Chapter 9

Monetary Policy and Nominal Exchange Rate Determination

Thus far, we have focused on the determination of *real* variables, such as consumption, the trade balance, the current account, and the real exchange rate. In this chapter, we study the determination of *nominal* variables, such as the nominal exchange rate, the price level, inflation, and the quantity of money.

We will organize ideas around using a theoretical framework (model) that is similar to the one presented in previous chapters, with one important modification: there is a demand for money.

An important question in macroeconomics is why households voluntarily choose to hold money. In the modern world, this question arises because money takes the form of unbacked paper notes printed by the government. This kind of money, one that the government is not obliged to exchange for goods, is called fiat money. Clearly, fiat money is intrinsically valueless. One reason why people value money is that it facilitates transactions. In the absence of money, all purchases of goods must take the form of barter. Barter exchanges can be very difficult to arrange because they require double coincident of wants. For example, a carpenter who wants to eat an ice cream must find an ice cream maker that is in need of a carpenter. Money eliminates the need for double coincidence of wants. In this chapter we assume that agents voluntarily hold money because it facilitates transactions.

9.1 The quantity theory of money

What determines the level of the nominal exchange rate? Why has the Euro been depreciating vis-a-vis the US dollar since its inception in 1999? The quantity theory of money asserts that a key determinant of the exchange rate is the quantity of money printed by central banks.

According to the quantity theory of money, people hold a more or less stable fraction of their income in the form of money. Formally, letting Y denote real income, M^d money holdings, and P the price level (i.e., the price of a representative basket of goods), then

$$M^d = \kappa P \cdot Y$$

This means that the *real* value of money, M^d/P, is determined by the level of real activity of the economy. Let $m^d \equiv M^d/P$ denote the demand for real money balances. The quantity theory of money then maintains that m^d is determined by nonmonetary or real factors such as aggregate output, the degree of technological advancement, etc.. Let M^s denote the nominal money *supply*, that is, M^s represents the quantity of bills and coins in circulation plus checking deposits. Equilibrium in the money market requires that money demand be equal to money supply, that is,

$$\frac{M^s}{P} = m^d \tag{9.1}$$

A similar equilibrium condition has to hold in the foreign country. Let M^{*s} denote the foreign nominal money supply, P^* the foreign price level, and m^{*d} the demand for real balances in the foreign country. Then,

$$\frac{M^{*s}}{P^*} = m^{*d} \tag{9.2}$$

Let E denote the nominal exchange rate, defined as the domestic-currency price of the foreign currency. So, for example, if E refers to the dollar/euro exchange rate, then stands for the number of US dollars necessary to purchase one euro. Let e denote the real exchange rate. As explained in previous chapters, e represents the relative price of a foreign basket of goods in terms of domestic baskets of goods. Formally,

$$e = \frac{E\,P^*}{P}$$

Using this expression along with (9.1) and (9.2), we can express the nominal exchange rate, E, as

$$E = \frac{M}{M^*}\left(\frac{e\ m^*}{m}\right) \tag{9.3}$$

According to the quantity theory of money, not only m and m^* but also e are determined by non-monetary factors. The quantity of money, in turn, depends on the exchange rate regime maintained by the respective central banks. There are two polar exchange rate arrangements: flexible and fixed exchange rate regimes.

9.1.1 Floating (or Flexible) Exchange Rate Regime

Under a floating exchange rate regime, the market determines the nominal exchange rate E. In this case the level of the money supplies in the domestic and foreign countries, M^s and M^{*s}, are determined by the respective central banks and are, therefore, exogenous variables. Exogenous variables are those that are determined outside of the model. By contrast, the nominal exchange rate is an endogenous variable in the sense that its equilibrium value is determined within the model.

Suppose, for example, that the domestic central bank decides to increase the money supply M^s. It is clear from equation (9.3) that, all other things constant, the monetary expansion in the home country causes the nominal exchange rate E to depreciate by the same proportion as the increase in the money supply. (i.e., E increases). The intuition behind this effect is simple. An increase in the quantity of money of the domestic country increases the relative scarcity of the foreign currency, thus inducing an increase in the relative price of the foreign currency in terms of the domestic currency, E. In addition, equation (9.1) implies that when M increases the domestic price level, P, increases in the same proportion as M. An increase in the domestic money supply generates inflation in the domestic country. The reason for this increase in prices is that when the central bank injects additional money balances into the economy, households find themselves with more money than they wish to hold. As a result households try to get rid of the excess money balances by purchasing goods. This increase in the demand for goods drives prices up.

Suppose now that the real exchange rate depreciates, (that is e goes up). This means that a foreign basket of goods becomes more expensive relative to a domestic basket of goods. A depreciation of the real exchange rate can be due to a variety of reason, such as a terms-of-trade shock or the removal of import barriers. If the central bank keeps the money supply unchanged, then by equation (9.3) a real exchange rate depreciation causes a depreciation (an increase) of the nominal exchange rate. Note that e and E increase by the same proportion. The price level P is unaffected because neither M nor m have changed (see equation (9.1)).

9.1.2 Fixed Exchange Rate Regime

Under a fixed exchange rate regime, the central bank determines E by intervening in the money market. So given E, M^{*s}, and $e\,m^{*s}/m^s$, equation (9.3) determines what M^s ought to be in equilibrium. Thus, under a fixed exchange rate regime, M^s is an endogenous variable, whereas E is exogenously determined by the central bank.

Suppose that the real exchange rate, e, experiences a depreciation. In this case, the central bank must reduce the money supply (that is, M^s must fall) to compensate for the real exchange rate depreciation. Indeed, the money supply must fall by the same proportion as the real exchange rate. In addition, the domestic price level, P, must also fall by the same proportion as e in order for real balances to stay constant (see equation (9.1)). This implies that we have a deflation, contrary to what happens under a floating exchange rate policy.

9.2 Fiscal deficits and the exchange rate

The quantity theory of money provides a simple and insightful view of the relationship between money, prices, the nominal exchange rate, and real variables. However, it leaves a number of questions unanswered. For example, what is the effect of fiscal policy on inflation? What role do expectations about future changes in monetary and fiscal policy play for the determination of prices, exchange rates and real balances? To address these questions, it is necessary to use a richer model; one that incorporates a more realistic money demand specification and one that explicitly considers the relationship between monetary and fiscal policy.

In this section, embed a money demand function into a model with a government sector similar to the one used in chapter 4 to analyze the effects of fiscal deficits on the current account. Specifically, we consider a small-open endowment economy with free capital mobility, a single traded good per period, and a government that levies lump-sum taxes to finance government purchases. For simplicity, we assume that there is no physical capital and hence no investment. Domestic output is given as an endowment. Besides the introduction of money demand, a further difference with the economy studied in chapter 4 is that now the economy is assumed to exist not just for 2 periods but for an infinite number of periods. Such an economy is called an *infinite horizon* economy.

We discuss in detail each of the four building blocks that compose our monetary economy: (1) The money demand; (2) Purchasing power parity;

(3) Interest rate parity; and (4) The government budget constraint.

9.2.1 Money demand

In the quantity theory, money demand is assumed to depend only on the level of real activity. In reality, however, the demand for money also depends on the nominal interest rate. In particular money demand is decreasing in the nominal interest rate. The reason is that money is a non-interest-bearing asset. As a result, the opportunity cost of holding money is the nominal interest rate on alternative interest-bearing liquid assets such as time deposits, government bonds, and money market mutual funds. Thus, the higher the nominal interest rate the lower is the demand for real money balances. Formally, we assume a money demand function of the form:

$$\frac{M_t}{P_t} = L(\bar{C}, i_t), \tag{9.4}$$

where $\bar{C}$ denotes consumption and i_t denotes the domestic nominal interest rate in period t. The function L is increasing in consumption and decreasing in the nominal interest rate. We assume that consumption is constant over time. Therefore C does not have a time subscript. We indicate that consumption is constant by placing a bar over C. The money demand function $L(\cdot, \cdot)$ is also known as the *liquidity preference function*. Those readers interested in learning how a money demand like equation (9.4) can be derived from the optimization problem of the household should consult the appendix to this chapter.

