Section 2.9 – Higher–Order Derivatives

When we take the derivative of a function, that is called the first derivative. If we differentiate the answer, we would get the second derivative. If we were to differentiate yet again, we would get the third derivative and so on. For example, consider the function $f(x) = 2x^5 - 4x^3 + 9x^2 - 6x - 2$.

The 1st derivative: $\frac{dy}{dx} = f'(x) = 10x^4 - 12x^2 + 18x - 6$

The 2nd derivative: $\frac{d^2y}{dx^2} = f''(x) = 40x^3 - 24x + 18$

The 3rd derivative: $\frac{d^3y}{dx^3} = f'''(x) = 120x^2 - 24$

The 4th derivative: $\frac{d^4y}{dx^4} = f^{IV}(x) = 240x$

We have seem an application that utilizes this idea. Recall that if $s(t)$ is the position function, then the velocity function is $v(t) = s'(t)$ and the acceleration function $a(t) = v'(t)$. But $v'(t)$ is the rate of change of the speed or the rate of change of the rate of change of the position function: $a(t) = v'(t) = \frac{d}{dt}[s'(t)] = s''(t)$. So, the acceleration function is the second derivative of the position function. Let's look at some examples.

Ex. 1 An object moves along a straight line so that after t seconds, its position is $s(t) = 0.1t^3 - 0.8t^2 + 5t - 4$ meters. Find the velocity and acceleration of the object after 3 seconds.

<u>Solution:</u>

$v(t) = \frac{d}{dt}[0.1t^3 - 0.8t^2 + 5t - 4] = 0.3t^2 - 1.6t + 5$ and

$a(t) = \frac{d}{dt}[0.3t^2 - 1.6t + 5] = 0.6t - 1.6$.

Thus, $v(3) = 0.3(3)^2 - 1.6(3) + 5 = 2.7 - 4.8 + 5 = 2.9$.

and $a(3) = 0.6(3) - 1.6 = 1.8 - 1.6 = 0.2$.

So, after 3 seconds, the object is traveling at 2.9 meters per second and its speed is increasing at a rate of 0.2 meters per second per second. This is not a typo. The units for acceleration are meters per second per second or meters per second^2.

Ex. 2 It is estimated that t months from now, the average price for Acme SuperJet Rocket Skis will be:

$p(t) = -\,0.3t^3 + 100t + 400$ dollars.

a) At what rate will the price per unit be increasing with respect to time 6 months from now?

b) At what rate will the rate of price increase be changing with respect to time 6 months from now?

<u>Solution:</u>

a) Computing p '(t), we get:

$p'(t) = \frac{d}{dt}[-\,0.3t^3 + 100t + 400] = -\,0.9t^2 + 100.$

Thus, $p'(6) = -\,0.9(6)^2 + 100 = -\,32.4 + 100 = 67.6$. The price per unit will be increasing at rate of \$67.60 per month six months from now.

b) Computing p "(t), we get:

$p''(t) = \frac{d}{dt}[-\,0.9t^2 + 100] = -\,1.8t$. Thus, $p''(6) = -\,1.8(6) = -\,10.8$. Six months from now, rate of the price increase per unit will be decreasing at a rate of \$10.80 per month per month.

Ex. 3 Given $y = 3x^2 - 8\sqrt{x}$, find y', y", y"', y^{iv}.

<u>Solution:</u>

Since $y = 3x^2 - 8\sqrt{x} = y = 3x^2 - 8x^{1/2}$

then $y' = \frac{d}{dx}(3x^2 - 8x^{1/2}) = 6x - 4x^{-\,1/2} = 6x - \frac{4}{\sqrt{x}}$

and $y'' = \frac{d}{dx}(6x - 4x^{-\,1/2}) = 6 + 2x^{-\,3/2} = 6 + \frac{2}{x\sqrt{x}}$

and $y''' = \frac{d}{dx}(6 + 2x^{-\,3/2}) = 0 - 3x^{-\,5/2} = -\,\frac{3}{x^2\sqrt{x}}$

and $y^{iv} = \frac{d}{dx}(-\,3x^{-\,5/2}) = \frac{15}{2}x^{-\,7/2} = \frac{15}{2x^3\sqrt{x}}$