Chapter 7

Changes in Aggregate Spending and the Real Exchange Rate: The TNT Model

In the Balassa-Samuelson model studied in section 6.1, the production possibility frontier (PPF) is a *straight line*, which means that the slope of the PPF is the same regardless of the level of production of tradables and nontradables. Because in equilibrium the relative price of tradables in terms of nontradables equals the slope of the PPF, it follows that in the Balassa-Samuelson model the real exchange rate is independent of the level of production of tradables and nontradables. In this section, we will study a more realistic version of the model, the TNT model, in which the PPF is a concave function. As a result of this modification, the slope of the PPF, and therefore the relative price P_T/P_N, depends on the composition of output, which in equilibrium will be determined by the level of aggregate spending.

The TNT model has three building blocks: The *production possibility frontier*, which describes the production side of the economy; the *income expansion path*, which summarizes the aggregate demand for goods; and *international borrowing and lending*, which allows agents to shift consumption across time.[1] In subsections 7.1 and 7.2 develop the first two building blocks. Then in subsection 7.3 we characterize a partial equilibrium by studying the

[1]In the Balassa-Samuelson model neither the second nor the third building blocks are needed for the determination of the real exchange rate because in that model the PPF alone determines the real exchange rate.

determination of production, consumption and the real exchange rate in a given period taking as given the level of international borrowing and lending (i.e., taking as given the level of the current account balance). Finally, in subsection 7.4 we consider the general equilibrium of the economy, in which all variables, including the current account, are determined endogenously.

7.1 The production possibility frontier

Consider an economy that produces traded and nontraded goods with labor as the only factor input. Specifically, the production functions are given by

$$Q_T = F_T(L_T) \tag{7.1}$$

$$Q_N = F_N(L_N) \tag{7.2}$$

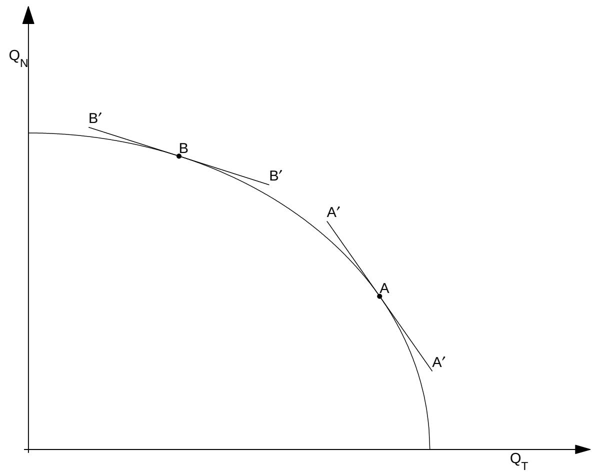
where Q_T and Q_N denote output of traded and nontraded goods, respectively and L_T and L_N denote labor input in the traded and nontraded sectors. The production functions $F_T(\cdot)$ and $F_N(\cdot)$ are assumed to be increasing and concave, that is, $F_T' > 0$, $F_N' > 0$, $F_T'' < 0$, $F_N'' < 0$. The assumption that the production functions are concave means that the marginal productivity of labor is decreasing in the amount of labor input used.[2] The total supply of labor in the economy is assumed to be equal to L, which is a positive constant. Therefore, the allocation of labor across sectors must satisfy the following resource constraint:

$$L_T + L_N = L \tag{7.3}$$

The two production functions along with this resource constraint can be combined into a single equation relating Q_N to Q_T. This relation is the production possibility frontier of the economy, which is shown in figure 7.1. The fact that production displays decreasing marginal productivity of labor implies that the PPF is concave toward the origin. The slope of the PPF, dQ_N/dQ_T, indicates the number of units of nontraded output that must be given up to produce an additional unit of traded output. That is, the slope of the PPF represents the cost of producing an additional unit of tradables in terms of nontradables. As Q_T increases, the PPF becomes steeper, which means that as Q_T increases, it is necessary to sacrify more units of nontraded

[2] Compare these production functions to those of the Balassa-Samuelson model. In the Balassa-Samuelson model, the production functions are $F_T(L_T) = a_T L_T$ and $F_N(L_N) = a_N L_N$. Thus, in that model $F_T' = a_T > 0$ and $F_N' = a_N > 0$, which means that the marginal product of labor is constant in both sectors, or, equivalently, that $F_T'' = F_N'' = 0$.

Figure 7.1: The production possibility frontier (PPF): the case of decreasing marginal productivity of labor



output to increase traded output by one unit. The slope of the PPF is given by the ratio of the marginal products of labor in the two sectors, that is,

$$\frac{dQ_N}{dQ_T} = -\frac{F'_N(L_N)}{F'_T(L_T)} \tag{7.4}$$

This expression makes it clear that the reason why the PPF becomes steeper as Q_T increases is that as Q_T increases so does L_T and thus the marginal productivity of labor in the traded sector, $F'_T(L_T)$ becomes smaller, while the marginal productivity of labor in the nontraded sector, $F'_N(L_N)$, increases as Q_N and L_N decline.

The slope of the PPF can be derived as follows. Differentiate the resource constraint (7.3) to get

$$dL_T + dL_N = 0$$

or

$$\frac{dL_N}{dL_T} = -1$$

This expression says that, because the total amount of labor is fixed, any increase in labor input in the traded sector must be offset by a one-for-one reduction of labor input in the nontraded sector. Now differentiate the production functions (7.1) and (7.2)

$$\begin{aligned} dQ_T &= F_T'(L_T)dL_T \\ dQ_N &= F_N'(L_N)dL_N \end{aligned}$$

Taking the ratio of these two equations and using the fact that $dL_N/dL_T = -1$ yields equation (7.4).

The slope of the PPF indicates how many units of nontradables it costs to produce one additional unit of tradables. In turn, the relative price of tradables in terms of nontradables, P_T/P_N, measures the relative revenue of selling one unit of traded good in terms of nontraded goods. Profit-maximizing firms will choose a production mix such that the relative revenue of selling an additional unit of tradables in terms of nontradables equals the relative cost of tradables in terms of nontradables. That is, firms will produce at a point at which the slope of the PPF equals (minus) the relative price of tradables in terms of nontradables:

$$\frac{F_N'(L_N)}{F_T'(L_T)} = \frac{P_T}{P_N} \tag{7.5}$$

Suppose that the real exchange rate, P_T/P_N is given by minus the slope of the line $A'A'$, which is $-P_T^o/P_N^o$ in figure 7.1. Then firms will choose to produce at point A, where the slope of the PPF is equal to the slope of $A'A'$. Consider now the effect of a real exchange rate appreciation, that is, a decline in P_T/P_N.[3] The new relative price is represented by the slope of the line $B'B'$, which is flatter than $A'A'$. In response to the decline in the relative price of tradables in terms of nontradables, firms choose to produce less tradables and more nontradables. Specifically, the new production mix is given by point B, located northwest of point A.

