Sect 10.3 - Absolute Value Equations

Concept #1 Solving Absolute Value Equations

Recall that the absolute value of a number is the distance that number is from zero. So, $|-4| = 4$ and $|4| = 4$. If we have an equation like $|x| = 4$, this means that $x = 4$ or $x = -4$. So, the solution to the equation is $\{-4, 4\}$. The absolute value has to be equal to a positive number. If we have an equation where the absolute value is equal to a negative number, then the equation has no solution.

Property #1: Absolute Value Equations in the form $|x| = a$
If a is a real number, then

1) If $a < 0$, then $|x| = a$ has no solution.
2) if $a \geq 0$, then $|x| = a$ has solutions of $x = a$ and $x = -a$.

Solve the following:

Ex. 1a $|x| = 7.3$

Ex. 1b $|x| - 5 = -5$

Ex. 1c $-3|y| = -27$

Ex. 1d $|b| + 13 = 7$

Solution:

a) Since $|x| = 7.3$, then the solutions are $\{-7.3, 7.3\}$.

b) We need to first get the absolute value by itself:

$|x| - 5 = -5$ (add 5 to both sides)

$|x| = 0$

Since $|x| = 0$, $x = 0$ and $x = -0$ which means $x = 0$. So, the solution is $\{0\}$.

c) We need to first get the absolute value by itself:

$-3|y| = -27$ (divide both sides by -3)

$|y| = 9$

Since $|y| = 9$, then the solutions are $\{-9, 9\}$.

d) We need to first get the absolute value by itself:

$|b| + 13 = 7$ (subtract 13 from both sides)

$|b| = -6$

But the absolute value cannot equal a negative number, so the solution is $\{\ \}$.

Solving Absolute Value Equations

1) Isolate the absolute value on one side of the equation and simplify the other side to get an equation in the form $|x| = a$.
2) Use property #1.
3) If there is no solution from property one, then we are finished. Otherwise, solve the resulting equations from property #1.
4) Check the solutions. It is possible to get a false solution.

Solve the following:

Ex. 2a $|3x + 4| = 5$ Ex. 2b $|2a - 3| - 6 = -9$

Ex. 2c $-2|3y - 1| = -16$ Ex. 2d $2|2x - 1| + 6 = 14x$

Solution:

a) $|3x + 4| = 5$ (the absolute value is already isolated) (use property #1)

$3x + 4 = 5$ or $3x + 4 = -5$

$3x = 1$ or $3x = -9$

$x = \frac{1}{3}$ or $x = -3$

Check

$|3x + 4| = 5$ $|3x + 4| = 5$

$|3(\frac{1}{3}) + 4| = 5$ $|3(-3) + 4| = 5$

$|1 + 4| = 5$ $|-9 + 4| = 5$

$|5| = 5$ $|-5| = 5$

Checks Checks

The solutions are $\{-3, \frac{1}{3}\}$.

b) $|2a - 3| - 6 = -9$ (isolate the absolute value)

$|2a - 3| = -3$ (the absolute value ≠ negative #)

No solution.

The solution is { }.

c) $-2|3y - 1| = -16$ (isolate the absolute value)

$|3y - 1| = 8$ (use property #1)

$3y - 1 = 8$ or $3y - 1 = -8$

$3y = 9$ or $3y = -7$

$y = 3$ or $y = -\frac{7}{3}$

Check

$-2\,|3(3)-1| = -16$ $\qquad$ $-2\,\left|3\left(-\frac{7}{3}\right)-1\right| = -16$

$-2\,|9-1| = -16$ $\qquad$ $-2\,|-7-1| = -16$

$-2\,|8| = -16$ $\qquad$ $-2\,|-8| = -16$

$-2(8) = -16$ $\qquad$ $-2(8) = -16$

Checks $\qquad$ Checks

The solutions are $\{-\frac{7}{3}, 3\}$.

d) $2|2x-1| + 6 = 14x$ (isolate the absolute value)

$2|2x-1| = 14x - 6$

$|2x-1| = 7x-3$ (use property #1)

$2x-1 = 7x-3$ or $2x-1 = -(7x-3)$

$-5x-1 = -3$ or $2x-1 = -7x+3$

$-5x = -2$ or $9x-1 = 3$

$x = 0.4$ or $9x = 4$

$x = 0.4$ or $x = \frac{4}{9}$

Check

$2|2(0.4)-1| + 6 = 14(0.4)$ or $2\left|2\left(\frac{4}{9}\right)-1\right| + 6 = 14\left(\frac{4}{9}\right)$

$2|0.8-1| + 6 = 5.6$ or $2\left|\frac{8}{9}-1\right| + 6 = \frac{56}{9}$

$2|-0.2| + 6 = 5.6$ or $2\left|\frac{8}{9}-\frac{9}{9}\right| + 6 = \frac{56}{9}$

$0.4 + 6 = 5.6$ or $2\left|-\frac{1}{9}\right| + 6 = \frac{56}{9}$

$6.4 = 5.6$ or $\frac{2}{9} + \frac{54}{9} = \frac{56}{9}$

Does Not Check. $\qquad$ Checks

The solution is $\left\{\frac{4}{9}\right\}$.

Concept #2 Solving equations that have two absolute values.

If we have an equation like $|a| = |b|$, this means that either a and b are equal or have opposites signs. In other words, either $a = b$ or $a = -b$.

Property #2: Absolute Value Equations in the form $|x| = |y|$

If $|x| = |y|$, then $x = y$ or $x = -y$

Solve the following:

Ex. 3a $|5x - 4| = |5x + 8|$ Ex. 3b $|2r - 7| = |3r + 11|$

Ex. 3c $-|4p - 1| = |p + 3|$

Solution:

a) Since $|5x - 4| = |5x + 8|$, then

$5x - 4 = 5x + 8$ or $5x - 4 = -(5x + 8)$

$-4 = 8$ or $5x - 4 = -5x - 8$

No Solution or $10x - 4 = -8$

$10x = -4$

$x = -0.4$

Check

$|5(-0.4) - 4| = |5(-0.4) + 8|$

$|-2 - 4| = |-2 + 8|$

$|-6| = |6|$

$6 = 6$

Checks

The solution is {– 0.4}.

b) Since $|2r - 7| = |3r + 11|$, then

$2r - 7 = 3r + 11$ or $2r - 7 = -(3r + 11)$

$-r - 7 = 11$ or $2r - 7 = -3r - 11$

$-r = 18$ or $5r - 7 = -11$

$r = -18$ or $5r = -4$

$r = -0.8$

Check

$|2(-18) - 7| = |3(-18) + 11|$

$|-36 - 7| = |-54 + 11|$

$|-43| = |-43|$

$43 = 43$ Checks

$|2(-0.8) - 7| = |3(-0.8) + 11|$

$|-1.6 - 7| = |-2.4 + 11|$

$|-8.6| = |8.6|$

$8.6 = 8.6$ Checks

The solutions are {– 18, – 0.8}.

c) Since $-|4p - 1|$ is negative and $|p + 3|$ cannot be negative, there is no solution. The solution is { }.