Sect 10.1 - Compound Inequalities

Concept #1 Union and Intersection

To understand the Union and Intersection of two sets, let's begin with an example. Let A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8}. **Union** of sets A and B, denoted $A \cup B$, is the set or elements that belong to set A or set B or both. Thus, $A \cup B$ = {1, 2, 3, 4, 5, 6, 8}. Think of the Union as a family reunion: you are bringing everyone together. The **intersection** of sets A and B, denoted $A \cap B$, is the set of all elements in both A and B. Thus, $A \cap B$ = {2, 4}. Think of the Intersection as the intersection of two roads; it is the part of the road that is in common with both streets. In terms of intervals, with the union of two intervals, we merge the two intervals together. With the intersection of two intervals, we take the pieces that overlap.

<u>Find the following sets:</u>

Ex. 1 $(-\infty, 4.5] \cup [-7, 6)$

<u>Solution:</u>

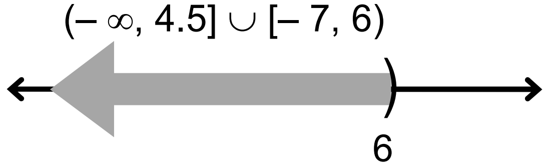
First, graph each interval:

$(-\infty, 4.5]$:

$[-7, 6)$:

Now, merge the two sets together:

$(-\infty, 4.5] \cup [-7, 6)$



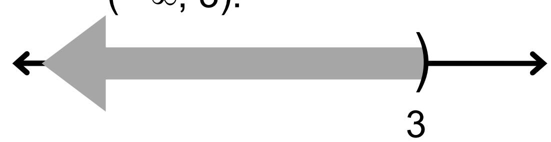
Thus, $(-\infty, 4.5] \cup [-7, 6)$ $= (-\infty, 6)$.

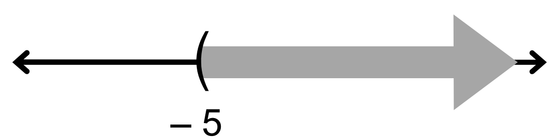
Ex. 2 $(-\infty, 3) \cap (-5, \infty)$

<u>Solution:</u>

First, graph each interval:

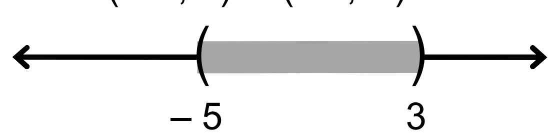
$(-\infty, 3)$:





Now, take the section where the two sets overlap:

$(-\infty, 3) \cap (-5, \infty)$



Thus, $(-\infty, 3) \cap (-5, \infty)$ $= (-5, 3)$.

Concept #2: Solving Compound Inequalities joined by the word: **and**

If we say that statement #1 *and* statement #2 are true, that means both statements have to be true. In terms of sets, if an element is in set A *and* in set B, that means the element has to be in both sets or in the intersection of A and B. Thus, the solution to two inequalities joined with the word "***and***" is the intersection of their solution sets. To solve a compound inequality with the word "and," we solve each inequality individual and then see where their solution sets overlap. That overlap will be the solution to the compound inequality:

<u>Solve the following:</u>

Ex. 3 $3x + 2 < -13$ and $-4x - 3 \geq -11$

<u>Solution:</u>

First, solve each inequality individually:

$3x + 2 < -13$ and $-4x - 3 \geq -11$

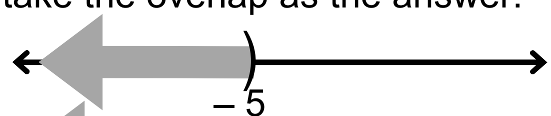
$3x < -15$ and $-4x \geq -8$

$x < -5$ and $x \leq 2$*

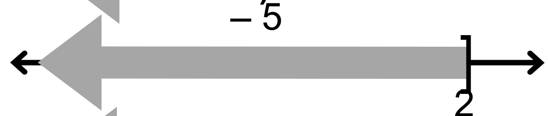
*Remember to switch the inequality sign whenever you multiply or divided both sides by a negative number.