The optimality condition (7.5) can be derived more formally as follows. Consider the problem faced by a firm in the traded sector. Its profits are given by revenues from sales of tradables, $P_T F_T(L_T)$, minus the cost of production, wL_T, where w denotes the wage rate, that is,

$$\text{profits in the traded sector} = P_T F_T(L_T) - wL_T$$

The firm will choose an amount of labor input that maximizes its profits. That is, it will choose L_T such that

$$P_T F_T'(L_T) - w = 0.$$

[3]Note that here we use the term ”real exchange rate” to refer to the relative price of tradables in terms of nontradables, P_T/P_N.

This first-order condition is obtained by taking the derivative of profits with respect to L_T and setting it equal to zero. The first-order condition says that the firm will equate the value of the marginal product of labor to the marginal cost of labor, w. A similar relation arises from the profit-maximizing behavior of firms in the nontraded sector:

$$P_N F_N'(L_N) - w = 0$$

Combining the above two first-order conditions to eliminate w yields equation (7.5).

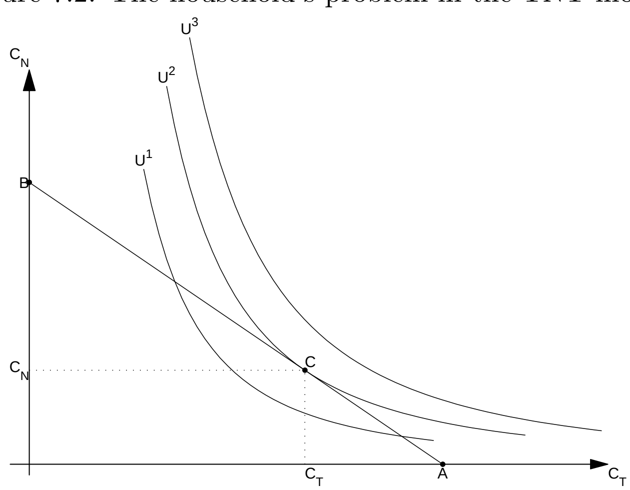
7.2 The income expansion path

Consider now the household's demand for tradable and nontradable consumption. In each period, households derive utility from consumption of traded and nontraded goods. In particular, their preferences are described by the following single-period utility function

$$U(C_T, C_N) \tag{7.6}$$

where $U(\cdot, \cdot)$ is increasing in both arguments. Figure 7.2 shows the indif-

Figure 7.2: The household's problem in the TNT model



ference curves implied by the utility function given in equation (7.6). The indifference curves are as usual downward sloping and convex toward the origin reflecting the fact that households like both goods and that the marginal rate of substitution of tradables for nontradables (the slope of the indifference curves) is decreasing in C_T. Also, because more is preferred to less, the level of utility increases as one moves northeast in the space (C_T, C_N). Thus, for example, in figure 7.2 the level of utility is higher on the indifference curve U^3 than on the indifference curve U^1.

Suppose the household has decided to spend the amount Y on consumption. How will the household allocate Y to purchases of each of the two goods? The household's budget constraint is given by

$$P_T C_T + P_N C_N = Y. \tag{7.7}$$

This constraint says that total expenditures on traded and nontraded consumption purchases must equal the amount the household chose to spend on consumption this period, Y. In figure 7.2 the budget constraint is given by the straight line connecting points A and B. If the household chooses to consume no nontraded goods, then it can consume Y/P_T units of traded goods (point A in the figure). On the other hand, if the household chooses to consume no traded goods, it can consume Y/P_N units of nontraded goods (point B in the figure). The slope of the budget constraint is given by $-P_T/P_N$.

The household chooses C_T and C_N so as to maximize its utility function (7.6) subject to its budget constraint (7.7). The maximum attainable level of utility is reached by consuming a basket of goods on an indifference curve that is tangent to the budget constraint, point C in the figure. At point C, the slope of the indifference curve equals the slope of the budget constraint. To derive this result algebraically, solve (7.7) for C_N and use the resulting expression, $C_N = Y/P_N - P_T/P_N C_T$, to eliminate C_N from (7.6). Then the household's problem reduces to choosing C_T so as to maximize

$$U\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N}C_T\right)$$

The first-order condition of this problem is obtained by taking the derivative with respect to C_T and equating it to zero:

$$U_T\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N}C_T\right) - \frac{P_T}{P_N}U_N\left(C_T, \frac{Y}{P_N} - \frac{P_T}{P_N}C_T\right) = 0$$

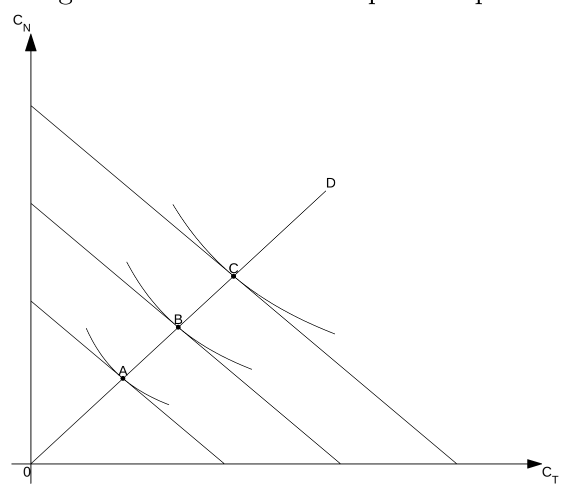
where $U_T(\cdot,\cdot)$ and $U_N(\cdot,\cdot)$ denote the partial derivatives of the utility function with respect to its first and second argument, respectively (or the marginal utilities of consumption of tradables and nontradables). Rearranging terms and using the fact that $Y/P_N - P_T/P_N C_T = C_N$ yields:

$$\frac{U_T(C_T, C_N)}{U_N(C_T, C_N)} = \frac{P_T}{P_N} \tag{7.8}$$

The left hand side of this expressions is (minus) the slope of the indifference curve (also known as the marginal rate of substitution between traded and nontraded goods). The right hand side is (minus) the slope of the budget constraint.