9.2.2 Purchasing power parity (PPP)

Because in the economy under consideration there is a single traded good and no barriers to international trade, purchasing power parity must hold. Let P_t be the domestic currency price of the good in period t, P_t^* the foreign currency price of the good in period t, and E_t the nominal exchange rate in period t, defined as the price of one unit of foreign currency in terms of domestic currency. Then PPP implies that in any period t

$$P_t = E_t P_t^*$$

For simplicity, assume that the foreign currency price of the good is constant and equal to 1 ($P_t^* = 1$ for all t). In this case, it follows from PPP that the domestic price level is equal to the nominal exchange rate,

$$P_t = E_t. \tag{9.5}$$

Using this relationship, we can write the liquidity preference function (9.4) as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t), \tag{9.6}$$

9.2.3 The interest parity condition

In this economy, there is no uncertainty and free capital mobility. Thus, the gross domestic nominal interest rate must be equal to the gross world nominal interest rate times the expected gross rate of devaluation of the domestic currency. This relation is called the *uncovered interest parity condition*. Formally, let E^e_{t+1} denote the nominal exchange rate that agents expect at time t to prevail at time $t+1$, and let i_t denote the domestic nominal interest rate, that is, the rate of return on an asset denominated in domestic currency and held from period t to period $t+1$. Then the uncovered interest parity condition is:

$$1 + i_t = (1 + r^*)\frac{E^e_{t+1}}{E_t} \tag{9.7}$$

In the absence of uncertainty, the nominal exchange rate that will prevail at time $t+1$ is known at time t, so that $E^e_{t+1} = E_{t+1}$. Then, the uncovered interest parity condition becomes

$$1 + i_t = (1 + r^*)\frac{E_{t+1}}{E_t} \tag{9.8}$$

This condition has a very intuitive interpretation. The left hand side is the gross rate of return of investing 1 unit of domestic currency in a domestic currency denominated bond. Because there is free capital mobility, this investment must yield the same return as investing 1 unit of domestic currency in foreign bonds. One unit of domestic currency buys $1/E_t$ units of the foreign bond. In turn, $1/E_t$ units of the foreign bond pay $(1+r^*)/E_t$ units of foreign currency in period $t+1$, which can then be exchanged for $(1 + r^*)E_{t+1}/E_t$ units of domestic currency.[1]

[1] Here two comments are in order. First, in chapter 5, we argued that free capital mobility implies that *covered* interest rate parity holds. The difference between covered and uncovered interest rate parity is that covered interest rate parity uses the forward exchange rate F_t to eliminate foreign exchange rate risk, whereas uncovered interest rate parity uses the expected future spot exchange rate, E^e_{t+1}. In general, F_t and E^e_{t+1} are not equal to each other. However, under certainty $F_t = E^e_{t+1} = E_{t+1}$, so covered and uncovered interest parity are equivalent. Second, in chapter 5 we further argued that

9.2.4 The government budget constraint

The government has three sources of income: tax revenues, T_t, money creation, $M_t - M_{t-1}$, and interest earnings from holdings of international bonds, $E_t r^* B^g_{t-1}$, where B^g_{t-1} denotes the government's holdings of foreign currency denominated bonds carried over from period $t-1$ into period t and r^* is the international interest rate. Government bonds, B^g_t, are denominated in foreign currency and pay the world interest rate r^*. The government allocates its income to finance government purchases, $P_t G_t$, where G_t denotes real government consumption of goods in period t, and to changes in its holdings of foreign bonds, $E_t(B^g_t - B^g_{t-1})$. Thus, in period t, the government budget constraint is

$$E_t(B^g_t - B^g_{t-1}) + P_t G_t = T_t + (M_t - M_{t-1}) + E_t r^* B^g_{t-1}$$

The left hand side of this expression represents the government's uses of revenue and the right hand side the sources. Note that B^g_t is not restricted to be positive. If B^g_t is positive, then the government is a creditor, whereas if it is negative, then the government is a debtor.[2] We can express the government budget constraint in real terms by dividing the left and right hand sides of the above equation by the price level P_t. After rearranging terms, the result can be written as

$$B^g_t - B^g_{t-1} = \frac{M_t - M_{t-1}}{P_t} - \left[G_t - \frac{T_t}{P_t} - r^* B^g_{t-1}\right] \tag{9.9}$$

The first term on the right hand side measures the government's real revenue from money creation and is called *seignorage revenue*,

$$\text{seignorage revenue} = \frac{M_t - M_{t-1}}{P_t}.$$

The second term on the right hand side of (9.9) is the difference between government expenditures and income from the collection of taxes and from interest payments on interest-bearing assets. This term is called *real secondary deficit* and we will denote it by DEF_t,

$$DEF_t = (G_t - T_t/P_t) - r^* B^g_{t-1}$$

free capital mobility implies that covered interest parity must hold for *nominal* interest rates. However, in equation (9.7) we used the world *real* interest rate r^*. In the context of our model this is okay because we are assuming that the foreign price level is constant ($P^* = 1$) so that, by the Fisher equation (5.3), the nominal world interest rate must be equal to the real world interest rate ($i^*_t = r^*_t$).

[2]Note that the notation here is different from the one used in chapter 4, where B^g_t denoted the level of government debt.

The difference between government expenditures and tax revenues ($G_t - T_t/P_t$) is called *primary deficit*. Thus, the secondary government deficit equals the difference between the primary deficit and interest income from government holdings of interest bearing assets.

Using the definition of secondary deficit and the fact that by PPP $P_t = E_t$, the government budget constraint can be written as

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t \tag{9.10}$$

This equation makes it transparent that a fiscal deficit ($DEF_t > 0$) must be associated with money creation ($M_t - M_{t-1} > 0$) or with a decline in the government's holdings of assets ($B_t^g - B_{t-1}^g < 0$), or both. To complete the description of the economy, we must specify the exchange rate regime, to which we turn next.

9.2.5 A fixed exchange rate regime

Under a fixed exchange rate regime, the government intervenes in the foreign exchange market in order to keep the exchange rate at a fixed level. Let that fixed level be denoted by E. Then $E_t = E$ for all t. When the government pegs the exchange rate, the money supply becomes an endogenous variable because the central bank must stand ready to exchange domestic for foreign currency at the fixed rate E. Given the nominal exchange rate E, the PPP condition, given by equation (9.5), implies that the price level, P_t, is also constant and equal to E for all t. Because the nominal exchange rate is constant, the expected rate of devaluation is zero. This implies, by the interest parity condition (9.8), that the domestic nominal interest rate, i_t, is constant and equal to the world interest rate r^*. It then follows from the liquidity preference equation (9.6) that the demand for nominal balances is constant and equal to $EL(\bar{C}, r^*)$. Since in equilibrium money demand must equal money supply, we have that the money supply is also constant over time: $M_t = M_{t-1} = EL(\bar{C}, r^*)$. Using the fact that the money supply is constant, the government budget constraint (9.10) becomes

$$B_t^g - B_{t-1}^g = -DEF_t \tag{9.11}$$

In words, when the government pegs the exchange rate, it loses one source of revenue, namely, seignorage. Therefore, fiscal deficits must be entirely financed through the sale of interest bearing assets.