Now, graph each solution set and take the overlap as the answer:

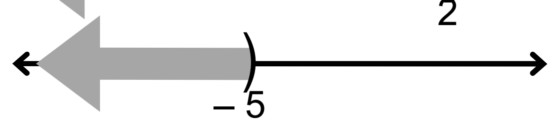
$x < -5$



$x \leq 2$



Thus, $(-\infty, -5) \cap (-\infty, 2]$
$= (-\infty, -5)$.



Ex. 4 $0.6x + 0.8 \leq x - 0.4$ and $\frac{4}{3}x + \frac{40}{3} < \frac{2}{3}x + 16$

<u>Solution:</u>

First, solve each inequality individually:

$0.6x + 0.8 \leq x - 0.4$ and $\mathbf{3}\left(\frac{4}{3}x\right) + \mathbf{3}\left(\frac{40}{3}\right) < \mathbf{3}\left(\frac{2}{3}x\right) + \mathbf{3}(16)$

$0.6x + 0.8 \leq x - 0.4$ and $4x + 40 < 2x + 48$

$-0.4x + 0.8 \leq -0.4$ and $2x + 40 < 48$

$-0.4x \leq -1.2$ and $2x < 8$

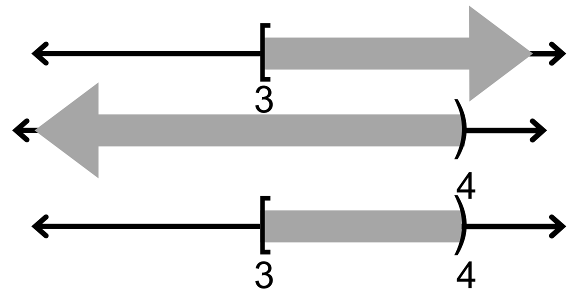
$x \geq 3$ and $x < 4$

Now, graph each solution set and take the overlap as the answer:

$x \geq 3$

$x < 4$

Thus, $[3, \infty) \cap (-\infty, 4)$
$= [3, 4)$.



Ex. 5 $5(x + 4) \geq 50$ and $6x + 9 < 4x + 3$

<u>Solution:</u>

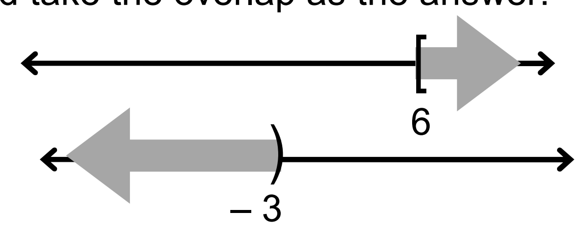
First, solve each inequality individually:

$5(x + 4) \geq 50$	and	$6x + 9 < 4x + 3$
$5x + 20 \geq 50$	and	$2x + 9 < 3$
$5x \geq 30$	and	$2x < -6$
$x \geq 6$	and	$x < -3$

Now, graph each solution set and take the overlap as the answer:

$x \geq 6$

$x < -3$



Thus, $[6, \infty) \cap (-\infty, -3)$ No overlap which means no solution
$= \{\ \}$.

Ex. 6 $-7 < 4x - 5 \leq 3$

<u>Solution:</u>

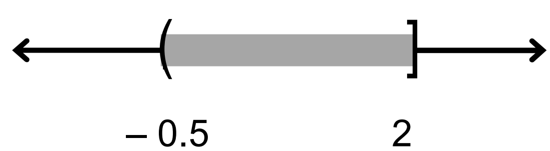
$-7 < 4x - 5 \leq 3$ (add 5 to all three sides)

$\underline{+5 = \quad +5 = +5}$

$-2 < 4x \leq 8$

$\dfrac{-2}{4} < \dfrac{4x}{4} \leq \dfrac{8}{4}$ (divide all three sides by 4)

$-0.5 < x \leq 2$



$(-0.5, 2]$

Notice that $-0.5 < x \leq 2$ means that $x > -0.5$ and $x \leq 2$. We could have solve the $-7 < 4x - 5 \leq 3$ as $-7 < 4x - 5$ and $4x - 5 \leq 3$.

