Sect 11.4 – Addition and Subtraction of Radicals

Concept #1 Definition of Like Radicals.

When adding or subtracting algebra expression, we were only able to add or subtract terms that were like terms. Two terms are like terms if they have the exactly same variables with exactly the same corresponding exponents. So, terms like $3x^2y$ and $-7x^2y$ are like terms, whereas terms like $6x^3y$ and $-7x^3z$ are not and terms like $4x^3y^2$ and $-3x^4y^2$ are not. Only like terms can be combined. The same is true for radicals; in order to combine two radicals, the indices (think exponent) have to be the same and the radicands (think variables) also have to match. Such radicals are called like radicals.

Definition of Like Radicals
Two radical terms are **like radicals** if the have the same indices and the same radicands.

Determine which pair of radicals are like radicals:

Ex. 1a $3\sqrt{11a}$ and $-5\sqrt{11a}$ Ex. 1b $-3\sqrt[3]{3x}$ and $5\sqrt[4]{3x}$

Ex. 1c $y\sqrt[4]{7}$ and $y\sqrt[4]{5}$ Ex. 1d $\sqrt[3]{5x^2y}$ and $4\sqrt[3]{5x^2y}$

Solution:

a) The indices are 2 and the radicands are 11a, so $3\sqrt{11a}$ and $-5\sqrt{11a}$ are like radicals.

b) The indices are different ($3 \neq 4$), so $-3\sqrt[3]{3x}$ and $5\sqrt[4]{3x}$ are not like radicals.

c) The radicands are different ($7 \neq 5$), so $y\sqrt[4]{7}$ and $y\sqrt[4]{5}$ are not like radicals.

d) The indices are 3 and the radicands are $5x^2y$, so $\sqrt[3]{5x^2y}$ and $4\sqrt[3]{5x^2y}$ are like radicals.

Concept #2 Addition and Subtraction of Radicals

Much in the same way as adding and subtracting terms, only like radicals can be combined by combining their numerical coefficients. The distributive property is what makes this process legal. If we are simplifying

$3\sqrt[4]{7} - \sqrt[4]{7} + 6\sqrt[4]{7}$, we can factor out $\sqrt[4]{7}$ from all three terms and simplify:

$$\sqrt[4]{7}\,(3 - 1 + 6) = \sqrt[4]{7}\,(8) = 8\sqrt[4]{7}.$$

We always write the numerical coefficient in front on the radical just as we write the numerical coefficient in front of a term.

Simplify the following:

Ex. 2a $5.4\sqrt[3]{3} - 6.3\sqrt[3]{3} - \sqrt[3]{3}$

Ex. 2b $6\sqrt{11} - 5\sqrt{11} + 7\sqrt[3]{11}$

Ex. 2c $-8\sqrt[5]{ab^2} + 2\sqrt[5]{ab^2} - 7\sqrt[5]{ab^2}$

Ex. 2d $\frac{3}{14}r\sqrt{7x} + \frac{2}{3}r\sqrt{7z} - \frac{5}{21}r\sqrt{7x}$

Ex. 2e $\sqrt[5]{8} + \sqrt[5]{8} + \sqrt[5]{8}$

Solution:

a) $5.4\sqrt[3]{3} - 6.3\sqrt[3]{3} - \sqrt[3]{3}$ (all the terms are like radicals)
Since $5.4 - 6.3 - 1 = -1.9$, then
$5.4\sqrt[3]{3} - 6.3\sqrt[3]{3} - \sqrt[3]{3} = -1.9\sqrt[3]{3}$.

b) Only the first two terms are like radicals. Since $6 - 5 = 1$, then
$6\sqrt{11} - 5\sqrt{11} + 7\sqrt[3]{11} = 1\sqrt{11} + 7\sqrt[3]{11} = \sqrt{11} + 7\sqrt[3]{11}$.

c) $-8\sqrt[5]{ab^2} + 2\sqrt[5]{ab^2} - 7\sqrt[5]{ab^2}$ (all the terms are like radicals)
Since $-8 + 2 - 7 = -13$, then
$-8\sqrt[5]{ab^2} + 2\sqrt[5]{ab^2} - 7\sqrt[5]{ab^2} = -13\sqrt[5]{ab^2}$.

d) Only the first and last terms are like radicals. Since $\frac{3}{14} - \frac{5}{21}$
$= \frac{9}{42} - \frac{10}{42} = -\frac{1}{42}$, then
$\frac{3}{14}r\sqrt{7x} + \frac{2}{3}r\sqrt{7z} - \frac{5}{21}r\sqrt{7x} = -\frac{1}{42}\sqrt{7x} + \frac{2}{3}r\sqrt{7z}$.

e) $\sqrt[5]{8} + \sqrt[5]{8} + \sqrt[5]{8}$ (all the terms are like radicals)
$\sqrt[5]{8} + \sqrt[5]{8} + \sqrt[5]{8} = 1\sqrt[5]{8} + 1\sqrt[5]{8} + 1\sqrt[5]{8} = 3\sqrt[5]{8}$.

A common mistake that people make is to add the radicands.

$\sqrt[5]{8} + \sqrt[5]{8} + \sqrt[5]{8} \neq \sqrt[5]{24}$ and

$\sqrt{x} + \sqrt{y} \neq \sqrt{x+y}$.

