assume that a negative shock (such as an earthquake) produces a decline in output in period 1 from $Q_1$ to $Q_1 - \Delta < Q_1$, but leaves output in period 2 unchanged. The situation is illustrated in figure 2.5, where A denotes the endowment before the shock $(Q_1, Q_2)$ and A$'$ the endowment after the shock $(Q_1 - \Delta, Q_2)$. As a consequence of the decline in $Q_1$, the budget constraint shifts toward the origin. The new budget constraint is parallel to the old one because the world interest rate is unchanged. The household could adjust to the output shock by reducing consumption in period 1 by exactly the amount of the output decline, $\Delta$, thus leaving consumption in period 2

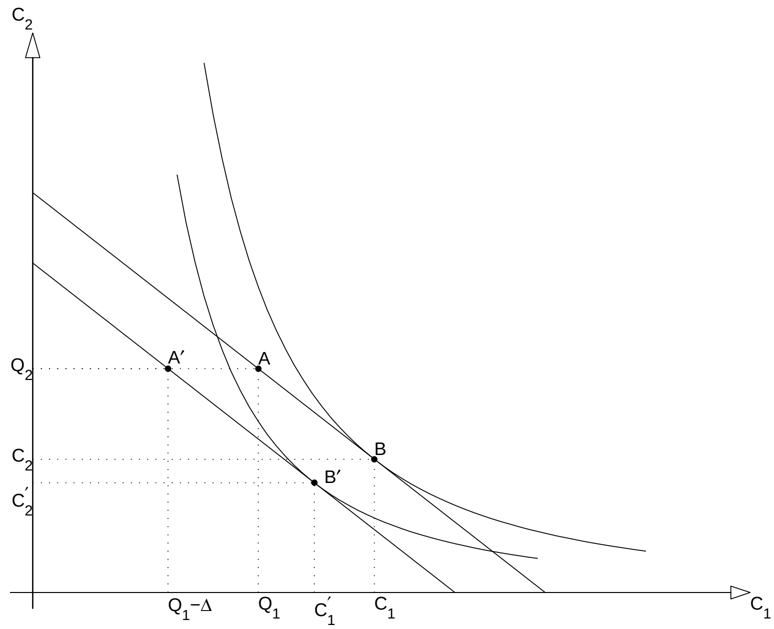
Figure 2.5: A temporary decline in output and the intertemporal budget constraint




unchanged. However, if both $C_1$ and $C_2$ are normal goods (i.e., goods whose consumption increases with income), the household will choose to smooth consumption by reducing both $C_1$ (by less than $\Delta$) and $C_2$. Figure 2.6 depicts the economy's response to the temporary output shock. As a result of the shock, the new optimal consumption bundle, B′, is located southwest of the pre-shock allocation, B. In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in the absence of the shock) and finances it by acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

The important principle to take away from this example is that temporary negative income shocks are smoothed out by borrowing from the rest of the world rather than by adjusting current consumption by the size of the shock. [*Question:* How would the economy respond to a temporary positive income shock?] The pattern of adjustment is quite different when the income shock is of a more permanent nature.

Figure 2.6: Adjustment to a temporary decline in output

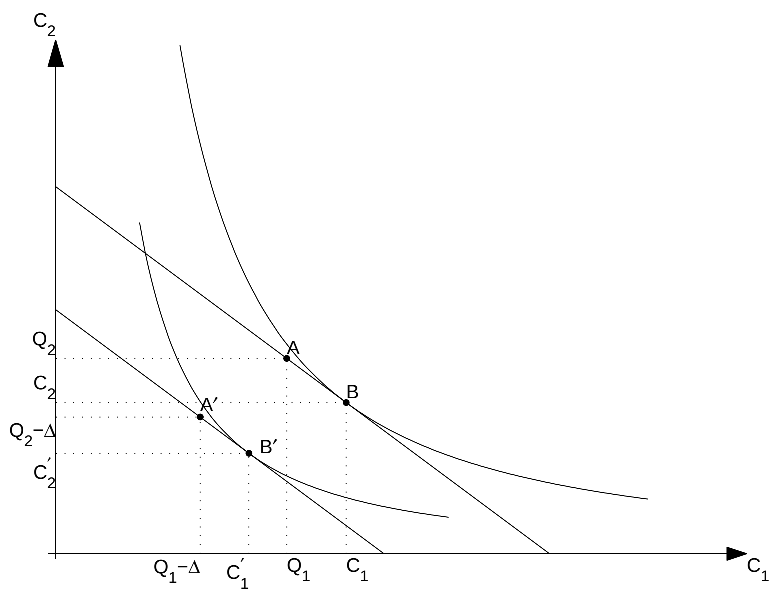



### 2.3.2 Permanent Output Shocks

Consider now a permanent negative output shock that reduces both $Q_1$ and $Q_2$ by $\Delta$. Figure 2.7 illustrates the situation. As a result of the decline in endowments, the budget constraint shifts to the left in a parallel fashion. The new budget constraint crosses the point $(Q_1 - \Delta, Q_2 - \Delta)$. As in the case of a temporary output shock, consumption-smoothing agents will adjust by reducing consumption in both periods. If consumption in each period fell by exactly $\Delta$, then the trade balance would be unaffected in both periods. In general the decline in consumption should be expected to be close to $\Delta$, implying that a permanent output shock has little consequences on the trade balance and the current account.