Fiscal deficits and the sustainability of currency pegs

For a fixed exchange rate regime to be sustainable over time, it is necessary that the government displays fiscal discipline. To see this, suppose that the government runs a perpetual secondary deficit, say $DEF_t = DEF > 0$ for all t. Equation (9.11) then implies that government assets are falling over time ($B^g_t - B^g_{t-1} = -DEF < 0$). At some point B^g_t will become negative, which implies that the government is a debtor. Suppose that there is an upper limit on the size of the public debt. Clearly, when the public debt hits that limit, the government is forced to either eliminate the fiscal deficit (i.e., set $DEF = 0$) or abandon the exchange rate peg. The latter alternative is called a *balance of payments crisis.* We will analyze balance of payments crises in more detail in section 9.3.

The fiscal consequences of a devaluation

Consider now the effects of a once-and-for-all devaluation of the domestic currency. By PPP, a devaluation produces an increase in the domestic price level of the same proportion as the increase in the nominal exchange rate. Given the households' holdings of nominal money balances the increase in the price level implies that real balances will decline. Thus, a devaluation acts as a tax on real balances. In order to rebuild their real balances, households will sell part of their foreign bonds to the central bank in return for domestic currency. The net effect of a devaluation is that the private sector is made poorer because it ends up with the same level of real balances but with less foreign assets. On the other hand, the government benefits because it increases its holdings of interest bearing assets.

To see more formally why a once-and-for-all devaluation of the domestic currency generates revenue for the government, assume that in period 1 the government unexpectedly announces an increase in the nominal exchange rate from E to $E' > E$, that is, $E_t = E'$ for all $t \geq 1$. By the PPP condition, equation (9.5), the domestic price level, P_t, jumps up in period 1 from E to E' and remains at that level thereafter. Because the nominal exchange rate is constant from period 1 on, the future rate of devaluation is zero, which implies, by the interest rate parity condition (9.8), that the domestic nominal interest rate is equal to the world interest rate ($i_t = r^*$ for all $t \geq 1$). Because the nominal interest rete was equal to r^* before period 1, it follows that an unexpected, once-and-for-all devaluation has no effect on the domestic nominal interest rate. The reason why the nominal interest rate remains unchanged is that it depends on the *expected future* rather than the *actual* rate of devaluation. In period 0, households did not expect the government to devalue the domestic currency in period 1. Therefore, the expected devaluation rate was zero and the nominal interest rate was equal to r^*. In period 1, households expect no further devaluations of the domestic currency in the future, thus the nominal interest rate is also equal to r^* from period 1 on.

Using the fact that the nominal interest rate is unchanged, the liquidity preference equation (9.6) then implies that in period 1 the demand for nominal money balances increases from $EL(\bar{C}, r^*)$ to $E'L(\bar{C}, r^*)$. This means that the demand for nominal balances must increase by the same proportion as the nominal exchange rate. Consider now the government budget constraint in period 1.

$$B_1^g - B_0^g = \frac{M_1 - M_0}{E'} - DEF_1.$$

The numerator of the first term on the right-hand side, $M_1 - M_0$, equals $E'L(\bar{C}, r^*) - EL(\bar{C}, r^*)$, which is positive. Thus, in period 1 seignorage revenue is positive. In the absence of a devaluation, seignorage revenue would be nil because in that case $M_1 - M_0 = EL(\bar{C}, r^*) - EL(\bar{C}, r^*) = 0$. Therefore, a devaluation increases government revenue in the period in which the devaluation takes place. In the periods after the devaluation, $t = 2, 3, 4, \ldots$, the nominal money demand, M_t, is constant and equal to $M_1 = E'L(\bar{C}, r^*)$, so that $M_t - M_{t-1} = 0$ for all $t \geq 2$ and seignorage revenue is nil.

9.2.6 Equilibrium under a floating exchange rate regime

Under a floating exchange rate regime, the nominal exchange rate is market determined, that is, the nominal exchange rate is an endogenous variable. We will assume that the central bank determines how much money is in circulation each period. Therefore, this monetary/exchange rate regime is exactly the opposite to the one studied in subsection 9.2.5, where the central bank fixed the nominal exchange rate and let the quantity of money be market (or endogenously) determined.

Consider a specific monetary policy in which the central bank expands the money supply at a constant, positive rate μ each period, so that

$$M_t = (1 + \mu)M_{t-1} \tag{9.12}$$

Our goal is to find out how the endogenous variables of the model, such as the nominal exchange rate, the price level, real balances, the domestic nominal interest rate, and so forth behave under the monetary/exchange rate regime specified by equation (9.12). To do this, we will conjecture (or guess) that in equilibrium the nominal exchange rate depreciates at the rate μ. We will then verify that our guess is correct. Thus, we are guessing that

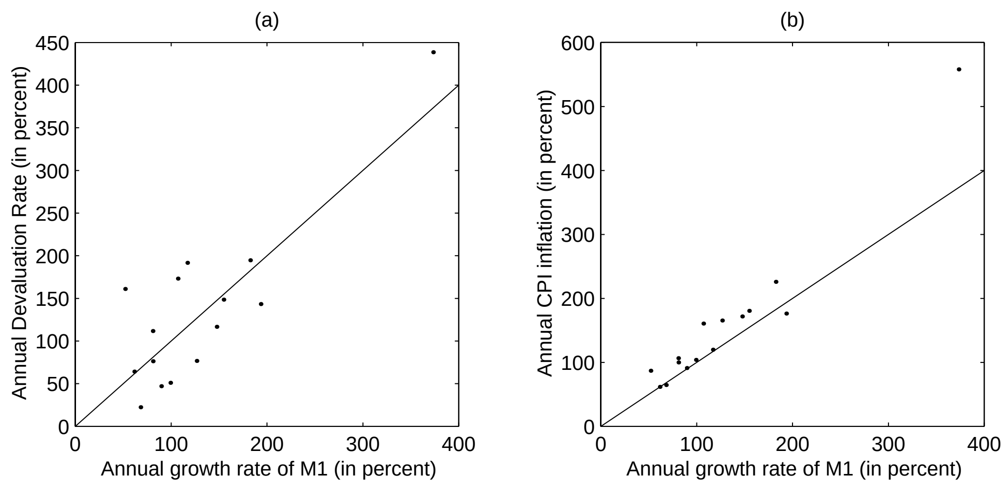
$$\frac{E_{t+1}}{E_t} = 1 + \mu$$

Because PPP holds and the foreign price level is one (i.e., $P_t = E_t$), the domestic price level must also grow at the rate of monetary expansion μ,

$$\frac{P_{t+1}}{P_t} = 1 + \mu.$$

This expression says that, given our guess, the rate of inflation must equal the rate of growth of the money supply. Panels (a) and (b) of figure 9.1 display annual averages of the rate of depreciation of the Argentine cur-

Figure 9.1: Devaluation, inflation, and money growth. Argentina 1975-89



rency vis-à-vis the U.S. dollar, the Argentine money growth rate, and the Argentine inflation rate for the period 1975-1990. The data is roughly consistent with the model in showing that there exists a close positive relationship between these three variables.[3]

[3]Strictly speaking, the model predicts that all points in both figures should lie on a straight line, which is clearly not the case. The reason for this discrepancy may be that the model abstracts from a number of real world factors that affect the relationship between money growth, inflation, and depreciation. For example, in the model we assume that there is no domestic growth, that all goods are traded, that PPP holds, and that foreign inflation is constant.

To determine the domestic nominal interest rate i_t, use the interest parity condition (9.8)

$$1 + i_t = (1 + r^*)\frac{E_{t+1}}{E_t} = (1 + r^*)(1 + \mu),$$

which implies that the nominal interest rate is constant and increasing in μ. When μ is positive, the domestic nominal interest rate exceeds the real interest rate r^* because the domestic currency is depreciating over time. We summarize the positive relationship between i_t and μ by writing

$$i_t = i(\mu)$$

The notation $i(\mu)$ simply indicates that i_t is a function of μ. The function $i(\mu)$ is increasing in μ. Substituting this expression into the liquidity preference function (9.6) yields

$$\frac{M_t}{E_t} = L(\bar{C}, i(\mu)). \tag{9.13}$$

Note that $\bar{C}$ is a constant and that because the money growth rate μ is constant, the nominal interest rate $i(\mu)$ is also constant. Therefore, the right hand side of (9.13) is constant. For the money market to be in equilibrium, the left-hand side of (9.13) must also be constant. This will be the case only if the exchange rate depreciates—grows—at the same rate as the money supply. This is indeed true under our initial conjecture that $E_{t+1}/E_t = 1 + \mu$. Equation (9.13) says that in equilibrium real money balances must be constant and that the higher the money growth rate μ the lower the equilibrium level of real balances.

