Sect 10.5 - Linear Inequalities in Two Variables

Concept #1 Graphing a Linear Inequality in Two Variables

Definition

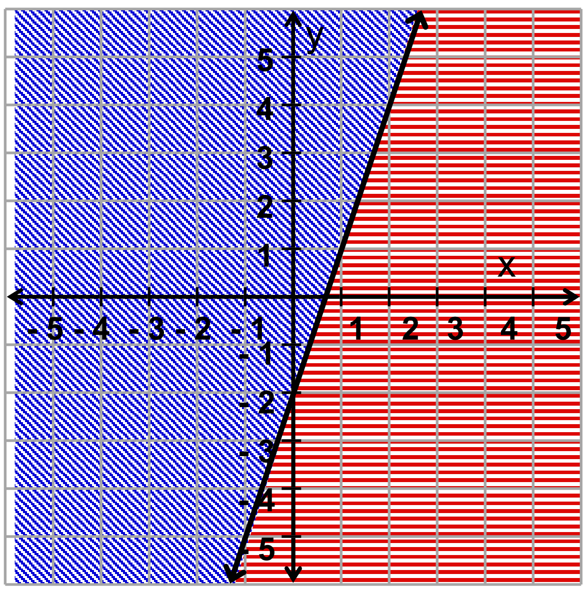
Let a, b, and c be real numbers where a and b are not both zero. Then an inequality that can be written in one of the following forms:

1) $ax + by > c$ 2) $ax + by < c$

3) $ax + by \geq c$ 4) $ax + by \leq c$

is called a **linear inequality in two variables**.

The graph a linear equation in two variables is a straight line. It divides the rectangular coordinate system into two regions. If we were to graph $y = 3x – 2$, its graph (black line) would look like the pictured below:



The line divides the coordinate system into two regions. The first region is above and to the left of the line (blue). The second region is below and to the right of the line (red). One region represents the points where $y > 3x – 2$ and the other region represents the points where $y < 3x – 2$. To determine which is which, we can pick a test point from one of the regions and substitute it into the equation to see which inequality is true. If we pick the point (0, 0) from the blue (left side) and substitute the values in for x and y, we get:

y ? 3x – 2

0 ? 3(0) – 2

0 ? – 2

For this to be true, $0 > – 2$, so the blue region represents the set of points such that $y > 3x – 2$ and the red region represents the set of points such that $y < 3x – 2$. Notice that if our inequality is solved for y, then $y > mx + b$ will be the region **above** the line and the $y < mx + b$ will be the region **below** the line. Whenever we are in doubt, we can always use a test point. This suggests a procedure for graphing linear inequalities.

Graphing Linear Inequalities in two variables

1) Graph the line as if there was an equal sign. Draw a solid line if the inequality is ≥ or ≤ (include the line as part of the solution) and a dashed line if the inequality is > or < (do not include the line as part of the solution).

2) a) If the inequality is solved for y, then shade the region above the line if the inequality is > or ≥ and below the line if the inequality is < or ≤.

b) If the inequality is not solved for y, then pick a test point that is clearly in one of the regions and evaluate the inequality at that point. If the inequality is true, then shade that region. If the inequality is false, shade the opposite region.

<u>Graph the following:</u>

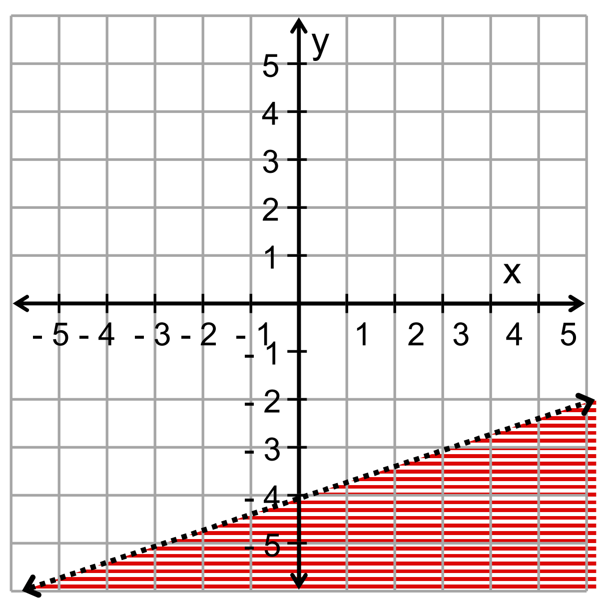
Ex. 1 $-\frac{1}{3}x + y < -4$

<u>Solution:</u>

The inequality is easy to solve for y:

$-\frac{1}{3}x + y < -4$

$y < \frac{1}{3}x - 4$



1) Graph the line as if we had "=" instead of <. Since this is a strict inequality, we will draw a dashed line.

$m = \frac{1}{3}$; y-int: (0, – 4).

2) Since the inequality is solved for y and since the inequality is <, we will shade below the line.