Ex. 7 $3x - 4 \le 5 < 4x + 13$

Solution:

Unlike example #6, we cannot get x by itself in the middle so we will have to solve this as two separate inequalities:

$3x - 4 \le 5 < 4x + 13$:

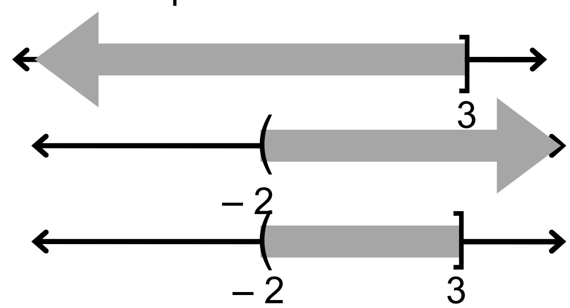
$3x - 4 \le 5$	and	$5 < 4x + 13$
$3x \le 9$	and	$-8 < 4x$
$x \le 3$	and	$-2 < x$ or $x > -2$

Now, graph each solution set and take the overlap as the answer:

$x \le 3$

$x > -2$

Thus, $(-\infty, 3] \cap (-2, \infty)$
$= (-2, 3]$.



Concept #3: Solving Compound Inequalities joined by the word: **or**

If we say that statement #1 *or* statement #2 is true, that means either the first or the second statement is true. In terms of sets, if an element is in set A *or* in set B, that means the element has to be in set A, set B, or both sets or in the union of A and B. Thus, the solution to two inequalities joined with the word "***or***" is the union of their solution sets. To solve a compound inequality with the word "or," we solve each inequality individual and then merge the solution sets together to get the solution to the inequality.

<u>Solve the following:</u>

Ex. 8 $\frac{4}{3}x - 2 < -26$ or $-8x - 3.1 \ge 0.5$

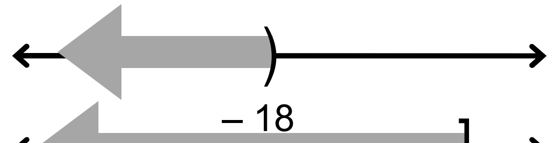
Solution:

First, solve each inequality individually:

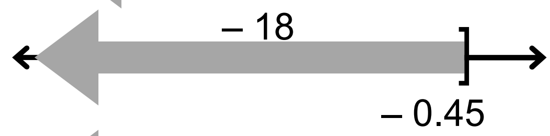
$\frac{4}{3}x - 2 < -26$	or	$-8x - 3.1 \ge 0.5$
$\frac{4}{3}x < -24$	or	$-8x \ge 3.6$
$x < -18$	or	$x \le -0.45$

Now, graph each solution set and merge the two solution sets:

$x < -18$



$x \le -0.45$



Thus, $(-\infty, -18) \cup (-\infty, -0.45]$

$= (-\infty, -0.45]$.

Ex. 9 $\quad -6x + 8 \le 2x - 45 \quad$ or $\quad 6(x + 5) < -13x + 87$

Solution:

First, solve each inequality individually:

$-6x + 8 \le 2x - 45 \quad$ or $\quad 6(x + 5) < -13x + 87$

$-6x + 8 \le 2x - 45 \quad$ or $\quad 6x + 30 < -13x + 87$

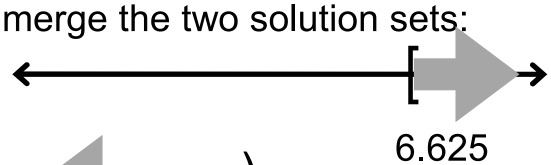
$-8x + 8 \le -45 \quad$ or $\quad 19x + 30 < 87$