Let's now return to the government budget constraint (9.10), which we reproduce below for convenience

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t$$

Let's analyze the first term on the right-hand side of this expression, seignorage revenue. Using the fact that $M_t = E_t L(\bar{C}, i(\mu))$ (equation (9.13)), we can write

$$\begin{aligned} \frac{M_t - M_{t-1}}{E_t} &= \frac{E_t L(\bar{C}, i(\mu)) - E_{t-1} L(\bar{C}, i(\mu))}{E_t} \\ &= L(\bar{C}, i(\mu)) \left(\frac{E_t - E_{t-1}}{E_t} \right) \end{aligned}$$

Using the fact that the nominal exchange rate depreciates at the rate μ, that is, $E_t = (1+\mu)E_{t-1}$, to eliminate E_t and E_{t-1} from the above expression, we can write seignorage revenue as

$$\frac{M_t - M_{t-1}}{E_t} = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1+\mu} \right) \tag{9.14}$$

Thus, seignorage revenue is equal to the product of real balances, $L(\bar{C}, i(\mu))$, and the factor $\mu/(1+\mu)$.

The right hand side of equation (9.14) can also be interpreted as the *inflation tax*. The idea is that inflation acts as a tax on the public's holdings of real money balances. To see this, let's compute the change in the real value of money holdings from period $t-1$ to period t. In period $t-1$ nominal money holdings are M_{t-1} which have a real value of M_{t-1}/P_{t-1}. In period t the real value of M_{t-1} is M_{t-1}/P_t. Therefore we have that the inflation tax equals $M_{t-1}/P_{t-1} - M_{t-1}/P_t$, or, equivalently,

$$\text{inflation tax} = \frac{M_{t-1}}{P_{t-1}} \frac{P_t - P_{t-1}}{P_t}$$

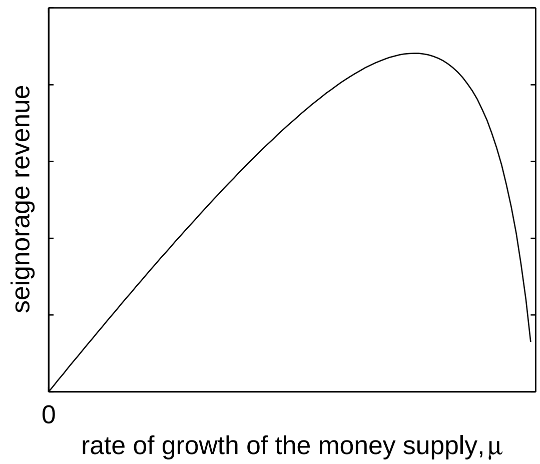
where M_{t-1}/P_{t-1} is the tax base and $(P_t - P_{t-1})/P_t$ is the tax rate. Using the facts that in our model real balances are equal to $L(\bar{C}, i(\mu))$ and that $P_t/P_{t-1} = 1+\mu$, the inflation tax can be written as

$$\text{inflation tax} = L(\bar{C}, i(\mu)) \frac{\mu}{1+\mu},$$

which equals seignorage revenue. In general seignorage revenue and the inflation tax are not equal to each other. They are equal in the special case that real balances are constant over time, like in our model when the money supply expands at a constant rate.

Because the tax base, real balances, is decreasing in μ and the tax rate, $\mu/(1+\mu)$, is increasing in μ, it is not clear whether seignorage increases or decreases with the rate of expansion of the money supply. Whether seignorage revenue is increasing or decreasing in μ depends on the form of the liquidity preference function $L(\cdot, \cdot)$ as well as on the level of μ itself. Typically, for low values of μ seignorage revenue is increasing in μ. However, as μ gets large the contraction in the tax base (the money demand) dominates the increase in the tax rate and therefore seignorage revenue falls as μ increases. Thus, there exists a *maximum level of revenue* a government can collect from printing money. The resulting relationship between the growth rate of the money supply and seignorage revenue has the shape of an inverted-U and is called the *inflation tax Laffer curve* (see figure 9.2).

Figure 9.2: The Laffer curve of inflation



Inflationary finance

We now use the theoretical framework developed thus far to analyze the link between fiscal deficits, prices, and the exchange rate. Consider a situation in which the government is running constant fiscal deficits $DEF_t = DEF > 0$ for all t. Furthermore, assume that the government has reached its borrowing limit and thus cannot finance the fiscal deficits by issuing additional debt, so that $B_t^g - B_{t-1}^g$ must be equal to zero. Under these circumstances, the government budget constraint (9.10) becomes

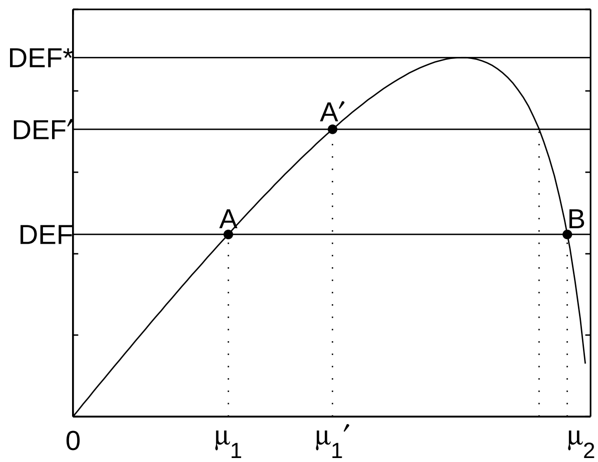
$$DEF = \frac{M_t - M_{t-1}}{E_t}$$

It is clear from this expression, that a country that has exhausted its ability to issue public debt must resort to printing money in order to finance the fiscal deficit. This way of financing the public sector is called *monetization of the fiscal deficit.* Combining the above expression with (9.14) we obtain

$$DEF = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1+\mu} \right) \tag{9.15}$$

Figure 9.3 illustrates the relationship between fiscal deficits and the rate of monetary expansion implied by this equation. The Laffer curve of inflation corresponds to the right hand side of (9.15). The horizontal line plots the left hand side (9.15), or DEF. There are two rates of monetary expansion, μ_1 and μ_2, that generate enough seignorage revenue to finance the fiscal

Figure 9.3: Inflationary finance and the Laffer curve of inflation



deficit DEF. Thus, there exist two equilibrium levels of monetary expansion associated with a fiscal deficit equal to DEF. In the μ_2 equilibrium, point B in the figure, the rates of inflation and of exchange rate depreciation are relatively high and equal to μ_2, whereas in the μ_1 equilibrium, point A in the figure, the rates of inflation and depreciation are lower and equal to μ_1. Empirical studies show that in reality, economies tend to be located on the upward sloping branch of the Laffer curve. Thus, the more realistic scenario is described by point A.

Consider now the effect of an increase in the fiscal deficit from DEF to $DEF' > DEF$. To finance the larger fiscal deficit, the government is forced to increase the money supply at a faster rater. At the new equilibrium, point A', the rate of monetary expansion, $\mu_1{}'$ is greater than at the old equilibrium. As a result, the inflation rate, the rate of depreciation of the domestic currency, and the nominal interest rate are all higher.