Ex. 2 $3x - 4y \le -12$

<u>Solution:</u>

1) Graph the line as if we had "=" instead of ≤. Since the inequality is ≤, we will draw a solid line. It may be easiest

x	y
0	3
– 4	0
4	6

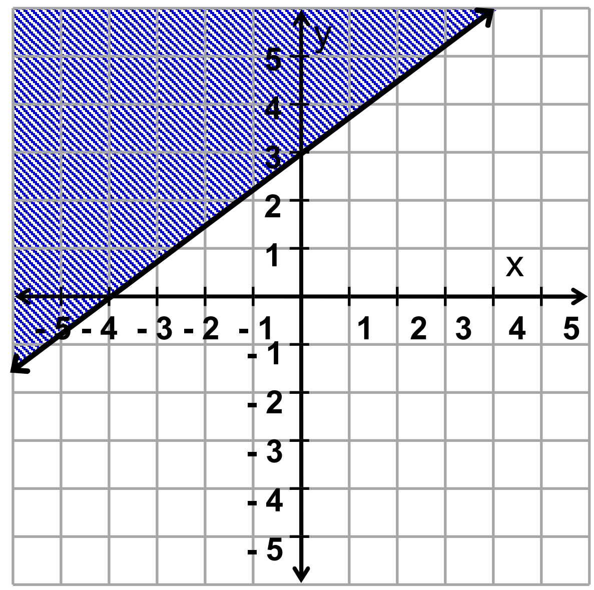
to make a table of values:
Now, plot the points and draw the line:

2) Since the inequality is not solve for y, we will need to use a test point: Let's use (0, 0). Plugging into the original inequality, we get:

$3(0) – 4(0) \le – 12$

$0 \le – 12$ False

So, we want to shade the opposite region:

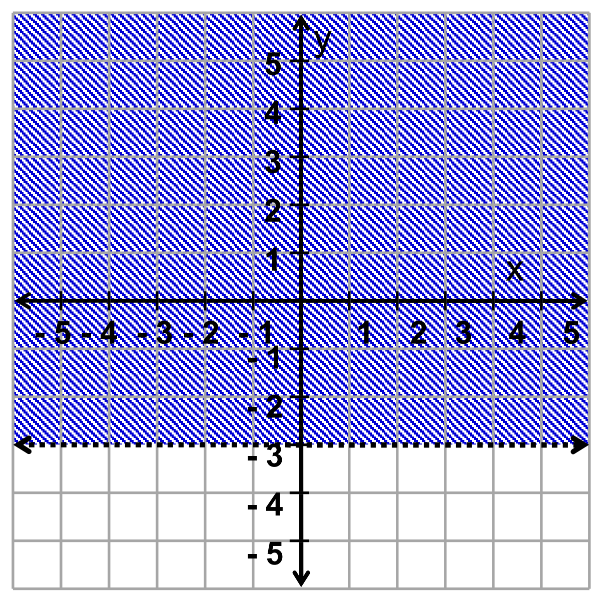


Ex. 3 $y > – 3$

Solution:

1) Graph the line as if we have "=" using a dashed line. The line $y = – 3$ is a horizontal line passing through $y = – 3$.

2) Since the inequality is already solved for y and the inequality is >, we will shade above the line:



Ex. 4 $– \frac{2}{3}x – \frac{3}{5}y \ge – \frac{6}{5}$

Solution:

First, clear fractions:

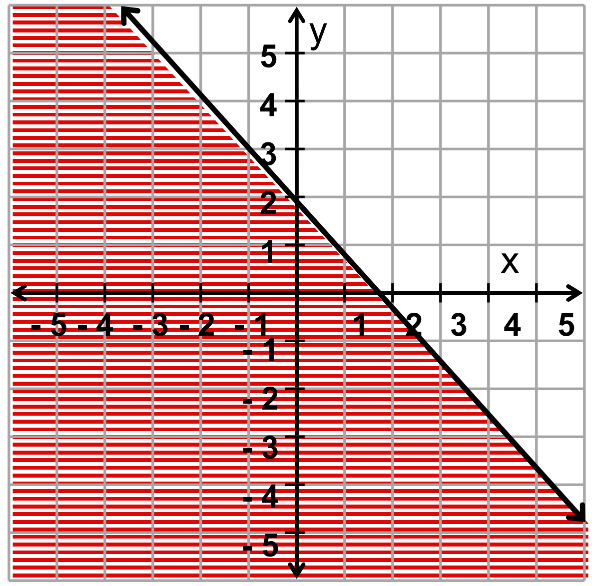
$\mathbf{– 15}(– \frac{2}{3}x) – (\mathbf{– 15})(\frac{3}{5}y) \ge \mathbf{– 15}(– \frac{6}{5})$

$10x + 9y \le 18$

1) Graph the line as if we had "=" instead of ≤. Since the inequality is ≤, we will draw a solid line. It may be easiest to make a table of values:

x	y
0	2
1.8	0
3.6	– 2

Now, plot the points and draw the line:



2) Since the inequality is not solve for y, we will need to use a test point: Let's use (0, 0). Plugging into the original inequality, we get:

