Section 6.1 – Consumer's and Producer's Surplus

Let's begin with some informal definitions of Consumer's Surplus and Producer's Surplus.

Consumer's Surplus is the difference between what consumers are willing to spend and the actual expenditure.

Producer's Surplus is the difference the actual expenditure and what the producer was willing to sell the merchandise for.

To understand these ideas better, consider the following example:

Ex. 1 Suppose Rachel, Juan, and LaTonya each want to buy a new 27-inch color T.V. Rachel is willing to pay \$300 for the T.V., Juan is willing to pay \$325 for the T.V., and LaTonya is willing to pay \$250 for the T.V. If Diode Town is selling the T.V. set for \$230, what is the total consumer surplus for the three people?

Solution:

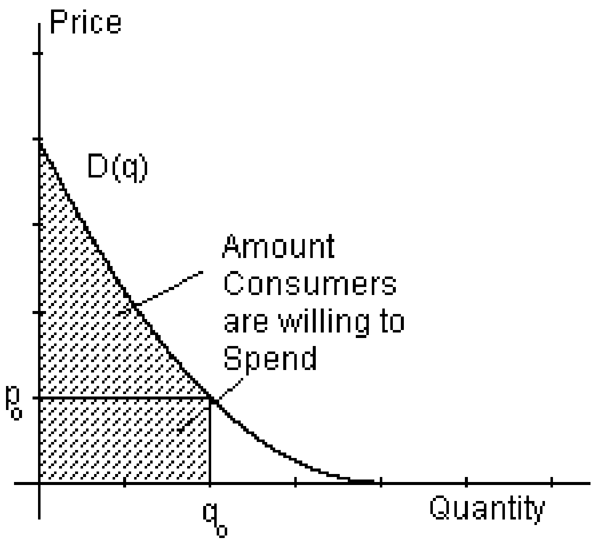
Since Rachel was willing to pay \$300, but the the T.V. was only \$230, she came out \$70 ahead. Similarly, Juan was willing to pay \$325, so he came out \$95 ahead and even LaTonya pocketed \$20. The total surplus for the three people is \$70 + \$95 + \$20 = \$185.

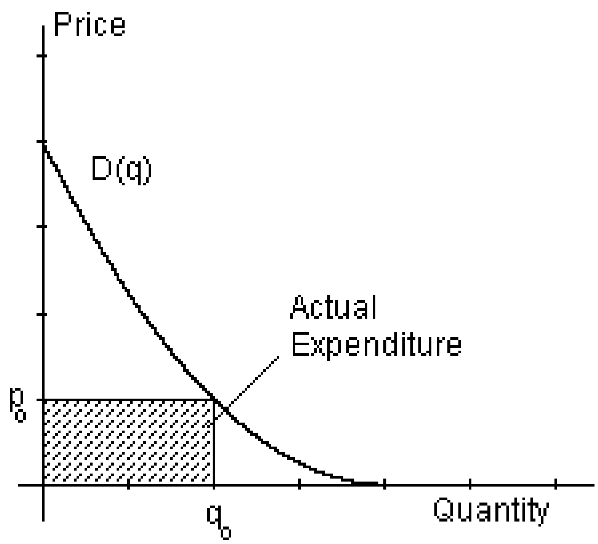
Ex. 2 Suppose that Diode Town is willing to sell the T.V. described in example seven for \$200 each. What is the producer's surplus if each of the three people paid \$230 for the T.V.?

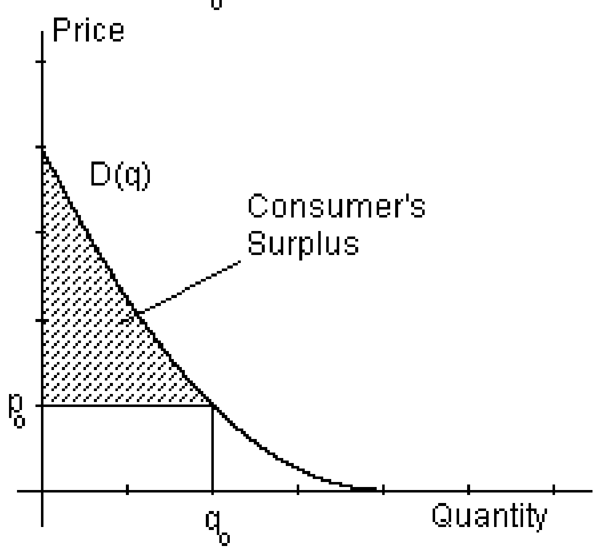
Solution:

Since each person paid \$230 for the T.V. and Diode Town was willing to sell the T.V. for \$200, their surplus per person was \$30. Thus, the total producer's surplus was \$30 + \$30 + \$30 = \$90.