The following numerical example provides additional insight on the connection between money creation and fiscal deficits. Suppose that the liquidity preference function is given by:

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{1 + i_t}{i_t} \right)$$

Suppose that the government runs a fiscal deficit of 10% of GDP ($DEF/Q = .1$), that the share of consumption in GDP is 65% ($\bar{C}/Q = .65$), that the world real interest rate is 2.5% per quarter ($r^* = .025$), and that γ is equal to .16. The question is what is the rate of monetary expansion necessary to monetize the fiscal deficit. Combining equations (9.2.6) and (9.15) and using the fact $1 + i_t = (1 + r^*)(1 + \mu)$ we have,

$$DEF = \gamma \bar{C} \frac{(1 + r^*)(1 + \mu)}{(1 + r^*)(1 + \mu) - 1} \frac{\mu}{1 + \mu}$$

Divide the left and right hand sides of this expression by Q and solve for μ to obtain

$$\mu = \frac{r^*(DEF/Q)}{(1 + r^*)(\gamma(\bar{C}/Q) - (DEF/Q))} = \frac{0.025 \times 0.1}{1.025 \times (0.16 \times 0.65 - 0.1)} = 0.61$$

The government must increase the money supply at a rate of 61% per quarter. This implies that both the rates of inflation and depreciation of the domestic currency in this economy will be 61% per quarter. The nominal interest rate is 65% per quarter. At a deficit of 10% of GDP, the Laffer curve is extremely flat. For example, if the government cuts the fiscal deficit by 1% of GDP, the equilibrium money growth rate falls to 16%.

In some instances, inflationary finance can degenerate into hyperinflation. Perhaps the best-known episode is the German hyperinflation of 1923. Between August 1922 and November 1923, Germany experienced an average monthly inflation rate of 322 percent.[4] More recently, in the late 1980s a number of hyperinflationary episodes took place in Latin America and Eastern Europe. One of the more severe cases was Argentina, where the inflation rate averaged 66 percent per month between May 1989 and March 1990.

A hyperinflationary situation arises when the fiscal deficit reaches a level that can no longer be financed by seignorage revenue alone. In terms of figure 9.3, this would be the case if the fiscal deficit would be larger than DEF^*, the level of deficit associated with the peak of the Laffer curve. What happens in practice is that the government is initially unaware of the fact that no rate of monetary expansion will suffice to finance the deficit. In its attempt to close the fiscal gap, the government accelerates the rate of money creation. But this measure is counterproductive because the government has entered the downward sloping side of the Laffer curve. The decline in seignorage revenue leads the government to increase the money supply at an even faster rate. These dynamics turn into a vicious cycle that ends in an accelerating inflationary spiral. The most fundamental step in ending

[4] A fascinating account of four Post World War I European hyperinflations is given in Sargent, "The End of Four Big Inflations," in Robert Hall, editor, *Inflation: Causes and Effects,* The University of Chicago Press, Chicago, 1982.

hyperinflation is to eliminate the underlying budgetary imbalances that are at the root of the problem. When this type of structural fiscal reforms is undertaken and is understood by the public, hyperinflation typically stops abruptly.

Money growth and inflation in a growing economy

Thus far, we have considered the case in which consumption is constant over time.[5] We now wish to consider the case that consumption is growing over time. Specifically, we will assume that consumption grows at a constant rate $\gamma > 0$, that is,

$$C_{t+1} = (1+\gamma)C_t.$$

We also assume that the liquidity preference function is of the form

$$L(C_t, i_t) = C_t l(i_t)$$

where $l(\cdot)$ is a decreasing function.[6] Consider again the case that the government expands the money supply at a constant rate $\mu > 0$. As before, we find the equilibrium by first guessing the value of the depreciation rate and then verifying that this guess indeed can be supported as an equilibrium outcome. Specifically, we conjecture that the domestic currency depreciates at the rate $(1+\mu)/(1+\gamma) - 1$, that is,

$$\frac{E_{t+1}}{E_t} = \frac{1+\mu}{1+\gamma}$$

Our conjecture says that given the rate of monetary expansion, the higher the rate of economic growth, the lower the rate of depreciation of the domestic currency. In particular, if the government wishes to keep the domestic currency from depreciating, it can do so by setting the rate of monetary expansion at a level no greater than the rate of growth of consumption ($\mu \leq \gamma$). By interest rate parity,

$$\begin{aligned}(1+i_t) &= (1+r^*)\frac{E_{t+1}}{E_t} \\ &= (1+r^*)\frac{(1+\mu)}{(1+\gamma)}\end{aligned}$$

[5]Those familiar with the appendix will recognize that the constancy of consumption is a direct implication of our assumption that the subjective discount rate is equal to the world interest rate, that is, $\beta(1+r^*) = 1$. It is clear from (9.19) that consumption will grow over time only if $\beta(1+r^*)$ is greater than 1.

[6]Can you show that this form of the liquidity preference function obtains when the period utility function is given by $\ln C_t + \theta \ln(M_t/E_t)$. Under this particular preference specification find the growth rate of consumption γ as a function of β and $1+r^*$.

This expression says that the nominal interest rate is constant over time. We can summarize this relationship by writing

$$i_t = i(\mu, \gamma), \quad \text{for all } t$$

where the function $i(\mu, \gamma)$ is increasing in μ and decreasing in γ. Equilibrium in the money market requires that the real money supply be equal to the demand for real balances, that is,

$$\frac{M_t}{E_t} = C_t l(i(\mu, \gamma))$$

The right-hand side of this expression is proportional to consumption, and therefore grows at the gross rate $1 + \gamma$. The numerator of the left hand side grows at the gross rate $1 + \mu$. Therefore, in equilibrium the denominator of the left hand side must expand at the gross rate $(1 + \mu)/(1 + \gamma)$, which is precisely our conjecture.

Finally, by PPP and given our assumption that $P_t^* = 1$, we have that the domestic price level, P_t, must be equal to the nominal exchange rate, E_t. It follows that the domestic rate of inflation must be equal to the rate of depreciation of the nominal exchange rate, that is,

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{E_t - E_{t-1}}{E_{t-1}} = \frac{1 + \mu}{1 + \gamma} - 1$$

This expression shows that to the extend that consumption growth is positive the domestic inflation rate is *lower* than the rate of monetary expansion. The intuition for this result is straightforward. A given increase in the money supply that is not accompanied by an increase in the demand for real balances will translate into a proportional increase in prices. This is because in trying to get rid of their excess nominal money holdings households attempt to buy more goods. But since the supply of goods is unchanged the increased demand for goods will be met by an increase in prices. This is a typical case of "more money chasing the same amount of goods." When the economy is growing, the demand for real balances is also growing. That means that part of the increase in the money supply will not end up chasing goods but rather will end up in the pockets of consumers.

9.3 Balance-of-payments crises

A balance of payments, or BOP, crisis is a situation in which the government is unable or unwilling to meet its financial obligations. These difficulties may manifest themselves in a variety of ways, such as the failure to honor the domestic and/or foreign public debt or the suspension of currency convertibility.

What causes BOP crises? Sometimes a BOP crisis arises as the inevitable consequence of unsustainable combinations of monetary and fiscal policies. A classic example of such a policy mix is a situation in which a government pegs the nominal exchange rate and at the same time runs a fiscal deficit. As we discussed in subsection 9.2.5, under a fixed exchange rate regime, the government must finance any fiscal deficit by running down its stock of interest bearing assets (see equation (9.11)). Clearly, to the extent that there is a limit to the amount of debt a government is able to issue, this situation cannot continue indefinitely. When the public debt hits its upper limit the government is forced to change policy. One possibility is that the government stops servicing the debt (i.e., stops paying interest on its outstanding financial obligations), thereby reducing the size of the secondary deficit. This alternative was adopted by Mexico in August of 1982, when it announced that it would be unable to honor its debt commitments according to schedule, marking the beginning of what today is known as the Developing Country Debt Crisis. A second possibility is that the government adopt a fiscal adjustment program by cutting government spending and raising regular taxes and in that way reduce the primary deficit. Finally, the government can abandon the exchange rate peg and resort to monetizing the fiscal deficit. This has been the fate of the vast majority of currency pegs adopted in developing countries. The economic history of Latin America of the past two decades is plagued with such episodes. For example, the currency pegs implemented in Argentina, Chile, and Uruguay in the late 1970s, also known as *tablitas*, ended with large devaluations in the early 1980s; similar outcomes were observed in the Argentine Austral stabilization plan of 1985, the Brazilian Cruzado plan of 1986, the Mexican plan of 1987, and, more recently the Brazilian Real plan of 1994.