$-\frac{2}{3}(0) + -\frac{3}{5}(0) \geq -\frac{6}{5}$

$0 \geq -\frac{6}{5}$ True

So, we want to shade the region containing (0, 0):

Ex. 5 $\frac{4}{5}x < y$

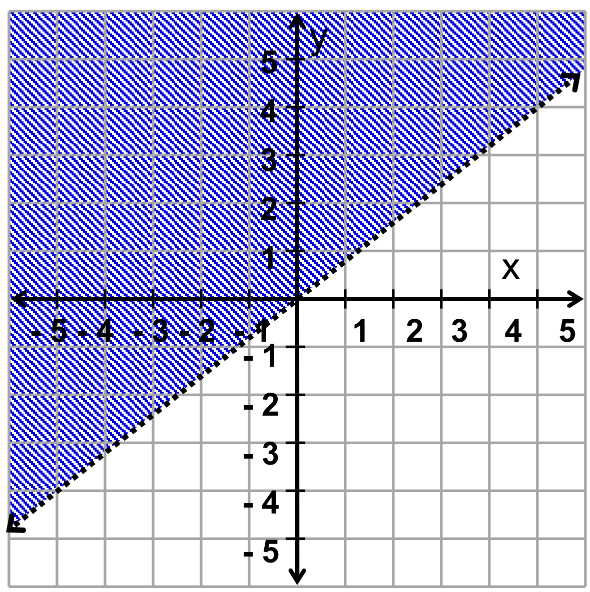
<u>Solution:</u>

Notice that $\frac{4}{5}x < y$ is the same as $y > \frac{4}{5}x$.

1) Graph the line as if we had "=" instead of >. Since this is a strict inequality, we will draw a dashed line.

$m = \frac{4}{5}$; y-int: (0, 0).

2) Since the inequality is already solved for y and the inequality is >, we will then shade above the line:



In the previous example, if we wanted to use a test point, we cannot use the point (0, 0) since it lies on the line. We will need to pick a point that is clearly in one region. We could use the point (0, 5):

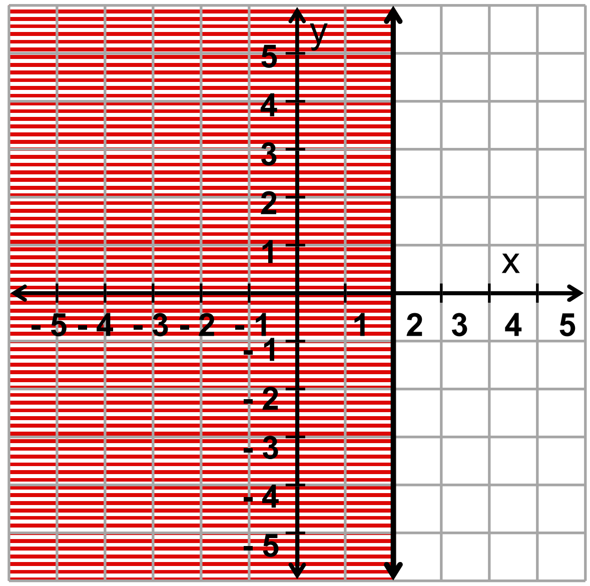
$\frac{4}{5}(0) < 5$

$0 < 5$ True.

Ex. 6 $x \leq 2$

Solution:

1) Graph the line as if we have "=" using a solid line. The line x = 2 is a vertical line passing through x = 2.
2) All the points that have x-coordinate less than 2 are to the left of the line, so we shade to the left.



Concept #2 Compound Linear Inequalities in Two Variables.

In graphing compound linear inequalities in two variables, we graph each linear inequality and either take the union or the intersection of the solutions depending upon the application.

Graph the following system:

Ex. 7 $y < \frac{2}{3}x - 4$ or $y \geq -2x + 4$

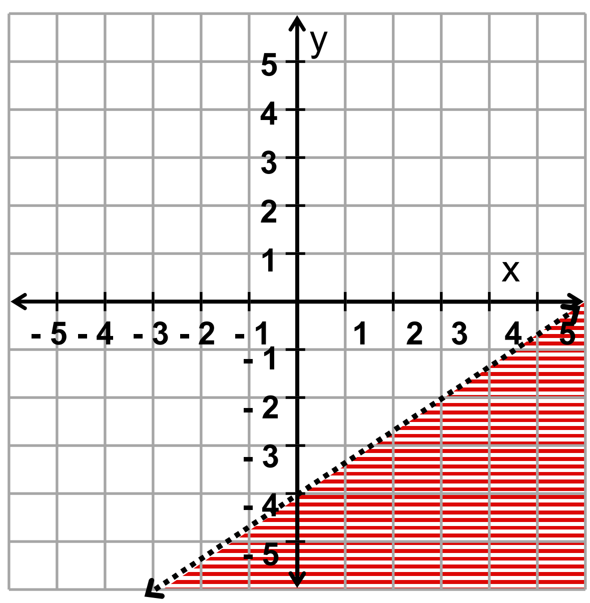
Solution:

First, graph each inequality individual:

$y < \frac{2}{3}x - 4$

1) Graph the line as if we had "=" instead of <. Since this is a strict inequality, we will draw a dashed line.