Ex. 3a $3\sqrt{175} - 6\sqrt{28} + \sqrt{98}$

Ex. 3b $8\sqrt{2x^9y^4} - 2x^4y^2\sqrt{32x}$

Ex. 3c $-5x\sqrt[3]{3y^4} - 17xy\sqrt[3]{24y} + 6y\sqrt[3]{375x^3y}$

Solution:

None of these have any like radicals at the outset. But, we can first simplify each radical and then see if any like radicals develop:

a) $3\sqrt{175} = 3\sqrt{25 \bullet 7} = 3\sqrt{5^2 \bullet 7} = 3 \bullet 5\sqrt{7} = 15\sqrt{7}$,

$-6\sqrt{28} = -6\sqrt{4 \bullet 7} = -6\sqrt{2^2 \bullet 7} = -6 \bullet 2\sqrt{7} = -12\sqrt{7}$, and

$\sqrt{98} = \sqrt{49 \bullet 2} = \sqrt{7^2 \bullet 2} = 7\sqrt{2}$. So,

$3\sqrt{175} - 6\sqrt{28} + \sqrt{98} = \mathbf{15}\sqrt{7} \mathbf{- 12}\sqrt{7} + 7\sqrt{2}$

$= \mathbf{3}\sqrt{7} + 7\sqrt{2}$.

b) $8\sqrt{2x^9y^4} = 8\sqrt{(x^4)^2(y^2)^2 \bullet 2x} = 8\sqrt{(x^4)^2(y^2)^2} \bullet \sqrt{2x}$

$= 8\,|\,x^4y^2\,|\,\sqrt{2x}$, but x^4 and y^2 are ≥ 0, so,

$8\,|\,x^4y^2\,|\,\sqrt{2x} = 8x^4y^2\sqrt{2x}$.

$-2x^4y^2\sqrt{32x} = -2x^4y^2\sqrt{16 \bullet 2x} = -2x^4y^2\sqrt{4^2 \bullet 2x}$

$= -2x^4y^2\sqrt{4^2} \bullet \sqrt{2x} = -2x^4y^2 \bullet 4\sqrt{2x} = -8x^4y^2\sqrt{2x}$.

Thus, $8\sqrt{2x^9y^4} - 2x^4y^2\sqrt{32x} = 8x^4y^2\sqrt{2x} - 8x^4y^2\sqrt{2x} = 0$.

c) $-5x\sqrt[3]{3y^4} = -5x\sqrt[3]{y^3 \bullet 3y} = -5x\sqrt[3]{y^3} \bullet \sqrt[3]{3y} = -5xy\sqrt[3]{3y}$,

$-17xy\sqrt[3]{24y} = -17xy\sqrt[3]{8 \bullet 3y} = -17xy\sqrt[3]{2^3 \bullet 3y}$

$= -17xy\sqrt[3]{2^3} \bullet \sqrt[3]{3y} = -17xy \bullet 2\sqrt[3]{3y} = -34xy\sqrt[3]{3y}$,

and $6y\sqrt[3]{375x^3y} = 6y\sqrt[3]{125x^3 \bullet 3y} = 6y\sqrt[3]{5^3x^3} \bullet \sqrt[3]{3y}$

$= 6y \bullet 5x\sqrt[3]{3y} = 30xy\sqrt[3]{3y}$. Thus,

$-5x\sqrt[3]{3y^4} - 17xy\sqrt[3]{24y} + 6y\sqrt[3]{375x^3y}$

$= -5xy\sqrt[3]{3y} - 34xy\sqrt[3]{3y} + 30xy\sqrt[3]{3y} = -9xy\sqrt[3]{3y}$.

Concept #3 Translations

Translate the following into an expression. Simplify if possible.

Ex. 4a The difference between the square root of a non-negative number cubed and the square of 7.

Ex. 4b Seven times the cube root of a number raised to the fourth power increased by the product of twice the same number and the cube root of the same number.

Solution:

a) Let's break it down into smaller pieces:

"**the square root of a non-negative number cubed**:" $\sqrt{x^3}$

"*the square of 7*:" 7^2

"The difference between **the square root of a non-negative number cubed** and *the square of 7*:" becomes:

"The difference between $\sqrt{x^3}$ and 7^2:"

$$\sqrt{x^3} - 7^2 = \sqrt{x^2 \bullet x} - 7^2 = |x|\sqrt{x} - 49$$

But, the number is non-negative, so $|x| = x$.

Thus, $|x|\sqrt{x} - 49 = x\sqrt{x} - 49$.

b) Let's break it down into smaller pieces:

"the cube root of a number raised to the fourth power:" $\sqrt[3]{x^4}$

"Seven times $\sqrt[3]{x^4}$:" $7\sqrt[3]{x^4}$

"twice the same number:" $2x$

"the cube root of the same number:" $\sqrt[3]{x}$

"the product of 2x and $\sqrt[3]{x}$:" $2x\sqrt[3]{x}$

"$7\sqrt[3]{x^4}$ increased by $2x\sqrt[3]{x}$:" $7\sqrt[3]{x^4} + 2x\sqrt[3]{x}$

But, $7\sqrt[3]{x^4} = 7\sqrt[3]{x^3 \bullet x} = 7x\sqrt[3]{x}$.

So, $7\sqrt[3]{x^4} + 2x\sqrt[3]{x} = 7x\sqrt[3]{x} + 2x\sqrt[3]{x} = 9x\sqrt[3]{x}$.