An empirical regularity associated with the collapse of fixed exchange rate regimes is that in the days immediately before the peg is abandoned, the central bank looses vast amounts of reserves in a short period of time. The loss of reserves is the consequence of a run by the public against the domestic currency in anticipation of the impending devaluation. The stampede of people trying to massively get rid of domestic currency in exchange for foreign currency is driven by the desire to avoid the loss of real value of domestic currency denominated assets that will take place when the currency is devalued.

The first formal model of the dynamics of a fixed exchange rate collapse is due to Paul R. Krugman of Princeton University.[7] In this section, we will analyze these dynamics using the tools developed in sections 9.2.5 and 9.2.6. These tools will helpful in a natural way because, from an analytical point of view, the collapse of a currency peg is indeed a transition from a fixed to a floating exchange rate regime.

Consider a country that is running a constant fiscal deficit $DEF > 0$ each period. Suppose that in period 1 the country embarks in a currency peg. Specifically, assume that the government fixes the nominal exchange rate at E units of domestic currency per unit of foreign currency. Suppose that in period 1, when the currency peg is announced, the government has a positive stock of foreign assets carried over from period 0, $B_0^g > 0$. Further, assume that the government does not have access to credit. That is, the government asset holdings are constrained to being nonnegative, or $B_t^g \geq 0$ for all t. It is clear from our discussion of the sustainability of currency pegs in subsection 9.2.5 that, as long as the currency peg is in effect, the fiscal deficit produces a continuous drain of assets, which at some point will be completely depleted. Put differently, if the fiscal deficit is not eliminated, at some point the government will be forced to abandon the currency peg and start printing money in order to finance the deficit. Let T denote the period in which, as a result of having run out of reserves, the government abandons the peg and begins to monetize the fiscal deficit.

The dynamics of the currency crisis are characterized by three distinct phases. (1) The pre-collapse phase: during this phase, which lasts from $t = 1$ to $t = T-2$, the currency peg is in effect. (2) The BOP crisis: It takes place in period $t = T-1$, and is the period in which the central bank faces a run against the domestic currency, resulting in massive losses of foreign reserves. (3) The post-collapse phase: It encompasses the period from $t = T$ onwards In this phase, the nominal exchange rate floats freely and the central bank expands the money supply at a rate consistent with the monetization of the fiscal deficit.

(1) The pre-crisis phase: from $t = 1$ to $t = T - 2$

From period 1 to period $T-2$, the exchange rate is pegged, so the variables of interest behave as described in section 9.2.5. In particular, the nominal exchange rate is constant and equal to E, that is, $E_t = E$ for $t = 1, 2, \ldots, T-2$. By PPP, and given our assumption that $P_t^* = 1$, the domestic price level is also constant over time and equal to E ($P_t = E$ for $t = 1, 2, \ldots, T-2$).

[7]The model appeared in Paul R. Krugman, "A Model of Balance-of-Payments Crisis," *Journal of Money, Credit and Banking,* 11, 1979, 311-325.

Because the exchange rate is fixed, the devaluation rate $(E_t - E_{t-1})/E_{t-1}$, is equal to 0. The nominal interest, i_t, which by the uncovered interest parity condition satisfies $1 + i_t = (1 + r^*)E_{t+1}/E_t$, is equal to r^*. Note that the nominal interest rate in period $T - 2$ is also equal to r^* because the exchange rate peg is still in place in period $T - 1$. Thus, $i_t = r^*$ for $t = 1, 2, \ldots, T - 2$.

As discussed in section 9.2.5, by pegging the exchange rate the government relinquishes its ability to monetize the deficit. This is because the nominal money supply, M_t, which in equilibrium equals $EL(\bar{C}, r^*)$, is constant, and as a result seignorage revenue, given by $(M_t - M_{t-1})/E$, is nil. Consider now the dynamics of foreign reserves. By equation (9.11),

$$B_t^g - B_{t-1}^g = -DEF; \quad \text{for } t = 1, 2, \ldots, T - 2.$$

This expression shows that the fiscal deficit causes the central bank to lose DEF units of foreign reserves per period. The continuous loss of reserves in combination with the lower bound on the central bank's assets, makes it clear that a currency peg is unsustainable in the presence of persistent fiscal imbalances.

(3) The post-crisis phase: from $t = T$ onwards

The government starts period T without any foreign reserves ($B_{T-1}^g = 0$). Given our assumptions that the government cannot borrow (that is, B_t^g cannot be negative) and that it is unable to eliminate the fiscal deficit, it follows that in period T the monetary authority is forced to abandon the currency peg and to print money in order to finance the fiscal deficit. Thus, in the post-crisis phase the government lets the exchange rate float. Consequently, the behavior of all variables of interest is identical to that studied in subsection 9.2.6. In particular, the government will expand the money supply at a constant rate μ that generates enough seignorage revenue to finance the fiscal deficit. In section 9.2.6, we deduced that μ is determined by equation (9.15),

$$DEF = L(\bar{C}, i(\mu))\left(\frac{\mu}{1 + \mu}\right)$$

Note that because the fiscal deficit is positive, the money growth rate must also be positive. In the post-crisis phase, real balances, M_t/E_t are constant and equal to $L(\bar{C}, i(\mu))$. Therefore, the nominal exchange rate, E_t, must depreciate at the rate μ. Because in our model $P_t = E_t$, the price level also grows at the rate μ, that is, the inflation rate is positive and equal to μ.