Comparing the effects of temporary and permanent shocks on the current account, the following general principle emerges: Economies will tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down). Thus, temporary shocks tend to produce large movements in the current account while permanent shocks tend to leave the current account largely unchanged.

Figure 2.7: Adjustment to a permanent decline in output




### 2.3.3 Terms-of-Trade Shocks

Thus far, we have assumed that the country's endowments $Q_1$ and $Q_2$ can be either consumed or exported. Assume now that the good households like to consume, say food, is different from the good they are endowed with, say oil. In such an economy, both $C_1$ and $C_2$ must be imported and $Q_1$ and $Q_2$ must be exported. Let $P^M$ and $P^X$ denote the price of imports and exports, respectively. A country's terms of trade, $TT$, is the price of a country's exports relative to the price of its imports, that is, $TT = P^X/P^M$. In terms of our example, $TT$ represents the price of oil in terms of food. Thus, $TT$ indicates the amount of food that the country can by from the sale of an additional barrel of oil. Assuming that foreign assets are expressed in units of consumption, the household's budget constraints in periods 1 and 2, respectively, are:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + TT_1 Q_1$$

and

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + TT_2 Q_2.$$

These budget constraints are identical to (2.1) and (2.2) except for the fact that the terms of trade are multiplying the endowments. Using the terminal condition $B_2^* = 0$, the above two equations can be combined to obtain the following lifetime budget constraint:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + TT_1Q_1 + \frac{TT_2Q_2}{1+r_1}$$

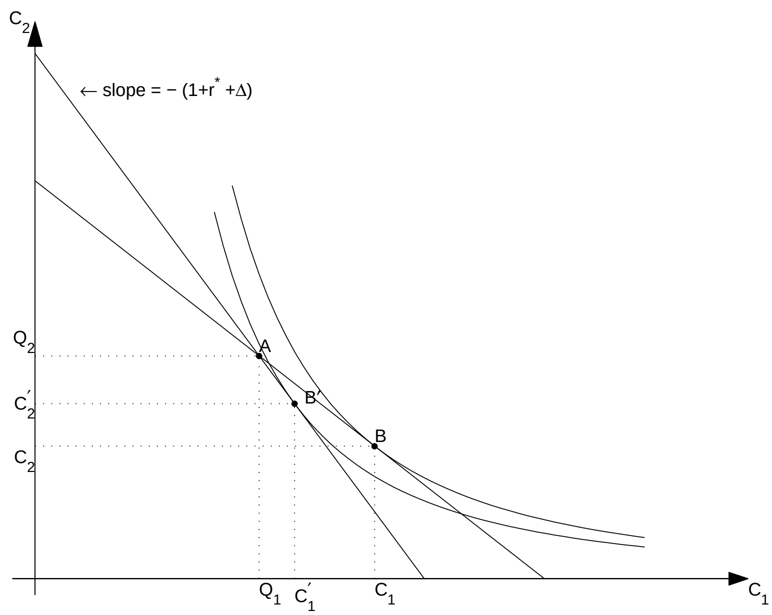
Comparing this lifetime budget constraint with the one given in equation (2.4), it is clear that terms of trade shocks are just like output shocks. Thus, in response to a transitory terms of trade deterioration (a transitory decline in TT), the economy will not adjust consumption much and instead will borrow on the international capital market, which will result in a current account deficit. On the other hand, in response to a permanent terms of trade deterioration (i.e., a fall in both $TT_1$ and $TT_2$), the country is likely to adjust consumption down, with little change in the trade balance or the current account.

### 2.3.4 World Interest Rate Shocks

An increase in the world interest rate, $r^*$, has two potentially opposing effects on consumption in period 1. On the one hand, an increase in the interest rate makes savings more attractive because the rate of return on foreign assets is higher. This effect is referred to as the substitution effect, because it induces people to substitute future for present consumption through saving. By the substitution effect, a rise in the interest rate induces consumption in period 1 to decline and therefore the current account to improve. On the other hand, an increase in the interest rate makes debtors poorer and creditors richer. This is called the income effect. By the income effect, an increase in the interest rate leads to a decrease in consumption in period 1 if the country is a debtor, reinforcing the substitution effect, and to an increase in consumption if the country is a creditor, offsetting (at least in part) the substitution effect. We will assume that the substitution effect is stronger than the income effect, so that savings increases in response to an increase in interest rates. Therefore, an increase in the world interest rate, $r^*$, induces a decline in $C_1$ and thus an improvement in the trade balance and the current account in period 1.