   $m = \frac{2}{3}$; y-int: (0, – 4).
2) Since the inequality is already solved for y and the inequality is <, we will then shade below the line:

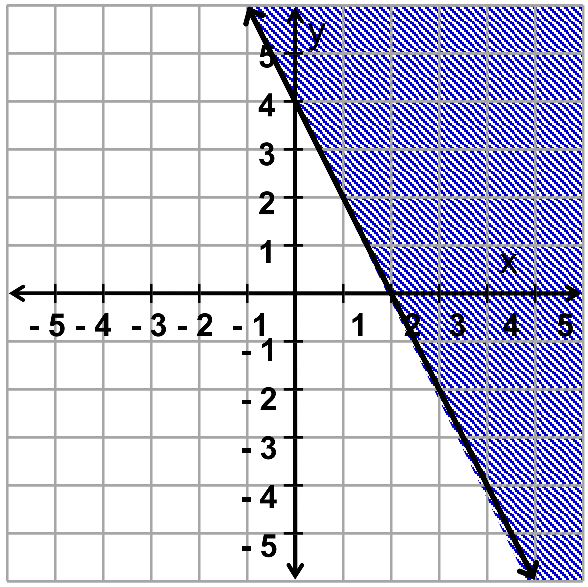


$y \geq -2x + 4$

1) Graph the line as if we had "=" instead of ≥. Since the inequality is ≥, we will draw a solid line.

   $m = \frac{-2}{1}$; y-int: (0, 4).

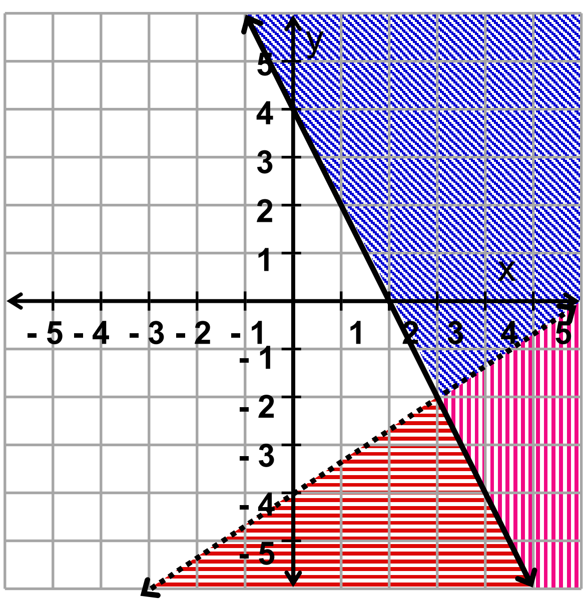
2) Since the inequality is already solved for y and the inequality is ≥, we will then shade above the line:



The word "or" means union so we will merge the two graphs together: $y < \frac{2}{3}x - 4$ or $y \geq -2x + 4$

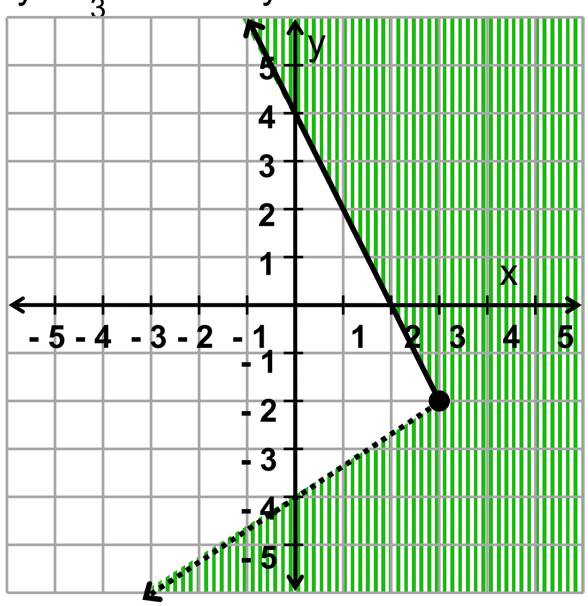
Merged graphs;

Take the Union:



Solution

$y < \frac{2}{3}x - 4$ or $y \geq -2x + 4$



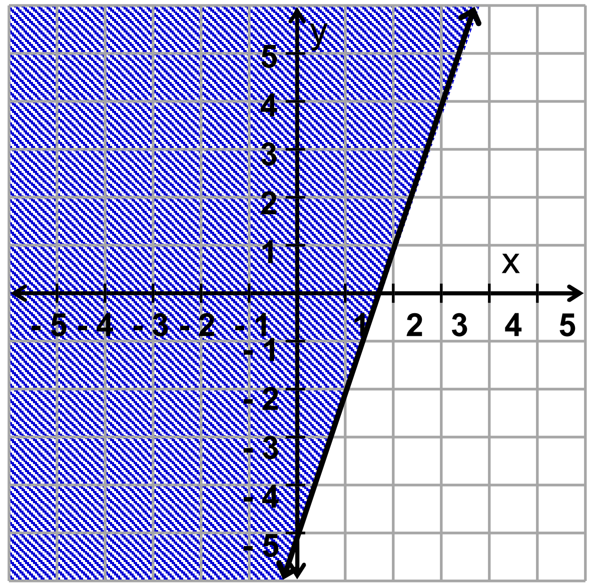
Ex. 8 $y \geq 3x - 5$ and $y > -x + 3$

<u>Solution:</u>

First, graph each inequality individual:

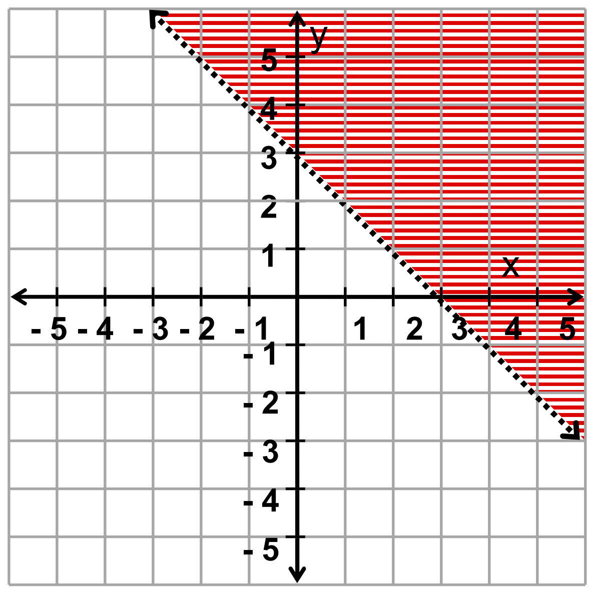
$y \geq 3x - 5$

1) Graph the line as if we had "=" instead of ≥. Since the inequality is ≥, we will draw a solid line.
   $m = 3$; y-int: $(0, -5)$.
2) Since the inequality is already solved for y and the inequality is ≥, we will then shade above the line:



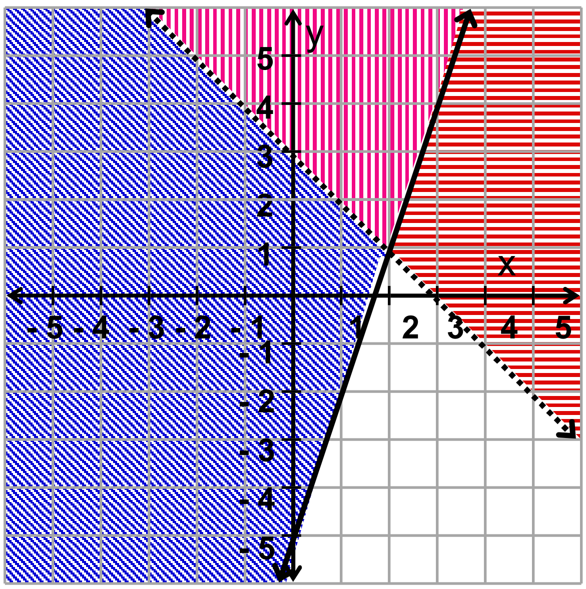
$y > -x + 3$

1) Graph the line as if we had "=" instead of >. Since this is a strict inequality, we will draw a dashed line.
   $m = \frac{-1}{1}$; y-int: $(0, 3)$.
2) Since the inequality is already solved for y and the inequality is >, we will then shade above the line:

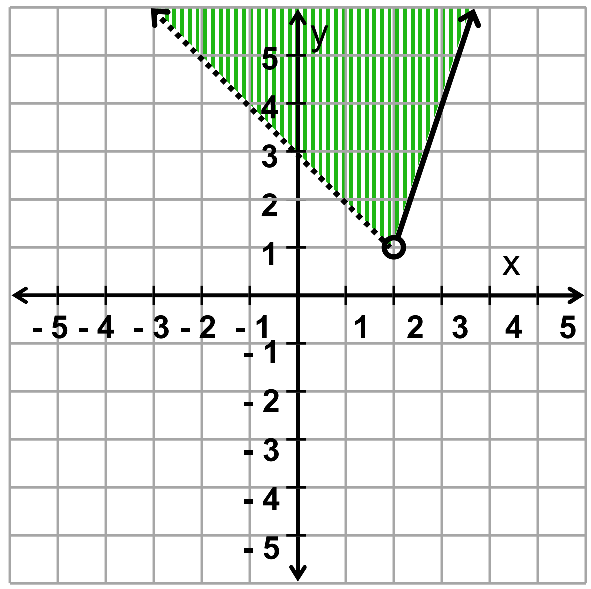


The word "and" means intersection so we see where the graphs overlap.