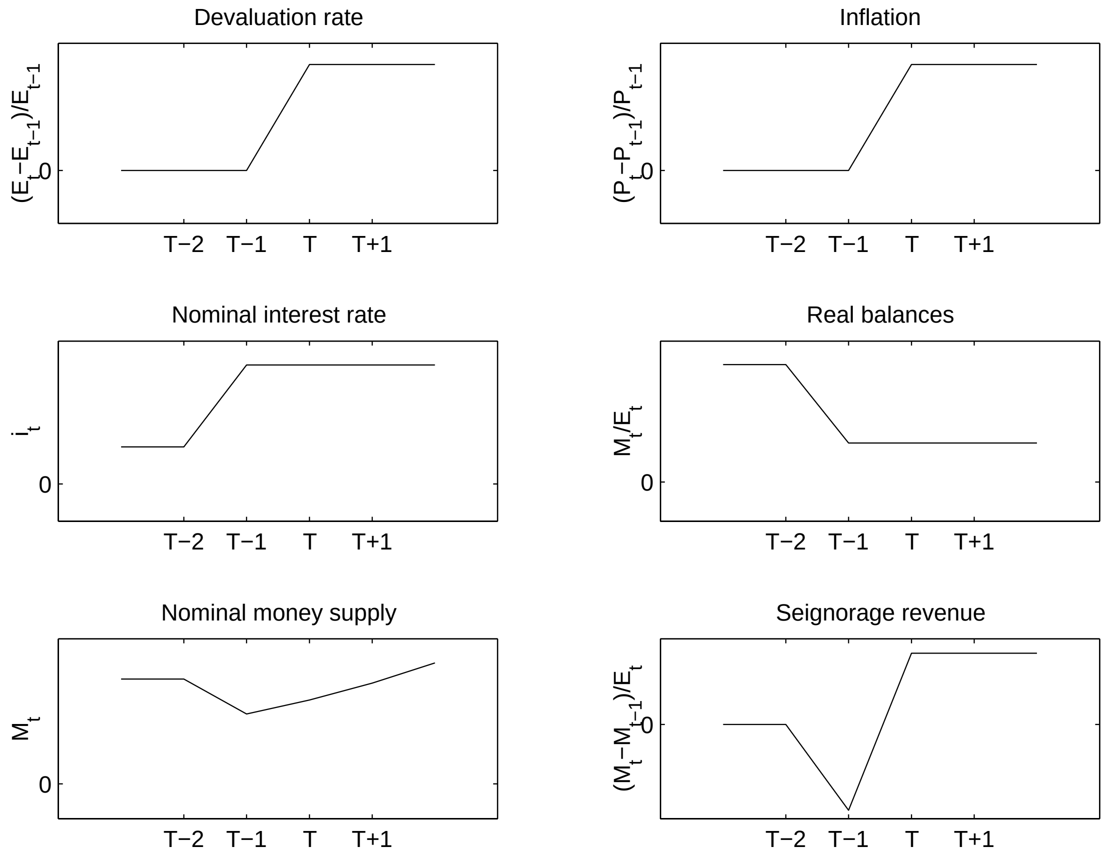
Finally, the nominal interest rate satisfies $1 + i_t = (1 + r^*)(1 + \mu)$. Let's compare the economy's pre- and post-crisis behavior. The first thing to note is that with the demise of the fixed exchange rate regime, price level stability disappears as inflation sets in. In the pre-crisis phase, the rate of monetary expansion, the rate of devaluation, and the rate of inflation are all equal to zero. By contrast, in the post-crisis phase these variables are all positive and equal to μ. Second, the sources of deficit finance are very different in each of the two phases. In the pre-crisis phase, the deficit is financed entirely with foreign reserves. As a result, foreign reserves display a steady decline during this phase. On the other hand, in the post-crisis phase the fiscal deficit is financed through seignorage income and foreign reserves are constant (and in our example equal to zero). Finally, in the post-crisis phase real balances are lower than in the pre-crisis phase because the nominal interest rate is higher.

(2) The BOP crisis: period $T - 1$

In period $T - 1$, the exchange rate peg has not yet collapsed. Thus, the nominal exchange rate and the price level are both equal to E, that is $E_{T-1} = P_{T-1} = E$. However, the nominal interest rate is not r^*, as in the pre-crisis phase, because in period $T-1$ the public expects a depreciation of the domestic currency in period T. The rate of depreciation of the domestic currency between periods $T-1$ and T is μ, that is, $(E_T - E_{T-1})/E_{T-1} = \mu$.[8] Therefore, the nominal interest rate in period $T - 1$ jumps up to its post-crisis level $i_{T-1} = (1 + r^*)(1 + \mu) - 1 = i(\mu)$. As a result of the increase in the nominal interest rate real balances fall in $T-1$ to their post-crisis level, that is, $M_{T-1}/E = L(\bar{C}, i(\mu))$. Because the nominal exchange rate does not change in period $T - 1$, the decline in real balances must be brought about entirely through a fall in nominal balances: the public runs to the central bank to exchange domestic currency for foreign reserves. Thus, in period $T - 1$ foreign reserves at the central bank fall by more than DEF. To see this more formally, evaluate the government budget constraint (9.10)

[8] *For technically inclined readers:* To see that $(E_T - E_{T-1})/E_{T-1} = \mu$, use the fact that in $T - 1$ real balances are given by $M_{T-1}/E_{T-1} = L(\bar{C}, (1 + r^*)E_T/E_{T-1} - 1)$ and that in period T the government budget constraint is $DEF = L(\bar{C}, i(\mu)) - (M_{T-1}/E_{T-1})(E_{T-1}/E_T)$. These are two equations in two unknowns, M_{T-1}/E_{T-1} and E_T/E_{T-1}. If we set $E_T/E_{T-1} = 1+\mu$, then the two equations collapse to (9.15) indicating that $E_T/E_{T-1} = 1 + \mu$ and $M_{T-1}/E_{T-1} = L(\bar{C}, i(\mu))$ are indeed the solution.

Figure 9.4: The dynamics of a balance-of-payments crisis



at $t = T - 1$ to get

$$\begin{aligned} B^g_{T-1} - B^g_{T-2} &= \frac{M_{T-1} - M_{T-2}}{E} - DEF \\ &= L(\bar{C}, i(\mu)) - L(\bar{C}, r^*) - DEF \\ &< -DEF \end{aligned}$$

The second equality follows from the fact that $M_{T-1}/E = L(\bar{C}, i(\mu))$ and $M_{T-2}/E = L(\bar{C}, r^*)$. The inequality follows from the fact that $i(\mu) = (1 + r^*)(1 + \mu) - 1 > r^*$ and the fact that the liquidity preference function is decreasing in the nominal interest rate. The above expression formalizes Krugman's original insight on why the demise of currency pegs is typically preceeded by a speculative run against the domestic currency and large losses of foreign reserves by the central bank: Even though the exchange rate is pegged in $T-1$, the nominal interest rate rises in anticipation of a devaluation in period T causing a contraction in the demand for real money balances. Because in period $T-1$ the domestic currency is still fully convertible, the central bank must absorb the entire decline in the demand for money by selling foreign reserves. Figure 9.4 closes this section by providing a graphical summary of the dynamics of Krugman-type BOP crises.

9.4 Appendix: A dynamic optimizing model of the demand for money

In this section we develop a dynamic optimizing model underlying the liquidity preference function given in equation (9.6). We motivate a demand money by assuming that money facilitates transactions. We capture the fact that money facilitates transactions by simply assuming that agents derive utility not only from consumption of goods but also from holdings of real balances. Specifically, in each period $t = 1, 2, 3, \ldots$ preferences are described by the following single-period utility function,

$$u(C_t) + z\left(\frac{M_t}{P_t}\right),$$

where C_t denotes the household's consumption in period t and M_t/P_t denotes the household's real money holdings in period t. The functions $u(\cdot)$ and $z(\cdot)$ are strictly increasing and strictly concave functions ($u' > 0$, $z' > 0$, $u'' < 0$, $z'' < 0$).

Households are assumed to be infinitely lived and to care about their entire stream of single-period utilities. However, households discount the future by assigning a greater weight to consumption and real money holdings the closer they are to the present. Specifically, their lifetime utility function is given by

$$\left[u(C_t) + z\left(\frac{M_t}{P_t}\right)\right] + \beta\left[u(C_{t+1}) + z\left(\frac{M_{t+1}}{P_{t+1}}\right)\right] + \beta^2\left[u(C_{t+2}) + z\left(\frac{M_{t+2}}{P_{t+2}}\right)\right] + \ldots$$

Here β is a number greater than zero and less than one called the *subjective discount factor*." The fact that households care more about the present than about the future is reflected in β being less than one.