Figure 2.8 describes the case of an increase in the world interest rate from $r^*$ to $r^* + \Delta$. We deduced before that the slope of the budget constraint is given by $-(1 + r^*)$. Thus, an increase in $r^*$ makes the budget constraint steeper. Because the household can always consume its endowment (recall

Figure 2.8: Adjustment to a world interest rate shock




that $B_0^*$ is assumed to be zero), point $A$ must lie on both the old and the new budget constraints. This means that in response to the increase in $r^*$, the budget constraint rotates clockwise through point A. The initial optimal consumption point is given by point B, where the household is borrowing in period 1. The new consumption allocation is point B′, which is located west of the original allocation, B. The increase in the world interest rate is associated with a decline in $C_1$ and thus an improvement in the trade balance and the current account in period 1. Note that because the household was initially borrowing, the income and substitution effects triggered by the rise in the interest rate reinforce each other, so savings increase unambiguously.

## 2.4 An algebraic example

Thus far, we have used a graphical approach to analyze the determination of the current account in the two-period economy. We now illustrate, by means of an example, the basic results using an algebraic approach. Let the utility function be of a log-linear type:

$$U(C_1, C_2) = \ln C_1 + \ln C_2,$$

where ln denotes the natural logarithm. In this case the marginal utility of consuming in the first period, $U_1(C_1, C_2)$, is given by

$$U_1(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_1} = \frac{\partial(\ln C_1 + \ln C_2)}{\partial C_1} = \frac{1}{C_1}$$

Similarly, the marginal utility of period 2 consumption, $U_2(C_1, C_2)$ is given by

$$U_2(C_1, C_2) = \frac{\partial U(C_1, C_2)}{\partial C_2} = \frac{\partial(\ln C_1 + \ln C_2)}{\partial C_2} = \frac{1}{C_2}$$

Here we used the fact that the derivative of the function $\ln x$ is $1/x$, that is, $\partial \ln x/\partial x = 1/x$. The household's first-order condition for utility maximization says that at the optimal consumption allocation

$$U_1(C_1, C_2) = (1 + r_1)U_2(C_1, C_2)$$

For the particular functional form for the utility function considered here, the above equilibrium condition becomes

$$\frac{1}{C_1} = (1 + r_1)\frac{1}{C_2} \tag{2.8}$$

Next, consider the intertemporal budget constraint of the economy (2.4):

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}.$$

Define $\bar{Y} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$. The variable $\bar{Y}$ represents the present discounted value of the household's total wealth, which is composed of his initial asset holdings and the stream of income $(Q_1, Q_2)$. Note that the household takes $\bar{Y}$ as given. We can rewrite the above expression as

$$C_1 = \bar{Y} - \frac{C_2}{1 + r_1}. \tag{2.9}$$

Combining this expression with (2.8), yields

$$C_1 = \frac{1}{2}\bar{Y}$$

In period 1, the trade balance is $TB_1 = Q_1 - C_1$ and the current account is $CA_1 = r_0 B_0^* + TB_1$. Using the definition of $\bar{Y}$ and the fact that in equilibrium $r_1 = r^*$, we have that $C_1$, $C_2$, $TB_1$, and $CA_1$ are given by

$$C_1 = \frac{1}{2}\left[(1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r^*}\right]$$

$$C_2 = \frac{1}{2}(1+r^*)\left[(1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r^*}\right]$$

$$TB_1 = \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*}\right] \tag{2.10}$$

$$CA_1 = r_0 B_0^* + \frac{1}{2}\left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*}\right] \tag{2.11}$$

Consider now the effects of temporary and permanent output shocks on the trade balance and the current account. Assume first that income falls temporarily by one unit, that is, $Q_1$ decreases by one and $Q_2$ is unchanged. It follows from (2.10) and (2.11) that the trade balance and the current account both fall by half a unit. This is because consumption in period 1 falls by only half a unit.

Suppose now that income falls permanently by one unit, that is, $Q_1$ and $Q_2$ both fall by one. Then the trade balance and the current account decline by $\frac{1}{2}\frac{r^*}{1+r^*}$. Consumption in period 1 falls by $\frac{1}{2}\frac{2+r^*}{1+r^*}$. For realistic values of $r^*$, the predicted deterioration in the trade balance and current account in response to the assumed permanent negative income shock is close to zero and in particular much smaller than the deterioration associated with the temporary negative income shock. For example, assume that the world interest rate is 10 percent, $r^* = .1$. Then, both the trade balance and the current account in period 1 fall by .046 in response to the permanent output shock and by .5 in response to the temporary shock. That is, the current account deterioration is 10 times larger under a temporary shock than under a permanent one.

Finally, consider the effect of an increase in the world interest rate $r^*$. Clearly, in period 1 consumption falls and both the trade balance and the current account improve. Note that the decline in consumption in period 1 is independent of whether the country is a net foreign borrower or a net foreign lender in period 1. This is because that for the particular preference specification considered in this example, the substitution effect always dominates the income effect.