Midterm Exam

ECON 4424

October 10, 2000

Problem 1 (35 points)

Consider the following game in normal form:

B A	**N**	**L**	**M**	**K**
S	1 2	1 2	0 5	1 4
T	2 1	2 4	1 6	2 4
R	1 0	2 1	5 4	3 3
V	2 2	2 3	2 7	3 3

Answer the following questions with regard to this game.

(a) Explain what a (strongly) dominated strategy is. Explain why it is well accepted that rational players should never play dominated strategies and that consequentially they can be eliminated from the game.

(b) Eliminate in an iterated fashion all (strongly) dominated strategies from the matrix above and deduce the most reduced matrix.

(c) Identify all Nash equilibria in the game given. Explain carefully why these Nash equilibria are exactly the Nash equilibria in the most reduced matrix of this game as derived under (b).

(d) In the irreducible matrix — or "most reduced" matrix — derived in (b) there are exactly three ways to further reduce this matrix by iterated elimination of weakly dominated strategies. Carefully reduce this game further for these three methods.
Identify which of the Nash equilibria computed in (c) are most plausible given these three ways to reduce the game further.

Problem 2 (40 points)

We consider the following game in extensive form:

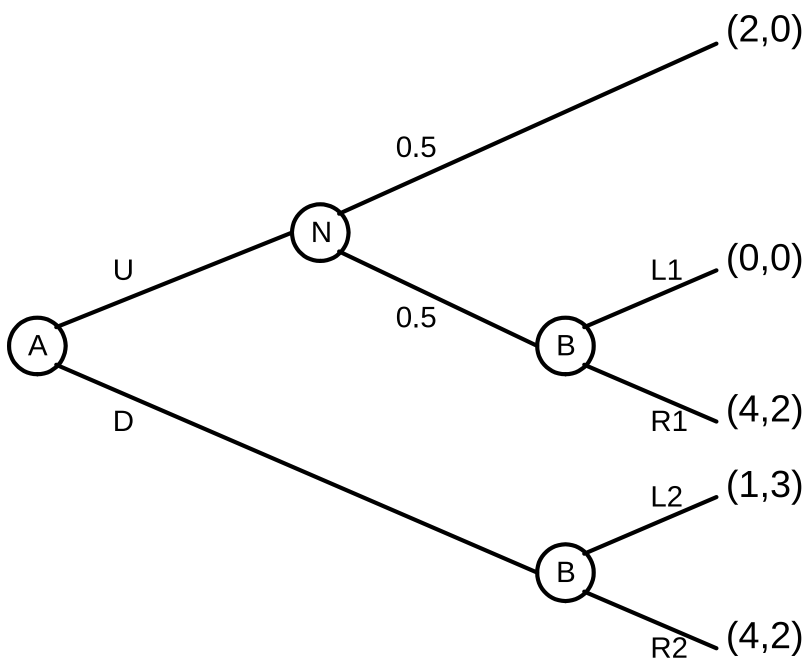

Figure 1: Game tree with Problem 2

There are two players in the game, A and B. Player A selects either action U or D at the beginning of the game. Nature (N) will reveal whether player A selected U with a probability of 50%. Subsequently player B selects between left (L) or right (R). Given that player B has complete information this leads to the choices L_1, L_2, R_1, and R_2.

Answer the following questions with regard to this game:

(a) Apply the method of backward induction to derive a solution to the game in extensive form as given by the game tree. Show that the game tree has a unique solution and given this solution as a strategy vector.

(b) Give the normal form of the game given in the game tree. Clearly label the strategies available to the players in this game.
Determine all Nash equilibria of this game. Show that there are two equilibria. Explain why the backward induction solution identified under (a) is one of the Nash equilibria in the normal form.

(c) Next consider that player B no longer has information of what player A selected. Hence, player B has incomplete information about player A's choice. Modify the game tree to reflect this change. Make your game tree sufficiently large so all nodes and actions are indicated well and clearly.
Can you still apply the method of backward induction to your modified game tree? Explain why.

(d) Give the normal form of the game tree that you developed in question (c). Determine the Nash equilibria of this game. Explain from your results why the lack of information is *beneficial* for player B in this game.

Problem 3 (25 points)

Consider the following normal form game:

A \ B	**N**	**W**	**S**	**E**
U	3, 1	2, 1	1, 0	3, 1
M	2, 0	1, 1	2, 3	4, 2
D	3, 0	0, 1	1, 8	3, 0

Solve the following questions for this particular game.

(a) Determine all Nash equilibria in this game. Show that there are in fact exactly three equilibria in this game.

(b) Give the definition of a *strict Nash equilibrium*. Which of the Nash equilibria identified under (a) are strict? Explain.

(c) Show that there is a unique way to eliminate weakly dominated strategies and that this game can be solved completely by iterated elimination of these weakly dominated strategies.

Why does this method identify the strict Nash equilibrium as the unique solution to this game? Give a detailed assessment that in fact *if* there exists a unique method to eliminate weakly dominated strategies in an iterated fashion *and* there exists a strict Nash equilibrium, *then* the iteration elimination method has to identify this strict Nash equilibrium as the unique solution.