Consider the household's optimal consumption choice for different levels of income. Figure 7.3 shows the household's budget constraint for three

Figure 7.3: The income expansion path

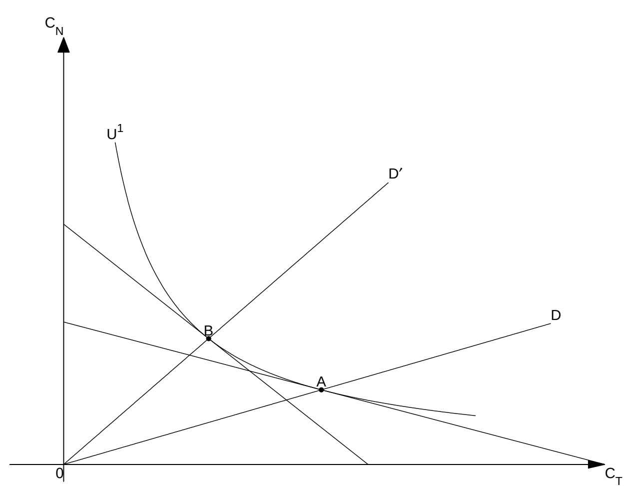


different levels of income, Y_1, Y_2, and Y_3, where $Y_1 < Y_2 < Y_3$. As income increases, the budget constraint shifts to the right in a parallel fashion. It shifts to the right because given for any given level of consumption of one of the goods, an increase in income allows the household to consume more of the other good. The shift is parallel because the relative price between tradables and nontradables is assumed to be unchanged (recall that the slope of the budget constraint is $-P_T/P_N$). We will assume that both goods are normal, that is, that in response to an increase in income, households choose to increase consumption of both goods. This assumption implies that the optimal consumption basket associated with the income level Y_2 (point B in the figure) contains more units of both tradable and nontradable goods than the consumption bundle associated with the lower income Y_1 (point A in the figure), that is, point B is located northeast of point A. Similarly, consumption of both traded and nontraded goods is higher when income is equal to Y_3 (point C in the figure) than when income is equal to Y_2. The *income expansion path* (IEP) is the locus of optimal consumption baskets corresponding to different levels of income, holding constant the relative price of traded and nontraded goods. Clearly, points A, B, and C must lie on the same income expansion path given by the line $\overline{\text{OD}}$ in figure 7.3.

Income expansion paths have four important characteristics: First, if both goods are normal, then income expansion paths are upward sloping. Second, income expansion paths must begin at the origin. This is because if income is nil, then consumption of both goods must be zero. Third, at the point of intersection with a given IEP, all indifference curves have the same slope. This is because each IEP is constructed for a given relative price P_T/P_N, and because at the optimal consumption allocation, the slope of the indifference curve must be equal to the relative price of the two goods. Fourth, an increase in the relative price of traded in terms of nontraded goods, P_T/P_N, produces a counterclockwise rotation of the IEP.

The intuition behind this last characteristic is that if the relative price of tradables in terms of nontradables goes up, households consume relatively less tradables and more nontradables. Figure 7.4 shows two income expan-

Figure 7.4: The income expansion path and a depreciation of the real exchange rate

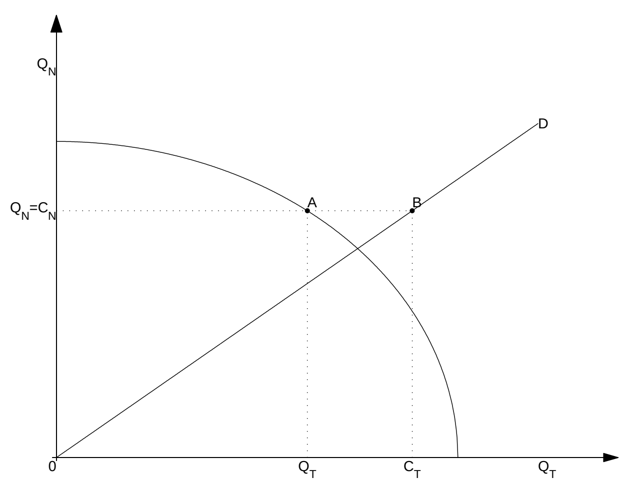


sion paths, $\overline{\text{OD}}$ and $\overline{\text{OD}'}$. The relative price underlying $\overline{\text{OD}}$ is lower than the relative price underlying $\overline{\text{OD}'}$. To see this, consider the slope of any indifference curve as it intersects each of the two IEPs. Take for example the indifference curve U^1 in figure 7.4. At the point of intersection with $\overline{\mathrm{OD}}$ (point A in the figure), U^1 is flatter than at the point of intersection with $\overline{\mathrm{OD}'}$ (point B). Because at point A the slope of U^1 is equal to the relative price underlying $\overline{\mathrm{OD}}$, and at point B the slope of U^1 is equal to the relative price underlying $\overline{\mathrm{OD}'}$, it follows that the relative price associated with $\overline{\mathrm{OD}'}$ is higher than the relative price associated with $\overline{\mathrm{OD}}$.

7.3 Partial equilibrium

We can now put together the first two building blocks of the model, the production possibility frontier and the income expansion path, to analyze the determination of production, consumption and the real exchange rate given the trade balance. Figure 7.5 illustrates a partial equilibrium. Suppose

Figure 7.5: Partial Equilibrium



that in equilibrium production takes place at point A on the PPF. The equilibrium real exchange rate, P_T/P_N, is given by the slope of the PPF at point A. Suppose that the IEP corresponding to the equilibrium real exchange rate is the line $\overline{\mathrm{OD}}$. By definition, nontraded goods cannot be imported or exported. Therefore, market clearing in the nontraded sector requires that production equals consumption, that is,

$$C_N = Q_N \tag{7.9}$$

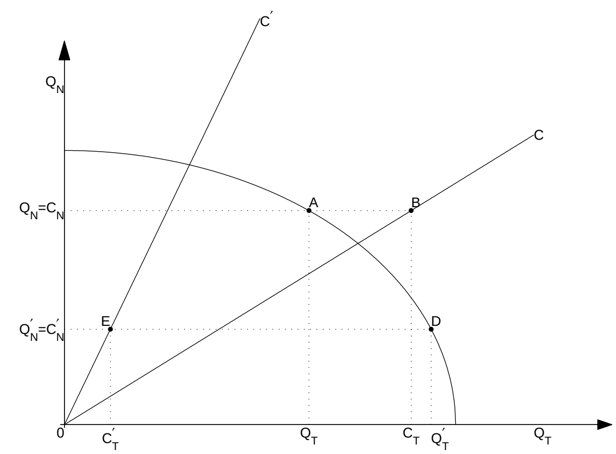
Given consumption of nontradables, the IEP determines uniquely the level of consumption of tradables (point B in the figure). Because our model does not feature investment in physical capital or government purchases, the trade balance is simply given by the difference between production and consumption of tradables,

$$TB = Q_T - C_T \tag{7.10}$$

In the figure, the trade balance is given by the horizontal distance between points A and B. Because in the figure consumption of tradables exceeds production, the country is running a trade balance deficit.