Now, let's define the consumer's and producer's surplus more formally. Let p = D(q) be the demand function and p = S(q) be the supply function. The Equilibrium Point (q_o, p_o) is where supply meets demand or when $S(q) = D(q)$. The total amount that consumers are willing to spend is the area under $D(q)$ on the interval $[0, q_o]$ or $\int_0^{q_o} D(q)\,dq$. In example one, the total amount that the consumers were willing to spend was \$300 + \$325 + \$250 = \$875. The actual expenditure at the equilibrium point is $q_o \cdot p_o$ or in example 1, 3•230 or \$690. The Consumer's Surplus is then $\left(\int_0^{q_o} D(q)\,dq\right) - q_o \cdot p_o$ or as in example 1, \$875 – \$690 = \$185.





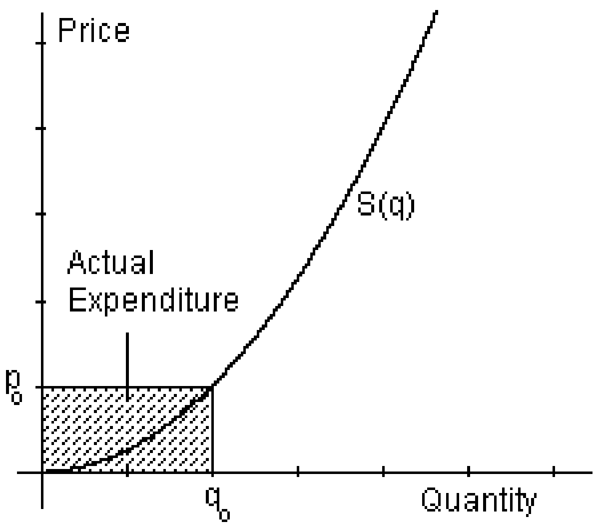


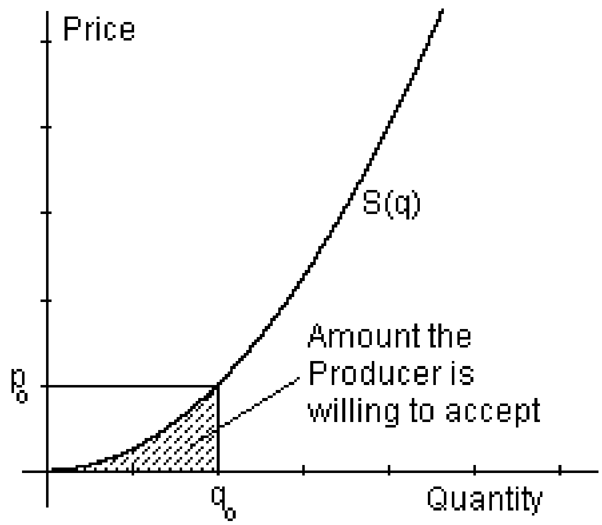
Consumer's Surplus

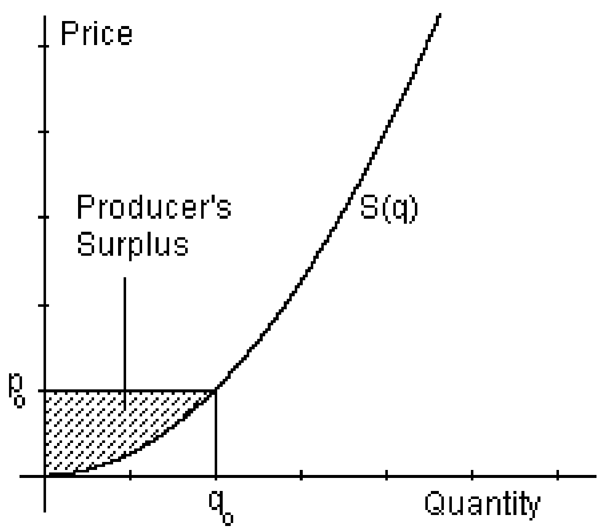
Let $D(q)$ be the demand function and let (q_o, p_o) be the equilibrium point. Then, the Consumer's Surplus is:

$$C.S. = \left[\int_0^{q_o} D(q)\,dq\right] - q_o \cdot p_o$$

The total amount that producers are willing to accept is the area under S(q) on the interval [0, q_o] or $\int_0^{q_o} S(q)\,dq$. In example two, the total amount that the producer was willing to accept was \$200 + \$200 + \$200 = \$600. The actual expenditure at the equilibrium point is $q_o \cdot p_o$ or $3 \cdot 230$ or \$690 in our example. The Producer's Surplus is then $q_o \cdot p_o - [\int_0^{q_o} S(q)\,dq]$ or as in example 2, \$690 – \$600 = \$90.