Answer key Problem 1

(a) **(7 points)** A strategy s_1 dominates a strategy s_2 for a certain player if it always gives a higher payoff irrespective of what the other player does. Hence, for every strategy t of the other player, strategy s_1 gives a strictly higher payoff than strategy s_2, i.e., $\pi_i(s_1, t) > \pi_i(s_2, t)$.
A rational player will never select a dominated strategy since the strategy that dominates it will always lead to a higher payoff irrespective of what the other player does. Thus, if maximization of one's payoff is one's objective, a dominated strategy can never fulfill that goal.

(b) **(8 points)** For player A strategy R is dominated by S. Eliminate R:

B **A**	**N**	**L**	**M**	**K**
S	1 2	1 2	0 5	1 4
T	2 1	2 4	1 6	2 4
V	2 2	2 3	2 7	3 3

In this reduced matrix, strategy M is dominated by strategy K for player B. Eliminate M:

B **A**	**N**	**L**	**K**
S	1 2	1 2	1 4
T	2 1	2 4	2 4
V	2 2	2 3	3 3

This 3×3 matrix is the resulting irreducible matrix. There are no further strong domination relationships between any of the remaining strategies.

(c) **(10 points)** In the original matrix given there are four Nash equilibria: (S, N), (T, L), (S, K), and (T, K).

A Nash equilibrium can never consist of a dominated strategy, because a dominated strategy can never be a best response. Thus, by eliminating dominated strategies one never throws away Nash equilibrium. In other words, iterated elimination of dominated strategies preserves all Nash equilibria of the original game and the Nash equilibria in the most reduced game are exactly the Nash equilibria of the original game.

(d) **(10 points)** In the reduced matrix K weakly dominates both N and L for player B. The three methods are now based on first eliminating strategy N, first eliminating strategy L, or first eliminating both strategies N and L.

First eliminate Strategy N:

A \ **B**	**L**	**K**
S	1 2	1 4
T	2 4	2 4
V	2 3	3 3

In this reduced matrix strategy T strongly dominates V and weakly dominates S for player A. Thus, both V and S can be eliminated simultaneously:

A \ **B**	**L**	**K**
T	2 4	2 4

In this reduced matrix we identify the two equilibria (T, L) and (T, K).

First eliminate strategy L:

A \ B	N	K
S	2, 1	4, 1
T	1, 2	4, 2
V	2, 2	3, 3

In this reduced matrix we identify strategy S as weakly dominant for player B. Hence, we can eliminate strategies T and V:[1]

A \ B	N	K
S	2, 1	4, 1

In this further reduced matrix we identify Nash equilibria (S, N) and (S, K).

Eliminate both strategies N and L simultaneously:

A \ B	K
S	4, 1
T	4, 2
V	3, 3

In this reduced matrix we can eliminate strategy V for player B. Thus, we identify Nash equilibria (S, K) and (T, K).

We conclude that the three methods have in common that they clearly identify the Nash equilibria (S, K) and (T, K) the most. Indeed these are the most plausible Nash equilibria to play since K is a weakly dominant strategy.

[1] You are invited to check that the order in which we eliminate these two strategies does not alter the outcome of the analysis.

Answer key to Problem 2

(a) **(8 points)** The backward induction solution is indicated in the game tree below. (The solution is given by the red links in the game tree.)The backward

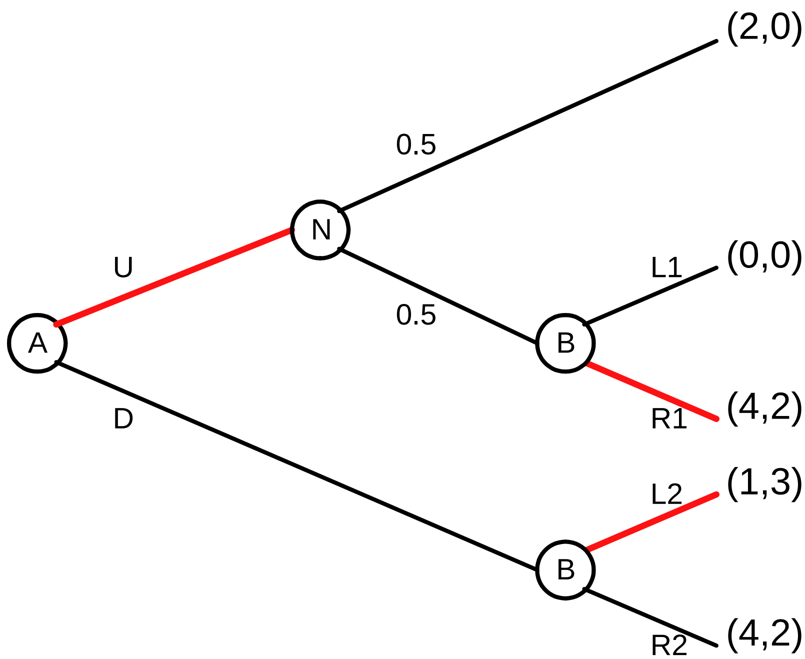

Figure 2: Backward induction solution

induction solution is given by (U, R_1L_2).