Let's now analyze the budget constraint of the household. In period t, the household allocates its wealth to purchase consumption goods, P_tC_t, to hold money balances, M_t, to pay taxes, T_t, and to purchase interest bearing foreign bonds, $E_t B_t^p$. Taxes are lump sum and denominated in domestic currency. The foreign bond is denominated in foreign currency. Each unit of foreign bonds costs 1 unit of the foreign currency, so each unit of the foreign bond costs E_t units of domestic currency. Foreign bonds pay the constant world interest rate r^* in foreign currency. Note that because the foreign price level is assumed to be constant, r^* is not only the interest rate in terms of foreign currency but also the interest rate in terms of goods. That is, r^* is the *real* interest rate.[9] The superscript p in B_t^p, indicates that these are bond holdings of *private* households, to distinguish them from the bond holdings of the government, which we will introduce later. In turn, the household's wealth at the beginning of period t is given by the sum of its money holdings carried over from the previous period, M_{t-1}, bonds purchased in the previous period plus interest, $E_t(1 + r^*)B_{t-1}^p$, and income from the sale of its endowment of goods, $P_t Q_t$, where Q_t denotes the household's endowment of goods in period t. This endowment is assumed to be exogenous, that is, determined outside of the model. The budget constraint of the household in period t is then given by:

$$P_t C_t + M_t + T_t + E_t B_t^p = M_{t-1} + (1 + r^*)E_t B_{t-1}^p + P_t Q_t \tag{9.16}$$

The left hand side of the budget constraint represents the uses of wealth and the right hand side the sources of wealth. The budget constraint is expressed in nominal terms, that is, in terms of units of domestic currency. To express the budget constraint in real terms, that is, in units of goods, we divide both the left and right hand sides of (9.16) by P_t, which yields

$$C_t + \frac{M_t}{P_t} + \frac{T_t}{P_t} + \frac{E_t}{P_t} B_t^p = \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + (1 + r^*) \frac{E_t}{P_t} B_{t-1}^p + Q_t$$

Note that real balances carried over from period $t-1$, M_{t-1}/P_{t-1}, appear multiplied by P_{t-1}/P_t. In an inflationary environment, P_t is greater than P_{t-1}, so inflation erodes a fraction of the household's real balances. This loss of resources due to inflation is called the inflation tax. The higher the rate of inflation, the larger the fraction of their income households must allocate to maintaining a certain level of real balances.

Recalling that P_t equals E_t, we can eliminate P_t from the utility function

[9] The domestic nominal and real interest rates will in general not be equal to each other unless domestic inflation is zero. To see this, recall the Fisher equation (5.3). We will return to this point shortly.

and the budget constraint to obtain:

$$\left[u(C_t) + z\left(\frac{M_t}{E_t}\right)\right] + \beta\left[u(C_{t+1}) + z\left(\frac{M_{t+1}}{E_{t+1}}\right)\right] + \beta^2\left[u(C_{t+2}) + z\left(\frac{M_{t+2}}{E_{t+2}}\right)\right] + \ldots \tag{9.17}$$

$$C_t + \frac{M_t}{E_t} + \frac{T_t}{E_t} + B_t^p = \frac{M_{t-1}}{E_t} + (1+r^*)B_{t-1}^p + Q_t \tag{9.18}$$

Households choose C_t, M_t, and B_t^p so as to maximize the utility function (9.17) subject to a series of budget constraints like (9.18), one for each period, taking as given the time paths of E_t, T_t, and Q_t. In choosing streams of consumption, money balances, and bonds, the households faces two tradeoffs. The first tradeoff is between consuming today and saving today to finance future consumption. The second tradeoff is between consuming today and holding money today.

Consider first the tradeoff between consuming one extra unit of the good today and investing it in international bonds to consume the proceeds tomorrow. If the household chooses to consume the extra unit of goods today, then its utility increases by $u'(C_t)$. Alternatively, the household could sell the unit of good for 1 unit of foreign currency and with the proceeds buy 1 unit of the foreign bond. In period $t+1$, the bond pays $1+r^*$ units of foreign currency, with which the household can buy $(1+r^*)$ units of goods. This amount of goods increases utility in period $t+1$ by $(1+r^*)u'(C_{t+1})$. Because households discount future utility at the rate β, from the point of view of period t, lifetime utility increases by $\beta(1+r^*)u'(C_{t+1})$. If the first alternative yields more utility than the second, the household will increase consumption in period t, and lower consumption in period $t+1$. This will tend to eliminate the difference between the two alternatives because it will lower $u'(C_t)$ and increase $u'(C_{t+1})$ (recall that $u(\cdot)$ is concave, so that $u'(\cdot)$ is decreasing). On the other hand, if the second alternative yields more utility than the first, the household will increase consumption in period $t+1$ and decrease consumption in period t. An optimum occurs at a point where the household cannot increase utility further by shifting consumption across time, that is, at an optimum the household is, in the margin, indifferent between consuming an extra unit of good today or saving it and consuming the proceeds the next period. Formally, the optimal allocation of consumption across time satisfies

$$u'(C_t) = \beta(1+r^*)u'(C_{t+1}) \tag{9.19}$$

We will assume for simplicity that the subjective rate of discount equals the world interest rate, that is,

$$\beta(1 + r^*) = 1 \tag{9.20}$$

Combining this equation with the optimality condition (9.19) yields,

$$u'(C_t) = u'(C_{t+1}) \tag{9.21}$$

Because $u(\cdot)$ is strictly concave, $u'(\cdot)$ is monotonically decreasing, so this expressions implies that $C_t = C_{t+1}$. This relationship must hold in all periods, implying that consumption is constant over time. Let $\bar{C}$ be this optimal level of consumption. Then, we have

$$C_t = C_{t+1} = C_{t+2} = \cdots = \bar{C}$$

Consider now the tradeoff between spending one unit of money on consumption and holding it for one period. If the household chooses to spend the unit of money on consumption, it can purchase $1/E_t$ units of goods, which yield $u'(C_t)/E_t$ units of utility. If instead the household chooses to keep the unit of money for one period, then its utility in period t increases by $z'(M_t/E_t)/E_t$. In period $t + 1$, the household can use the unit of money to purchase $1/E_{t+1}$ units of goods, which provide $u'(C_{t+1})/E_{t+1}$ extra utils. Thus, the alternative of keeping the unit of money for one period yields $z'(M_t/E_t)/E_t + \beta u'(C_{t+1})/E_{t+1}$ additional units of utility. In an optimum, the household must be indifferent between keeping the extra unit of money for one period and spending it on current consumption, that is,

$$\frac{z'(M_t/E_t)}{E_t} + \beta \frac{u'(C_{t+1})}{E_{t+1}} = \frac{u'(C_t)}{E_t} \tag{9.22}$$

Using the facts that $u'(C_t) = u'(C_{t+1}) = u'(\bar{C})$ and that $\beta = 1/(1 + r^*)$ and rearranging terms we have

$$z'\left(\frac{M_t}{E_t}\right) = u'(\bar{C})\left[1 - \frac{E_t}{(1 + r^*)E_{t+1}}\right] \tag{9.23}$$

Using the uncovered interest parity condition (9.8) we can write

$$z'\left(\frac{M_t}{E_t}\right) = u'(\bar{C})\left(\frac{i_t}{1 + i_t}\right) \tag{9.24}$$

This equation relates the demand for real money balances, M_t/E_t, to the level of consumption and the domestic nominal interest rate. Inspecting equation (9.24) and recalling that both u and z are strictly concave, reveals that the demand for real balances, M_t/E_t, is decreasing in the level of the nominal interest rate, i_t, and increasing in consumption, $\bar{C}$. This relationship is called the *liquidity preference function.* We write it in a compact form as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t)$$

which is precisely equation (9.6).

The following example derives the liquidity preference function for a particular functional form of the period utility function. Assume that

$$u(C_t) + z(M_t/E_t) = \ln C_t + \gamma \ln(M_t/E_t).$$

Then we have $u'(\bar{C}) = 1/\bar{C}$ and $z'(M_t/E_t) = \gamma/(M_t/E_t)$. Therefore, equation (9.24) becomes

$$\frac{\gamma}{M_t/E_t} = \frac{1}{\bar{C}} \left(\frac{i_t}{1+i_t} \right)$$

The liquidity preference function can be found by solving this expression for M_t/E_t. The resulting expression is in fact the liquidity preference function given in equation (9.2.6), which we reproduce here for convenience.

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{i_t}{1+i_t} \right)^{-1}$$

In this expression, M_t/E_t is linear and increasing in consumption and decreasing in i_t.