$-8x \le -53 \quad$ or $\quad 19x < 57$

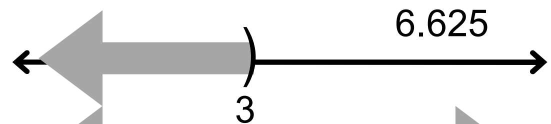
$x \ge 6.625 \quad$ or $\quad x < 3$

Now, graph each solution set and merge the two solution sets:

$x \ge 6.625$

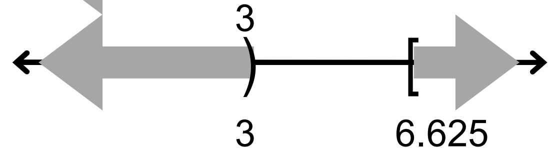


$x < 3$



Thus, $[6.625, \infty) \cup (-\infty, 3)$

$= (-\infty, 3) \cup [6.625, \infty)$.



Ex. 10 $\quad 7x + 4 \ge -24 \quad$ or $\quad 5x < x + 3$

Solution:

First, solve each inequality individually:

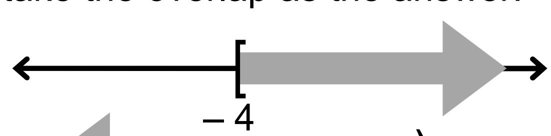
$7x + 4 \ge -24 \quad$ or $\quad 5x < x + 3$

$7x \ge -28 \quad$ or $\quad 4x < 3$

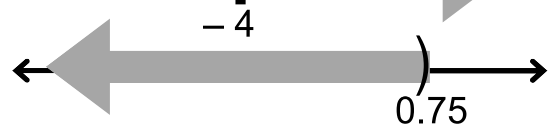
$x \ge -4 \quad$ or $\quad x < 0.75$

Now, graph each solution set and take the overlap as the answer:

$x \ge -4$

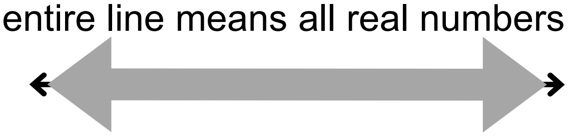


$x < 0.75$



Thus, $[-4, \infty) \cup (-\infty, 0.75)$. The entire line means all real numbers.

$= (-\infty, \infty)$



Concept #4 Applications

Recall some key phrases from chapter 2:

Inequalities

r is greater than 7	$r > 7$
v exceeds w	$v > w$
4 is greater than or equal to d	$4 \geq d$
g is at least 21	$g \geq 21$
e is less than 6	$e < 6$
b is less than or equal to 9	$b \leq 9$
a is at most 12	$a \leq 12$
9 cannot exceed u	$9 \leq u$
d is no more than x	$d \leq x$
e is no less than 4	$e \geq 4$
r is between 8 and 11	$8 < r < 11$
y is between – 5 and 7 inclusively	$-5 \leq y \leq 7$

<u>Set-up a compound inequality and solve the following:</u>

Ex. 11 The original Macintosh can operate safely between the temperatures of 10˚ C and 40˚ C. Find the equivalent range of temperatures (to the nearest whole number) in Fahrenheit using $C = \frac{5}{9}(F - 32^\circ)$.

<u>Solution:</u>

The operation temperature is between 10˚ C and 40˚ C which means:

$$10 < \frac{5}{9}(F - 32^\circ) < 40$$

$$10 < \frac{5}{9}F - \frac{160}{9} < 40$$

$$\mathbf{9}(10) < \mathbf{9}\left(\frac{5}{9}F\right) - \mathbf{9}\left(\frac{160}{9}\right) < \mathbf{9}(40)$$

$$90 < 5F - 160 < 360 \qquad \text{(add 160 to all sides)}$$

$$\underline{+160 = \quad +160 = +160}$$

$$250 < 5F < 520$$

$$\frac{250}{5} < \frac{5F}{5} < \frac{520}{5} \qquad \text{(divide all sides by 5)}$$

$$50^\circ\,F < F < 104^\circ\,F$$

The operational temperature is between 50˚ F and 104˚ F.