The purple area is (the vertical lines) where the two inequalities overlap:



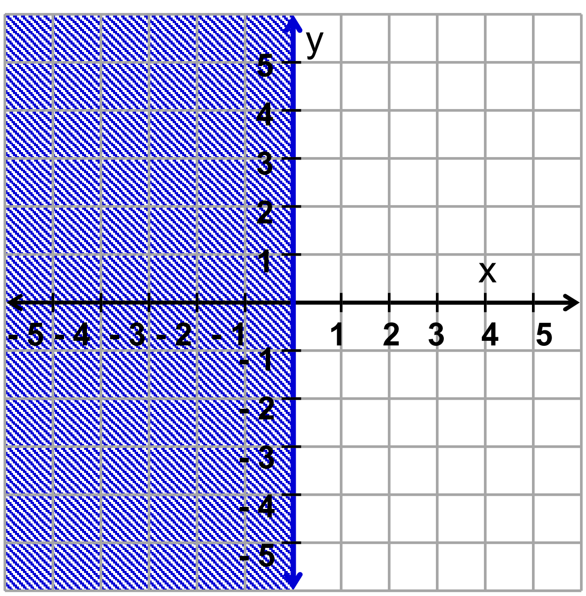
The solution is:
$y \geq 3x - 5$ and $y > -x + 3$



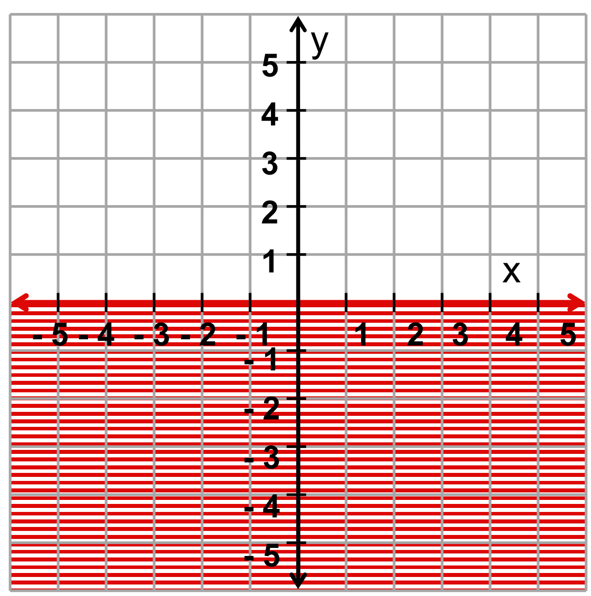
Ex. 9 $x \leq 0$ and $y \leq 0$

<u>Solution:</u>

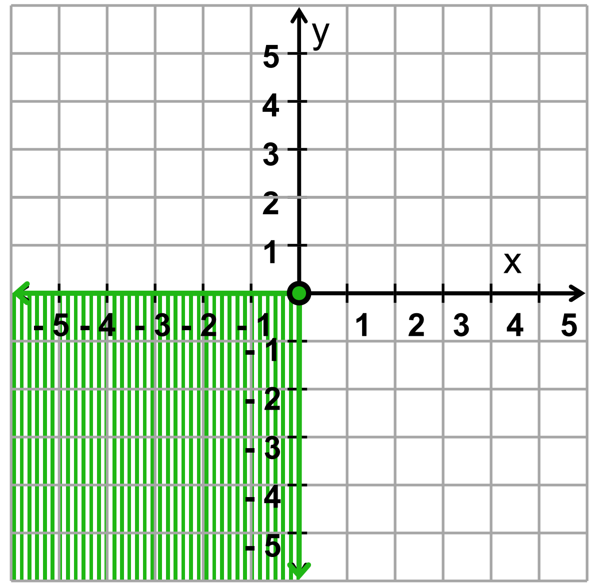
$x \leq 0$ is the y-axis and every piont to the left of y-axis.



$y \leq 0$ is the x-axis and every point below the x-axis.



The intersection of these two inequalities will give us the third quadrant union the negative x-axis union the the negative y-axis union the origin.



Concept #3 Graphing a Feasible Region

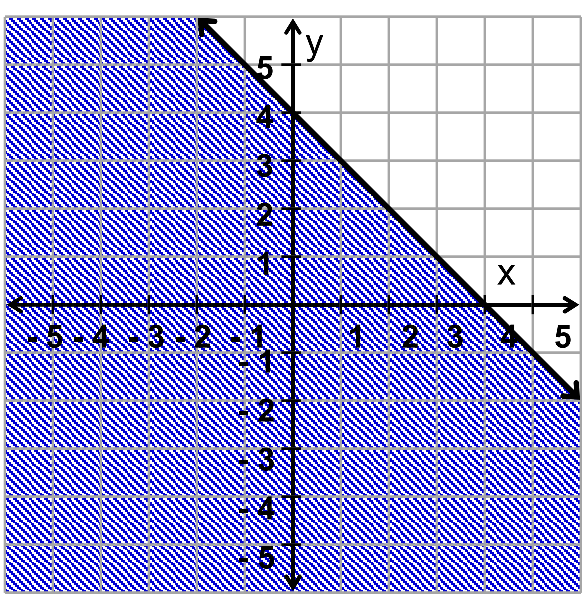
In an application problem, there may be some physical constraints on the inequality or on the system of inequalities. In production of chairs and tables for instance, the number of chairs and the number of tables cannot be negative. The graph of the inequality in this situation would have to be in the first quadrant. Let's try some examples:

<u>Graph the feasible regions:</u>

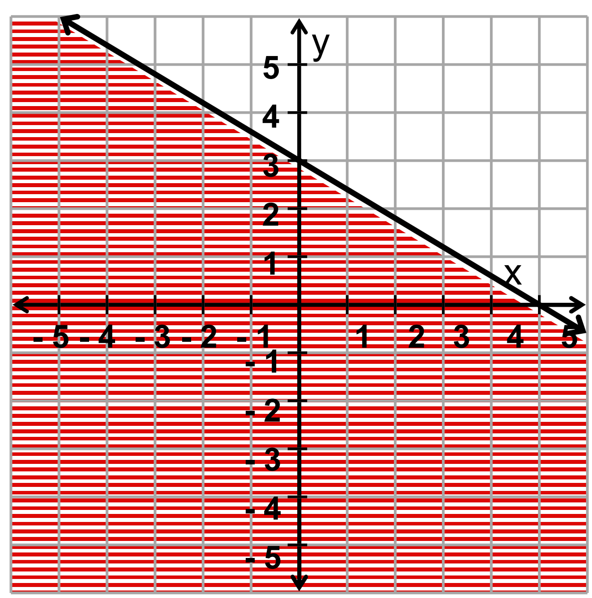
Ex. 10 $x + y \leq 4$, $3x + 5y \leq 15$, $x \geq 0$, and $y \geq 0$

<u>Solution:</u>

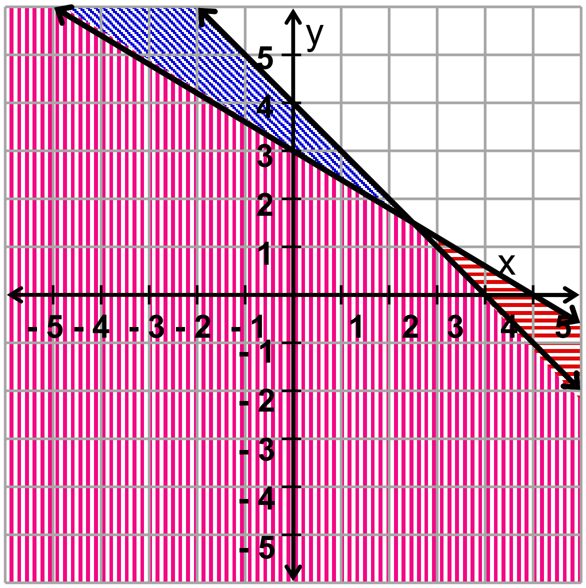
Graph $x + y \leq 4$



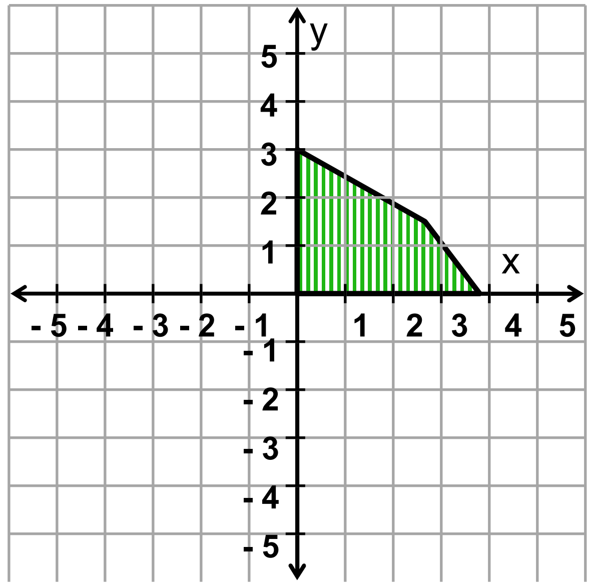
Graph $3x + 5y \leq 15$



Now, intersect the two graphs: The purple part (vertical lines) is where the two graphs overlap:



Since $x \geq 0$ and $y \geq 0$ is the 1st quadrant and the non-negative x and y-axes, then we only use that part of the overlap as our solution:



Ex. 11 The No Points in Space Corporation makes DVD recorders and DVD players. The total number of recorders and players the company produces in a day cannot exceed 600 units due to production limitations. Also, the number of players must be least equal to double the number of recorders produced.

a) Graph the feasible region.

b) Is the point (225, 150) in the feasible region?