Consider now the effect of a depreciation of the real exchange rate, that is, an increase in P_T/P_N. Figure 7.6 illustrates this situation. The economy

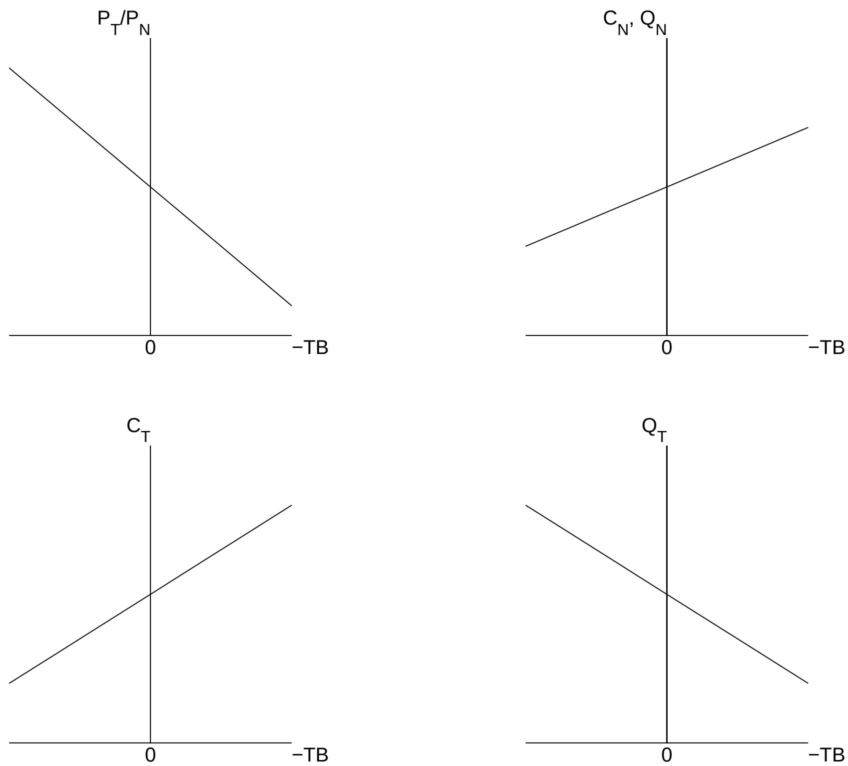
Figure 7.6: Partial equilibrium: a real exchange rate depreciation



is initially producing at point A and consuming at point B. Because in equilibrium the slope of the PPF must equal the real exchange rate, the depreciation of the real exchange rate induces a change in the production mix to a point like D, where the PPF is steeper than at point A. This shift in the composition of production has a clear intuition: as the price of tradables goes up relative to that of nontradables, firms find it profitable to expand production of traded goods at the expense of nontraded goods. On the demand side of the economy, the real exchange rate depreciation causes a counterclockwise rotation in the income expansion path from $\overline{\text{OC}}$ to $\overline{\text{OC'}}$. Having determined the new production position and the new IEP, we can easily determine the new equilibrium consumption basket (point E in the figure) and trade balance (the horizontal distance between points D and E).

Summing up, in response to the real exchange rate depreciation, the economy produces more tradables and less nontradables, and consumes less tradables as well as nontradables. As a result of the expansion in the production of tradables and the contraction in consumption of tradables, the economy ends up generating a smaller trade balance deficit. In fact, in the case shown in figure 7.6 the trade balance becomes positive. Figure 7.7 depicts the relationship between trade deficits, the real exchange rate, consumption, and production.

Figure 7.7: Partial equilibrium: endogenous variables as functions of the trade deficit



The TNT model can help understand the effects of external shocks that force countries to sharply adjust their current accounts. An example of this type of shock is the Debt Crisis of Developing Countries of the early 1980s, which we will discuss in more detail in chapter 8. In 1982, adverse conditions in international financial markets caused credit to dry up for highly indebted countries, particularly in Latin America. As a consequence, debtor countries, which until that moment were running large current account deficits, were all of the sudden forced to generate large trade balance surpluses in order to be able to service their debts. As predicted by the TNT model, the required external adjustment produced sharp real exchange rate depreciations, large contractions in aggregate spending, and costly reallocations of production away from the nontraded sector and toward the traded sector. Table 7.1 illustrates the effect of the Debt Crisis on Chile's trade balance and real exchange rate. In terms of the TNT model, the intuition

Table 7.1: Chile, trade balance and real exchange rate depreciation, 1979-1985

Year	Δe %	$\frac{TB}{GDP}$ %
1979		-1.7
1980		-2.8
1981		-8.2
1982	20.6	0.3
1983	27.5	5.0
1984	5.1	1.9
1985	32.6	5.3

behind the effect of the Debt Crisis on the affected developing countries is clear. In response to the shutdown of external credit, countries needed to generate trade balance surpluses to pay interest and principal on existing foreign debt. In order to generate a trade balance surplus, aggregate spending must decline. Given the relative price of tradables in terms of nontradables, P_T/P_N, households will cut consumption of both traded and nontraded goods. At the same time, given the relative price of tradables in terms of nontradables, production of nontradables should be unchanged. This means that an excess supply of nontradables would emerge. The only way that the market for nontradables can clear is if the relative price of nontradables falls—that is, if the real exchange rate depreciates—inducing firms to produce less nontradables and households to consume more nontradables.