(b) **(10 points)** The normal form representation of the game tree is given by

B **A**	**L$_1$L$_2$**	**L$_1$R$_2$**	**R$_1$L$_2$**	**R$_1$R$_2$**
U	0 1	0 1	1 3	1 3
D	3 1	2 4	3 1	2 4

This matrix is derived using the expected utility hypothesis.
The two Nash equilibria identified are (D, L_1L_2) and (U, R_1L_2).

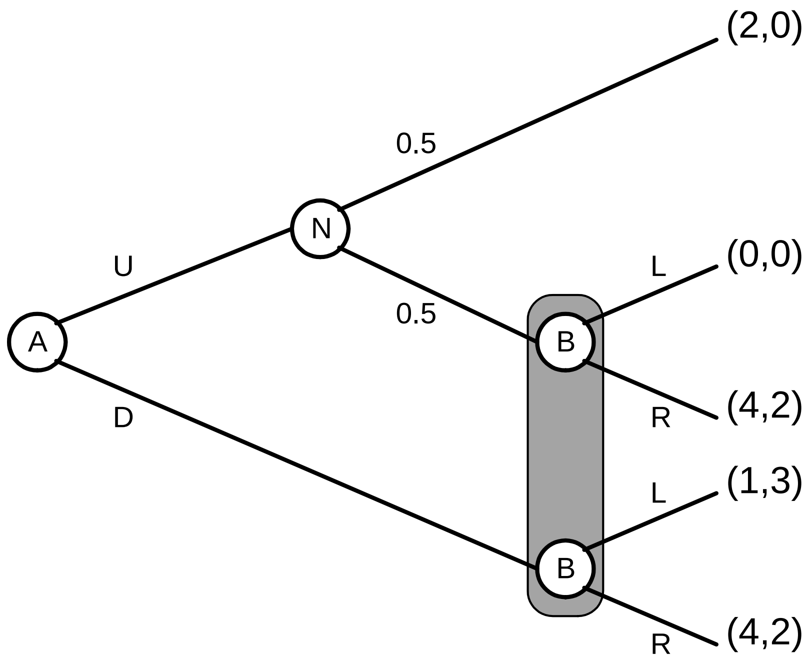

Figure 3: Modified game tree with Problem 2.

(c) **(10 points)** We have to introduce an information set for player B to reflect his deficiency of knowledge about player A's actions. This is reflected as follows:The method of backward induction can no longer be applied due to the information set in this modified game tree.

(d) **(12 points)** The normal form of the game tree derived under (c) is:

B **A**	**L**	**R**
U	0 1	1 3
D	3 1	2 4

This game has a unique Nash equilibrium given by (D, L). Note that in this unique Nash equilibrium player B's payoff of 3 is higher than his (expected) payoff of 1 in the backward induction solution as derived under (a). Thus, we may conclude that here indeed an information deficiency is beneficial for the player involved![2]

Answer key to Problem 3

(a) **(5 points)** The three equilibria are (U, N), (U, W), and (M, S).

(b) **(8 points)** A strategy vector (s^*, t^*) is a strict Nash equilibrium if

$$\begin{aligned} \pi_1(s^*, t^*) &> \pi_1(s, t^*) \text{ for every } s \neq s^* \text{ and} \\ \pi_2(s^*, t^*) &> \pi_2(s^*, t) \text{ for every } t \neq t^*. \end{aligned}$$

Hence, in a strict Nash equilibrium the strategies played are unique best responses to each other.

The unique strict Nash equilibrium is (M, S).

(c) **(12 points) Step 1:** In the matrix given E strongly dominates N and W weakly dominates N for player B. Thus we might eliminate N:

B **A**	**W**	**S**	**E**
U	1 2	0 1	1 3
M	1 1	3 2	2 4
D	1 0	8 1	0 3

Step 2: In this reduced matrix only U weakly dominates D and M strongly dominates D for player A. Thus, we can eliminate D:

B **A**	**W**	**S**	**E**
U	1 2	0 1	1 3
M	1 1	3 2	2 4

[2] In this particular case the information deficiency excludes the derived backward solution as a Nash equilibrium in the modified game. That backward induction solution was a "bad" equilibrium for player B. Instead the remaining Nash equilibrium becomes the unique one remaining, which is the "good" equilibrium for player B.

Step 3: In this matrix only E weakly dominates W for player B; so eliminate strategy W:

A \ B	S	E
U	1, 0	3, 1
M	2, 3	4, 2

Step 4: In this reduced matrix M strongly dominates U for player A. Thus, we eliminate U:

A \ B	S	E
M	2, 3	4, 2

Step 5: In this further reduced matrix we identify (M, S) as the unique solution to the method of iterated elimination of weakly dominated strategies developed here. In other words, this game is indeed "dominance solvable."