<u>Solution:</u>

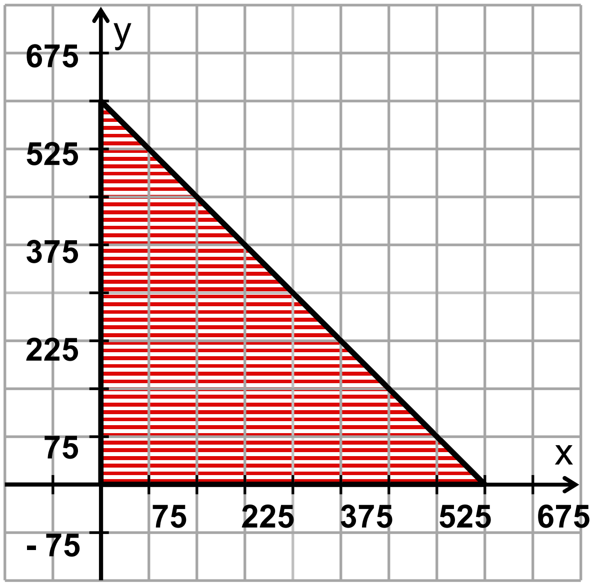
Let x = the number of DVD players produced
and y = the number of DVD recorders produced

Since a negative number of players and recorders cannot be produced, then $x \geq 0$ and $y \geq 0$. This represents the first quadrant and the non-negative x and y-axes. Rather than draw all four quadrants, we will only draw the first quadrant of our graph.

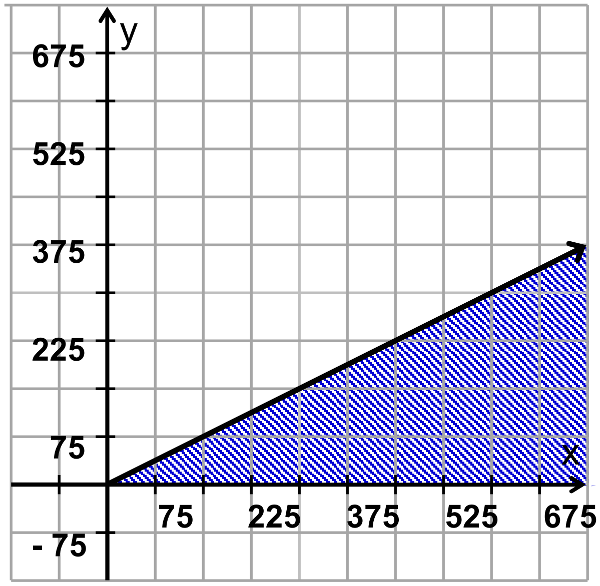
The total number of recorders and players cannot exceed 600 means: $x + y \leq 600$

The number of players must be least equal to double the number of recorders produced: $x \geq 2y$ or $y \leq \frac{1}{2}x$

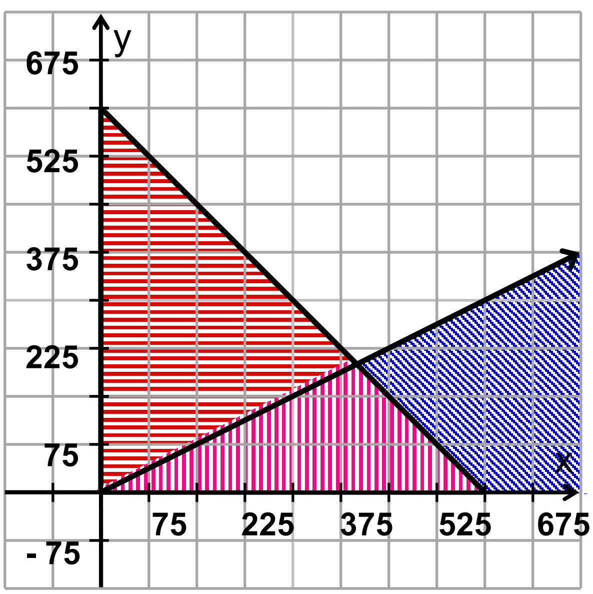
Graph $x + y \le 600$ in the 1st quadrant:



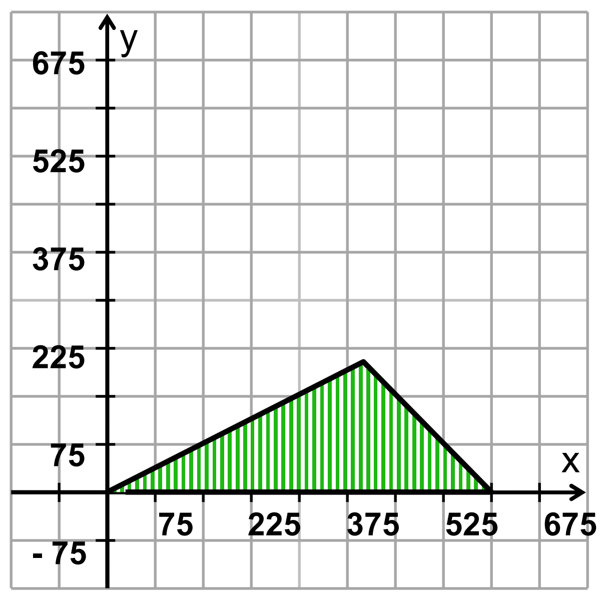
Graph $y \le \frac{1}{2}x$ in the 1st quadrant:



Now, intersect the two graphs. The purple region (vertical lines) is where they overlap.



Thus, the feasible region is:



b) No, (225, 150) is above the feasible region. The company cannot produce 225 players and 150 recorders in one day.