The tools developed thus far allow us to determine all variables of interest given the trade deficit, but do not tell us how the trade deficit itself is determined. Another way of putting this is that our model has more variables than equations. The equilibrium conditions of our model are: equations (7.1), (7.2), and (7.3) describing the PPF, equation (7.5), which ensures that the real exchange rate equals the slope of the PPF, equation (7.8) describing the IEP, equation (7.9), which guarantees market clearing in the nontraded sector, and equation (7.10), which defines the trade balance. These are 7 equations in 8 unknowns: Q_N, Q_T, L_N, L_T, C_N, C_T, TB, and P_T/P_N. To "close" the model, we need a theory to determine TB. More specifically, we need a theory that explains households' consumption decisions over time. In the next section, we merge the *static* partial equilibrium model developed in this section with the *intertemporal* approach to the current account studied in earlier chapters to obtain a *dynamic general equilibrium* model.

7.4 General equilibrium

To determine the equilibrium level of the trade balance, we introduce an intertemporal dimension to the TNT model. Assume that households live for two periods and have preferences described by the following intertemporal utility function

$$U(C_{T1}, C_{N1}) + \beta U(C_{T2}, C_{N2}),$$

where C_{T1} and C_{N1} denote, respectively, consumption of tradables and nontradables in period 1, and C_{T2} and C_{N2} denote the corresponding variables in period 2. The function $U(\cdot,\cdot)$ is the single period utility function given in (7.6), and $0 < \beta < 1$ is a constant parameter, called subjective discount factor, which determines the value households assign to future utility.

In the previous section, we deduced that, all other things constant, in equilibrium both C_T and C_N are increasing functions of the trade deficit, $-TB$ (see figure 7.7). Thus, we can define an *indirect* utility function $\tilde{U}(-TB) \equiv U(C_T, C_N)$ with C_T and C_N replaced by increasing functions of $-TB$. Clearly, the indirect utility function is increasing in $-TB$, because both C_T and C_N are increasing in $-TB$. We can therefore write the intertemporal utility function as

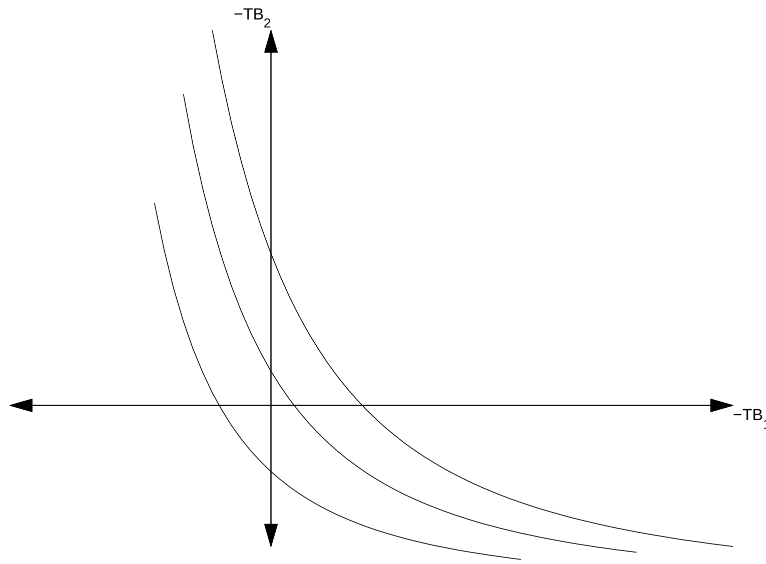
$$\tilde{U}(-TB_1) + \beta\tilde{U}(-TB_2) \tag{7.11}$$

Figure 7.8 shows the indifference curves associated with the indirect utility function (7.11). The indifference curves have the conventional form. They are downward sloping and convex to the origin. As one moves northeast in the space $(-TB_1, -TB_2)$ utility increases.

The household's budget constraint in period 1 is given by

$$C_{T1} + \frac{P_{N1}}{P_{T1}} C_{N1} + B_1^* = (1 + r_0)B_0^* + Q_{T1} + \frac{P_{N1}}{P_{T1}} Q_{N1}$$

Figure 7.8: The indirect utility function: indifference curves



The right hand side of this expression represents the sources of wealth of the household measured in terms of tradables. The households initial asset holdings including including interest are $(1 + r_0)B_0^*$, where B_0^* are initial holdings of foreign bonds denominated in units of traded goods, and r_0 is the return on the initial holdings of foreign bonds. The second source of wealth is the value of output in period 1, $Q_{T1} + (P_{N1}/P_{T1})Q_{N1}$, measured in terms of tradables. Note that we are measuring nontraded output in terms of tradables by multiplying it by the relative price of nontradables in terms of tradables. The left hand side of the budget constraint represents the uses of wealth. The household allocates its wealth to purchases of consumption goods, $C_{T1} + \frac{P_{N1}}{P_{T1}}C_{N1}$, and to purchases of foreign bonds, B_1^*. In equilibrium the market clearing condition in the nontraded sector requires that consumption of nontradables be equal to production of nontradables, that is, $C_{N1} = Q_{N1}$ (equation (7.9)). In addition, we have that $TB_1 = Q_{T1} - C_{T1}$ (equation (7.10)). Thus, the household's budget constraint in period 1 can be written as

$$-TB_1 + B_1^* = (1 + r_0)B_0^*$$

Similarly, in period 2 the budget constraint takes the form

$$-TB_2 + B_2^* = (1 + r_1)B_1^*,$$

where r_1 denotes the domestic interest rate paid on holdings of the foreign bond between periods 1 and 2. Foreign bonds are measured in terms of tradables. Thus, r_1 is the real interest rate in terms of tradables. [4] We will assume that the economy is small and that there is free capital mobility, so that the domestic interest rate on tradables must be equal to the world interest rate, r^*, that is,

$$r_1 = r^*.$$

By the no-Ponzi-game constraint $B_2^* \geq 0$ and the fact that no household is willing leave outstanding assets in period 2, we have