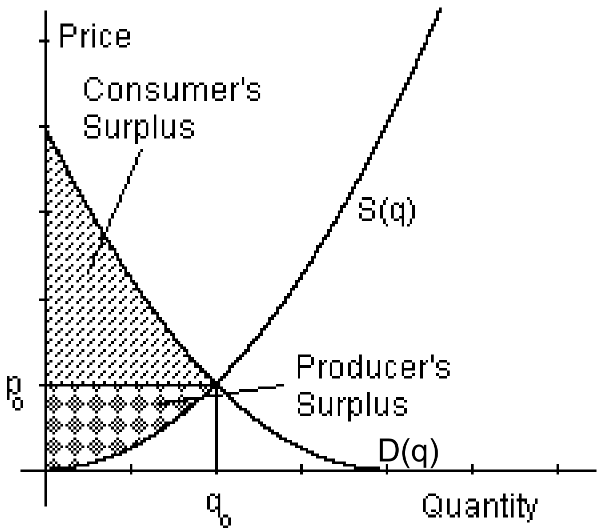




Producer's Surplus
Let S(q) be the supply function and let (q_o, p_o) be the equilibrium point. Then, the Producer's Surplus is:

$$\text{P.S.} = q_o \cdot p_o - [\int_0^{q_o} S(q)\,dq]$$

Graphing both D(q) and S(q) gives us the total picture.



You can find the Consumer's and Producer's Surplus at other point aside from the equilibrium point, but typically we are interested in know what is going on when supply meets demand.

Ex. 3 The demand function for a certain commodity is $D(q) = 110 - q^2$ and the supply function is $S(q) = 14q + 15$.

a) Find the equilibrium point.

b) Find the Consumer's Surplus at the equilibrium point.

c) Find the Producer's Surplus at the equilibrium point.

<u>Solution:</u>

a) Setting $S(q) = D(q)$ and solving yields:

$14q + 15 = 110 - q^2$

$q^2 + 14q - 95 = 0$

$(q - 5)(q + 19) = 0$

$q = 5$ or $q = -19$, reject $q = -19$

Thus, $q_o = 5$ units. Evaluating D(5) yields:

$D(5) = 110 - (5)^2 = 110 - 25 = \$85.$

Hence, the equilibrium point is (5 units, \$85).

b) $\text{C.S.} = [\int_0^{q_o} D(q)\,dq] - q_o \bullet p_o$

$= [\int_0^5 (110 - q^2)\,dq] - 5 \bullet 85 = [110q - \frac{q^3}{3}]\Big|_0^5 - 425$

$= [110(5) - \frac{(5)^3}{3}] - [0] - 425 = 550 - 41\frac{2}{3} - 425$

$\approx 83.33.$

Thus, the Consumer's Surplus is about \$83.33.

c) $\text{P.S.} = q_o \cdot p_o - [\int_0^{q_o} S(q)\,dq]$

$= 5 \cdot 85 - [\int_0^{5} (14q + 15)\,dq] = 425 - [7q^2 + 15q] \Big|_0^5$

$= 425 - \{[7(5)^2 + 15(5)] - [0]\} = 425 - [175 + 75]$

$= 425 - 250 = 175.$

Thus, the Producer's Surplus is \$175.

Ex. 4 Given the demand and the supply functions:

$p = d(q) = (q - 30)^2$ and $p = s(q) = q^2 + 10q + 200$, find

a) The equilibrium point.

b) The consumer's surplus.

c) The producer's surplus.

Solution:

a) Setting $S(q) = D(q)$ and solving yields:

$q^2 + 10q + 200 = (q - 30)^2$

$q^2 + 10q + 200 = q^2 - 60q + 900$

$10q + 200 = -60q + 900$

$70q = 700$

$q = 10$

Thus, $q_o = 10$ units. Evaluating $D(10)$ yields:

$D(10) = (10 - 30)^2 = (-20)^2 = \$400.$

Hence, the equilibrium point is (10 units, \$400).

b) $\text{C.S.} = [\int_0^{q_o} D(q)\,dq] - q_o \cdot p_o$

$= [\int_0^{10} (q^2 - 60q + 900)\,dq] - 10 \cdot 400$

$= [\frac{q^3}{3} - 30q^2 + 900q] \Big|_0^{10} - 4000$

$= [\frac{(10)^3}{3} - 30(10)^2 + 900(10)] - [0] - 4000$

$= \frac{1000}{3} - 3000 + 9000 - 0 - 4000$

$\approx 2333.33.$

Thus, the Consumer's Surplus is about \$2333.33.

c) $\text{P.S.} = q_o \cdot p_o - [\int_0^{q_o} S(q)\, dq]$

$= 10 \cdot 400 - [\int_0^{10} (q^2 + 10q + 200)\, dq]$

$= 4000 - [\frac{q^3}{3} + 5q^2 + 200q] \Big|_0^{10}$

$= 4000 - \{[\frac{(10)^3}{3} + 5(10)^2 + 200(10)] - [0]\}$

$= 4000 - [\frac{1000}{3} + 500 + 2000]$

$= 4000 - 2833.3333... = 1166.666....$

Thus, the Producer's Surplus is \$1166.67.