Section 4.6 - Elasticity of Demand

Elasticity of Demand – measures the sensitivity of demand x(p) to changes in the price p.

$$\varepsilon(p) = -\frac{\%\text{ change in } x(p)}{\%\text{ change in } p} = -\frac{\dfrac{100\left(\dfrac{dx}{dp}\right)}{x(p)}}{\dfrac{100\left(\dfrac{dp}{dp}\right)}{p}} = -\frac{p}{x(p)}\bullet\frac{dx}{dp} = -\frac{pD'(p)}{D(p)}$$

If $\varepsilon(p) > 1$, the demand is said to be elastic (A small % change in price makes for a larger % change in demand). The total revenue is decreasing. (Lower prices)

If $\varepsilon(p) < 1$, the demand is said to be inelastic (A small % change in price makes for a smaller % change in demand). The total revenue is increasing. (Raise prices)

If $\varepsilon(p) = 1$, the demand is said to have a unit of elasticity (A small % change in price makes for the same % change in demand). The total revenue is maximized.

Ex. 1 Given $x = D(p) = \frac{750}{(p+2)^2}$ and p = \$23, find the following:

a) The elasticity
b) The elasticity at the given value of p. Is demand elastic or inelastic?
c) The values of p that the total revenue is maximized. What is the maximum revenue?

Solution:

a) First, we calculate the derivative:

$$D(p) = \frac{750}{(p+2)^2} = 750(p+2)^{-2}$$

$$D'(p) = 750(-2)(p+2)^{-3} = \frac{-1500}{(p+2)^3}.$$

Thus, the elasticity is:

$$\varepsilon(p) = -\frac{pD'(p)}{D(p)} = -\frac{p\left(\dfrac{-1500}{(p+2)^3}\right)}{\dfrac{750}{(p+2)^2}} = \frac{1500p}{(p+2)^3} \div \frac{750}{(p+2)^2}$$

$= \frac{1500p}{(p+2)^3} \bullet \frac{(p+2)^2}{750} = \frac{2p}{p+2}.$

So, $\varepsilon(p) = \frac{2p}{p+2}$

b) Evaluating $\varepsilon(p)$ at p = \$23 yields:

$\varepsilon(23) = \frac{2(23)}{(23)+2} = \frac{46}{25} = 1.84$

$\varepsilon(23) > 1$ so, the demand is elastic. A small percent change in price makes for a larger percent change in demand (lower prices).

c) Set $\varepsilon(p) = 1$ and solve:

$\varepsilon(p) = \frac{2p}{p+2} = 1$

$\frac{2p}{p+2} = 1$ Multiply both sides by p + 2:

$\mathbf{(p+2)}\frac{2p}{p+2} = 1\mathbf{(p+2)}$

$2p = p + 2$

$p = 2$

Thus, the revenue is maximized when p = \$2. Since the revenue is function R(p) = p•D(p)

$= p\left(\frac{750}{(p+2)^2}\right) = \frac{750p}{(p+2)^2}$, then the maximum revenue is $R(2) = \frac{750(2)}{(2+2)^2} = \frac{1500}{16} = \$93.75$

Ex. 2 Given $x = D(p) = 1500e^{-0.45p}$ and p = \$2, find:

a) The elasticity

b) The elasticity at the given value of p. Is demand elastic or inelastic?

c) The values of p that the total revenue is maximized. What is the maximum revenue?

<u>Solution:</u>

a) First, we calculate the derivative:

$D(p) = 1500e^{-0.45p}$

$D'(p) = 1500e^{-0.45p}\frac{d}{dp}(-0.45p)$

$= 1500e^{-0.45p}(-0.45) = -675e^{-0.45p}$

Thus, the elasticity is:

$\varepsilon(p) = -\frac{pD'(p)}{D(p)} = -\frac{p\left(-675e^{-0.45p}\right)}{1500e^{-0.45p}} = \frac{675p}{1500} = \frac{9p}{20}$

So, $\varepsilon(p) = \frac{9p}{20}$

b) Evaluating $\varepsilon(p)$ at p = \$2 yields:

$\varepsilon(2) = \frac{9(2)}{20} = \frac{18}{20} = \frac{9}{10}$

$\varepsilon(2) < 1$ so, the demand is inelastic. A small percent change in price makes for a smaller percent change in demand (raise prices).

c) Set $\varepsilon(p) = 1$ and solve:

$\varepsilon(p) = \frac{9p}{20} = 1$

$\frac{9p}{20} = 1$ Multiply both sides by 20:

$\mathbf{(20)}\ \frac{9p}{20} = 1\mathbf{(20)}$

$9p = 20$

$p \approx 2.22$

Thus, the revenue is maximized when p ≈ \$2.22.

Since the revenue is function $R(p) = p \bullet D(p)$

$= p\left(1500e^{-0.45p}\right) = 1500pe^{-0.45p}$, then the maximum revenue is

$R(2.22) = 1500(2.22)e^{-0.45(2.22)} = 3330e^{-0.999}$

$= 3330(0.368247505...) = 1226.264... \approx \$1226.26$