$$B_2^* = 0$$

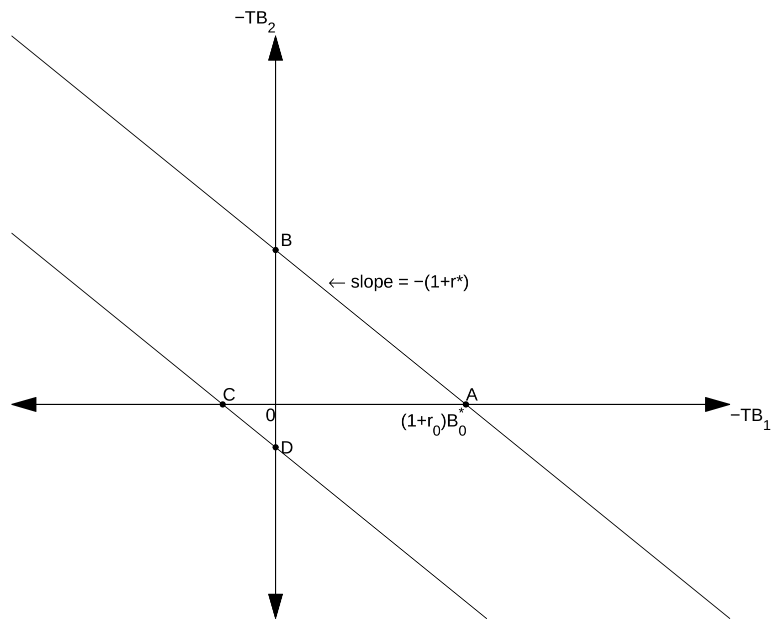
Combining the above four equations to eliminate B_1^*, B_2^*, and r_1, we get the following lifetime budget constraint

$$-TB_1 - \frac{TB_2}{1+r^*} = (1+r_0)B_0^* \tag{7.12}$$

This budget constraint says that the present discounted value of current and future trade deficits must be equal to the household's initial foreign asset holdings including interest payments. This way of writing the lifetime budget constraint should be familiar from earlier lectures. Indeed, we derived an identical expression in the context of a single-good, endowment economy (equation (2.7)). We also derived a budget constraint of this type in the context of an infinite horizon economy (equation (**??**)). Figure 7.9 shows the lifetime budget constraint (7.12). The slope of the budget constraint is negative and given by $-(1+r^*)$. If $-TB_2 = 0$, then in period 1 the economy can run a trade deficit equal to its entire initial wealth, that is, $-TB_1 = (1+r_0)B_0^*$ (point A in the figure). Alternatively, if $-TB_1 = 0$, then $-TB_2 = (1+r^*)(1+r_0)B_0^*$ (point B). The fact that at point A the trade deficit in period 1, $-TB_1$, is positive means initial asset holdings are positive ($(1+r_0)B_0^* > 0$). But this need not be the case. If the country was an initial debtor ($(1+r_0)B_0^* < 0$), then the budget constraint would be

[4] The interest rate in terms of tradables indicates how many units of tradables one receives next periods for each unit of tradables invested today. On the other hand, the interest rate in terms of nontradables represents the amount of nontradables one receives tomorrow per unit of nontradables invested today, and is given by $(1+r_1)(P_{N1}/P_{T1})/(P_{N2}/P_{T2})$. To see why this is so, note that 1 unit of nonntradables in period 1 buys P_{N1}/P_{T1} units of tradables in period 1, which can be invested at the rate r_1 to get $(1+r_1)P_{N1}/P_{T1}$ units of tradables in period 2. In turn each unit of tradables in period 2 can be exchanged for P_{T2}/P_{N2} units of nontradables in that period.

Figure 7.9: The intertemporal budget constraint



a line like the one connecting points C and D. In this case, point C is on the negative range of the horizontal axis indicating that even if the trade balance is zero in period 2, the country must generate a trade surplus in period 1 in order to pay back its initial debt.

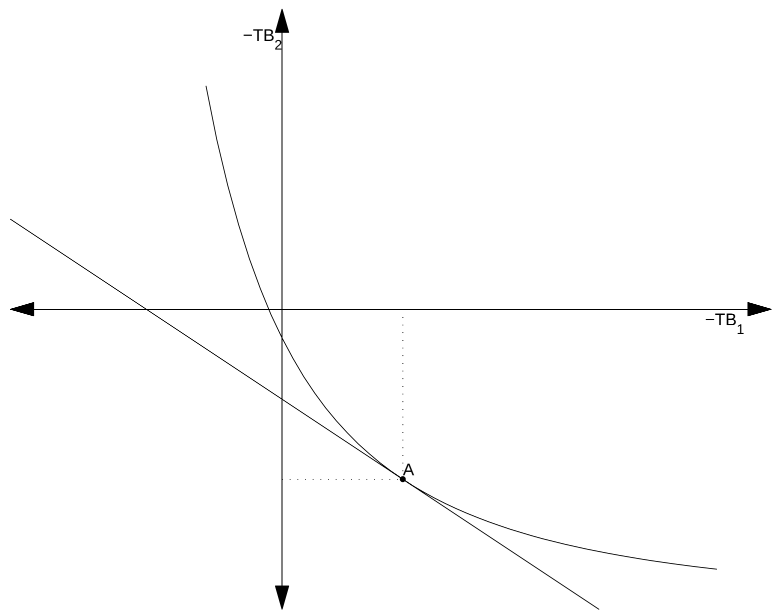
In equilibrium, households choose trade deficits in periods 1 and 2 so as to maximize their lifetime utility. This situation is attained at a point on the budget constraint that is tangent to an indifference curve (point A in figure 7.10). This implies that at the equilibrium allocation, the slope of the indifference curve is equal to the slope of the budget constraint. To derive this result formally, solve the budget constraint (7.12) for $-TB_1$ and use the result to eliminate $-TB_1$ from the indirect utility function (7.11), which yields

$$\tilde{U}\left((1+r_0)B_0^* - \frac{-TB_2}{1+r^*}\right) + \beta\tilde{U}(-TB_2).$$

To find the optimal level of the trade deficit in period 2, take the derivative of this expression with respect to $-TB_2$ and set it equal to zero, to get

$$\tilde{U}'\left((1+r_0)B_0^* - \frac{-TB_2}{1+r^*}\right)\left(\frac{-1}{1+r^*}\right) + \beta\tilde{U}'(-TB_2) = 0$$

Figure 7.10: General equilibrium



Rearranging terms and taking into account that $(1+r_0)B_0^* - (-TB_2)/(1+r^*) = -TB_1$ we obtain

$$\frac{\tilde{U}'(-TB_1)}{\beta\tilde{U}'(-TB_2)} = 1 + r^*. \tag{7.13}$$

The left hand side of this equation is (minus) the slope of the indifference curve, and the right hand side is (minus) the slope of the budget constraint.

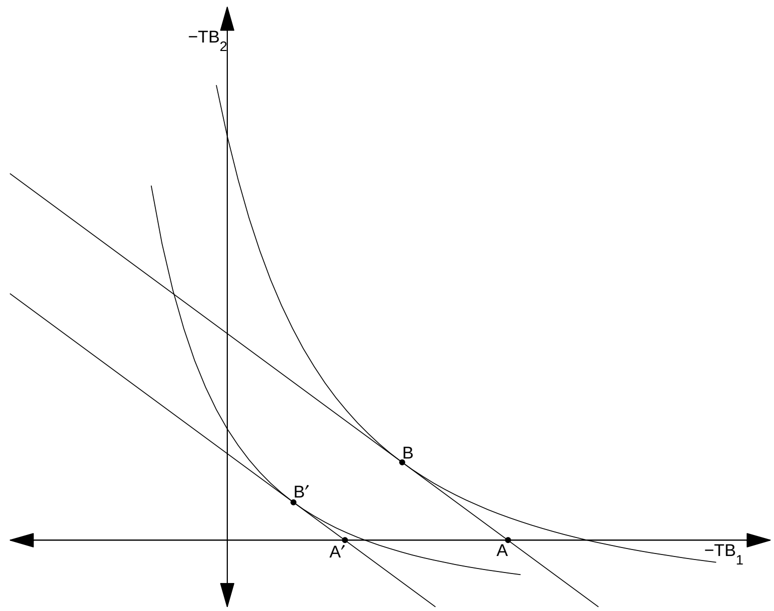
With this optimality condition we have "closed" the model. By closing the model we mean that we now have as many equilibrium conditions as we have endogenous variables. To recapitulate, in the previous subsection we obtained 7 equilibrium conditions for each period (equations (7.1), (7.2), (7.3), (7.5), (7.8), (7.9), and (7.10)) and 8 unknowns for each period (Q_N, Q_T, L_N, L_T, C_N, C_T, TB, and P_T/P_N). In this subsection, we obtained 2 additional equilibrium conditions, equations (7.12) and (7.13), by studying the intertemporal choice problem of the household.[5] Therefore, we now have 16 equations in 16 unknowns, so that the model is closed. In the next subsection we put the model to work by using it to address a number of real life questions.

[5]Note that equations (7.12) and (7.13) do not introduce any additional unknowns.

7.5 Wealth shocks and the real exchange rate

Consider the effect of a decline in a country's net foreign asset position on the real exchange rate and the trade balance. Figure 7.11 depicts the

Figure 7.11: A negative wealth shock



situation of a country that has a positive initial net foreign asset position ($(1 + r_0)B_0^*$) given by point A. The equilibrium is given by point B where the intertemporal budget constraint is tangent to an indifference curve. A decline in the initial net foreign asset position causes a parallel shift in the budget constraint to the left. In the figure, the change in the initial wealth position is given by the distance between points A and A'. The new equilibrium is given by point B', where the trade deficits in both periods are lower. The intuition behind this result is straightforward: as the country becomes poorer it must reduce aggregate spending. Households choose to adjust in both periods because in that way they achieve a smoother path of consumption over time.

Having established the effect of the wealth shock on the trade balance, we can use figure 7.7 to deduce the response of the remaining endogenous variables of the model. The negative wealth effect produces a decline in consumption of tradables and nontradables in both periods. This result makes sense, given that the economy has become poorer. In addition, the real exchange rate depreciates, or tradables become more expensive relative to nontradables. This change in relative prices is necessary in order to induce firms to produce less nontradables when the demand for this type of good falls. Finally, output increases in the traded sector and declines in the nontraded sector. Thus, the improvement in the trade balance is the result of both a decline in consumption and an expansion in production of tradables.

Wealth shocks provide an example of long-lasting deviations from PPP that arise even if productivity is not changing, and thus represent an alternative explanation of movements in the real exchange rate to the one offered by the Balassa-Samuelson model.

Are the predictions of the TNT model consistent with the observed response of countries that faced large wealth shocks? An example of a large negative wealth shock is World War II. For example, in Great Britain large military spending and structural damage wiped out much of the country's net foreign asset position and resulted in a protracted depreciation of the pound vis-à-vis the U.S. dollar.

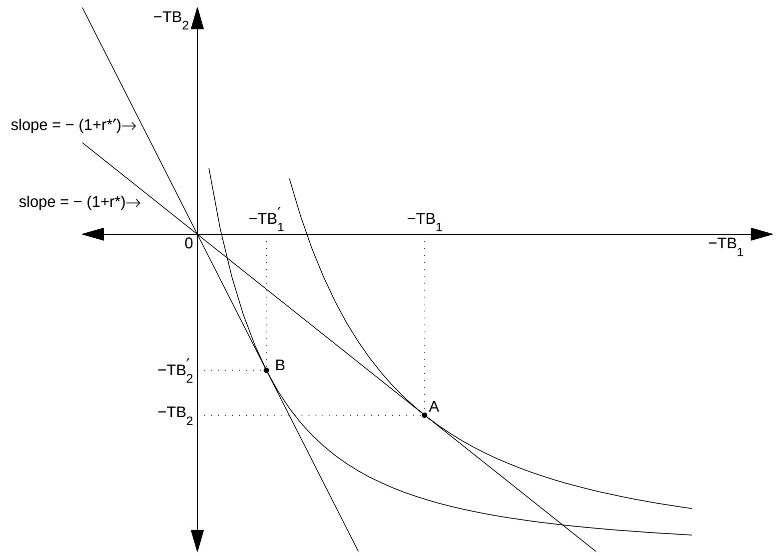
7.6 World interest rate shocks

It has been argued that in developing countries, variations in the real exchange rate are to a large extent due to movements in the world interest rate. For example, Guillermo Calvo, Leonardo Leiderman, and Carmen Reinhart studied the commovement between real exchange rates and U.S. interest rates for ten Latin American countries between 1988 and 1992.[6] They find that around half of the variance in real exchange rates can be explained by variations in U.S. interest rates. In particular, they find that in periods in which the world interest rate is relatively low, the developing countries included in their study experience real exchange rate appreciations. Conversely, periods of high world interest rates are associated with depreciations of the real exchange rate.

Is the TNT model consistent with the observed negative correlation between interest rates and the real exchange rate? Consider a small open economy, which, for simplicity, is assumed to start with zero initial wealth. Suppose further that the country is borrowing in period 1. The situation is illustrated in figure 7.12. The budget constraint crosses the origin, reflect-

[6] G. Calvo, L. Leiderman, and C. Reinhart, "Capital Inflows and Real Exchange Rate Appreciation in Latin America: The Role of External Factors," *International Monetary Fund Staff Papers,* Vol. 40, March 1993, 108-151.

Figure 7.12: An increase in the world interest rate



ing the fact that the initial net foreign asset position is nil. In the initial situation, the world interest rate is r^*. The equilibrium allocation is given by point A. The country is running a trade balance deficit in period 1 and a surplus in period 2. Suppose now that the world interest rate increases from r^* to $r^{*\prime} > r^*$. The higher interest rate causes a clockwise rotation of the budget constraint. The new equilibrium is point B, where the steeper budget constraint is tangent to an indifference curve. At point B, the economy is running a smaller trade deficit in period 1 than at point A. The improvement in the trade balance is the consequence of two reinforcing effects. First, the increase in the interest rate produces a substitution effect that induces households to postpone consumption and increase savings. Second, because the economy is borrowing in period 1, the increase in the interest rate makes domestic households poorer, thus causing a decline in aggregate spending.

It follows from figure 7.7 that the decline in the trade balance in period 1 caused by the interest rate hike is accompanied by a decline in consumption of tradables and nontradables, an expansion in traded output and a contraction in the nontraded sector. Finally, the real exchange rate depreciates. The TNT model is therefore consistent with the observation that high interest rates are associated with real depreciations of the exchange rate.

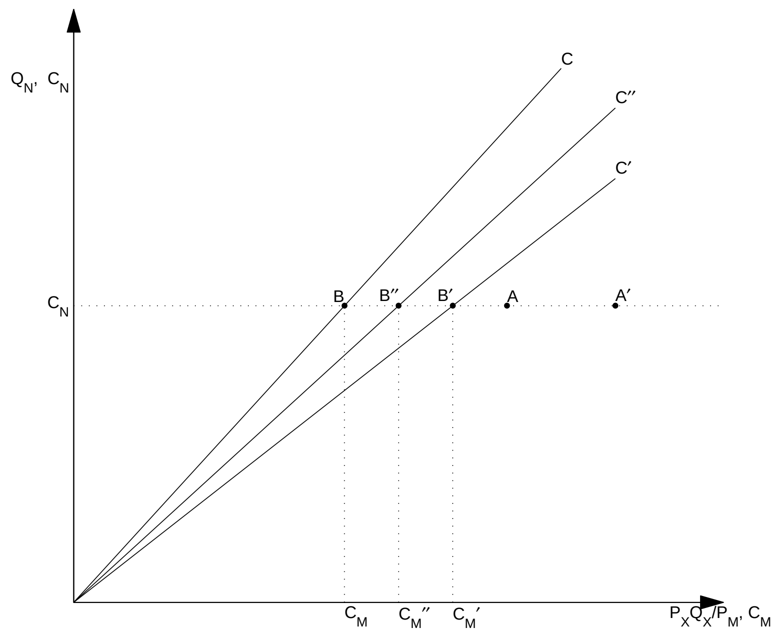
7.7 Terms-of-trade shocks

In order to incorporate terms-of-trade (TOT), we must augment the model to allow for two kinds of traded goods: importables and exportables. We will assume, as we did in our earlier discussion of terms of trade (subsection 2.3.3), that the country's supply of tradables is exported and not consumed, and that all traded goods consumed by domestic households are imported. The distinction between importables and exportables makes matters more complicated. To compensate, we will simplify the model's structure by assuming that the supplies of tradables and nontradables are exogenous. That is, we will study the effects of TOT shocks in an endowment economy. The only difference with our earlier treatment of TOT shocks is therefore the presence of nontradable goods.

Households consume importable goods and nontraded goods, and are endowed with fixed quantities of exportables and nontradables. Let C_M denote consumption of importables and Q_X the endowment of exportable goods. Let P_X/P_M denote the terms of trade, defined as the relative price of exportables in terms of importables. In this endowment economy, the PPF collapses to a single point, namely, the endowment of tradables and nontradables (Q_X, Q_N). Point A in figure 7.13 represents the value of the economy's endowment. In order to measure imports and exports in the same units on the horizontal axis, the endowment of exportables is expressed in terms of importables by multiplying Q_X by the terms of trade, P_X/P_M. Suppose that in equilibrium the economy is running a trade surplus equal to the horizontal distance between points A and B. It follows that the income expansion path, given by the locus $\overline{\text{OC}}$, must cross point B. The real exchange rate, now defined as P_M/P_N, can be read of the slope of the indifference curve at point B.

Suppose that the economy experiences a permanent improvement in the terms of trade, that is, an increase in P_X/P_M in both periods. Because the value of the endowment of exportables went up, point A in figure 7.13 shifts horizontally to the right to point A'. At the same time, the permanent TOT shock is likely to have a negligible effect on the trade balance. The reason is that a permanent increase in the TOT is equivalent to a permanent positive income shock, to which households respond by increasing consumption in both periods in the same magnitude as the increase in income, thus leaving the trade balance unchanged. The fact that the trade balance is unchanged implies that in the new equilibrium consumption of importables must increase in the same magnitude as the increase in the value of the endowment of tradables. The new consumption point is given by B' in the figure. The

Figure 7.13: An improvement in the terms of trade



distance between A and B is the same as the distance between A' and B'. The new income expansion path must go through point B'. This means that the IEP rotates clockwise, or, equivalently, that the real exchange rate appreciates (P_M/P_N goes down) in response to the improvement in TOT. The intuition behind this result is clear. The permanent increase in income caused by the improvement in TOT induces households to demand more of both goods, importables and nontradables. Because the supply of nontradables is fixed, the relative price of nontradables (the reciprocal of the real exchange rate) must increase to discourage consumption of nontradables, thereby restoring equilibrium in the nontraded sector.

Suppose now that the improvement in the terms of trade is temporary rather than permanent, that is, that P_X/P_M increases only in period 1. In this case, households will try to smooth consumption by saving part of the positive income shock in period 1. As a result the trade balance in period 1 improves. In terms of figure 7.13, the new consumption position, point B", is such that the distance between B" and B is smaller than the distance between A′ and A, reflecting the improvement in the trade balance. Therefore, as in the case of a permanent TOT shock, in response to a temporary TOT shock the IEP shifts clockwise. However, the rotation is smaller than under a permanent TOT shock. Consequently, the real exchange rate appreciation is also smaller under a temporary TOT shock than under a permanent one.