Sect 3.4 - The Slope-Intercept Form

Concepts #1 and #2 Slope-Intercept Form of a line

Recall the following definition from the beginning of the chapter:

Let a, b, and c be real numbers where a and b are not both zero. Then an equation that can be written in the form:

ax + by = c is called a **linear equation in two variables**

If a, b, and c happen to integers with a > 0, then we call this the standard form of a linear equation.

A linear equation in two variables is said to be written in **standard form** if it is in the form: Ax + By = C, where A, B, and C are integers and both A and B cannot be zero.

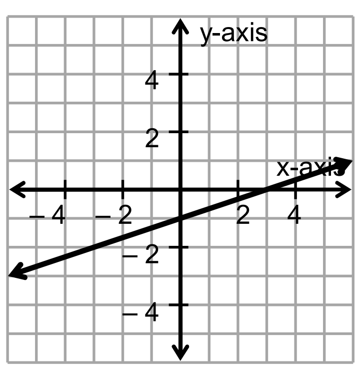
We also saw that if the linear equation is solved for y, the constant term was the y-coordinate of the y-intercept. In other words, if y = mx + b, then (0, b) is the y-intercept. Now, let us find the slope of the line. The y-intercept, (0, b), is one point on the line. If we let x = 1, then y = m(1) + b = m + b. So, (1, m + b) is a second point on the line. The slope is equal to $\frac{y_2 - y_1}{x_2 - x_1} = \frac{m + b - b}{1 - 0} = \frac{m}{1}$ = m. Thus, the coefficient of the x-term is m.

Slope-intercept form

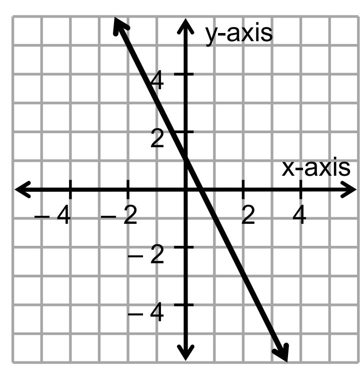
A linear equation in two variables is said to be written in **slope-intercept form** if it is in the form y = mx + b where m is the slope of the line and the point (0, b) is the y-intercept.

Given the graph below, find a) the slope, b) y-intercept, and c) the equation of the line:

Ex. 1



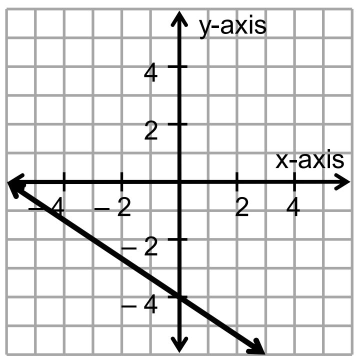
Ex. 2



Solution:
a) The graph cross the y-axis at – 1, so the y-intercept is (0, – 1).
b) Moving from the point (0, – 1) to (3, 0), we have to rise 1 unit and run 3 units. So, the slope is $\frac{\text{"rise"}}{\text{"run"}} = \frac{1}{3}$.
c) Since m = $\frac{1}{3}$ and b = – 1, the equation is y = $\frac{1}{3}$x – 1.

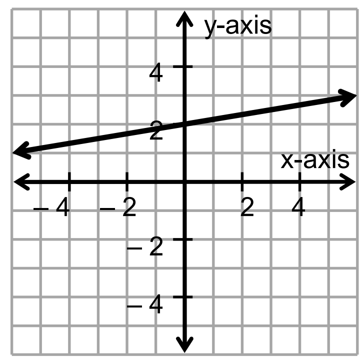
Solution:
a) The graph cross the y-axis at 1, so the y-intercept is (0, 1).
b) Moving from the point (0, 1) to (1, – 1), we have to fall 2 units and run 1 unit. So, the slope is $\frac{\text{"rise"}}{\text{"run"}} = \frac{-2}{1}$ = – 2.
c) Since m = – 2 and b = 1, the equation is y = – 2x + 1.

Ex. 3



Solution:
a) The graph cross the y-axis at – 4, so the y-intercept is (0, – 4).
b) Moving from the point (0, – 4) to (3, – 6), we have to fall 2 units and run 3 units. So, the slope is $\frac{\text{"rise"}}{\text{"run"}} = \frac{-2}{3} = -\frac{2}{3}$.
c) Since m = $-\frac{2}{3}$ and b = – 4, the equation is y = $-\frac{2}{3}$x – 4.

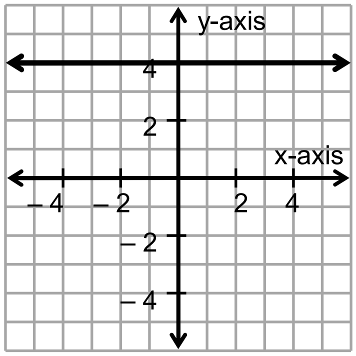
Ex. 4



Solution:
a) The graph cross the y-axis at 2, so the y-intercept is (0, 2).
b) Moving from the point (0, 2) to (6, 3), we have to rise 1 unit and run 6 units. So, the slope is $\frac{\text{"rise"}}{\text{"run"}} = \frac{1}{6}$.
c) Since m = $\frac{1}{6}$ and b = 2, the equation is y = $\frac{1}{6}$x + 2.

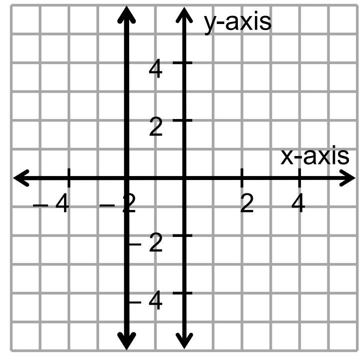
We can also go “backwards” to find the slope. In example #3, if we move from (0, – 4) to (– 3, – 2), we would rise 2 units and run 3 units backwards, So, our slope would be $\frac{\text{"rise"}}{\text{"run"}} = \frac{2}{-3} = -\frac{2}{3}$. Likewise, in example #4, if we went from (0, 2) to (– 6, 1), we would fall 1 unit and run 6 units backwards. So, our slope would be $\frac{\text{"rise"}}{\text{"run"}} = \frac{-1}{-6} = \frac{1}{6}$.

Ex. 5



Solution:
a) The graph cross the y-axis at 4, so the y-intercept is (0, 4).
b) This is a horizontal line so, m = 0.
c) The equation of a horizontal line is in the form y = #. Since the line crosses the y-axis at 4, then the equation is y = 4.

Ex. 6



Solution:
a) The graph does not cross the y-axis, so there is no y-intercept.

b) This is a vertical line so, m is undefined.
c) The equation of a vertical line is in the form x = #. Since the line crosses the x-axis at – 2, then the equation is x = – 2.

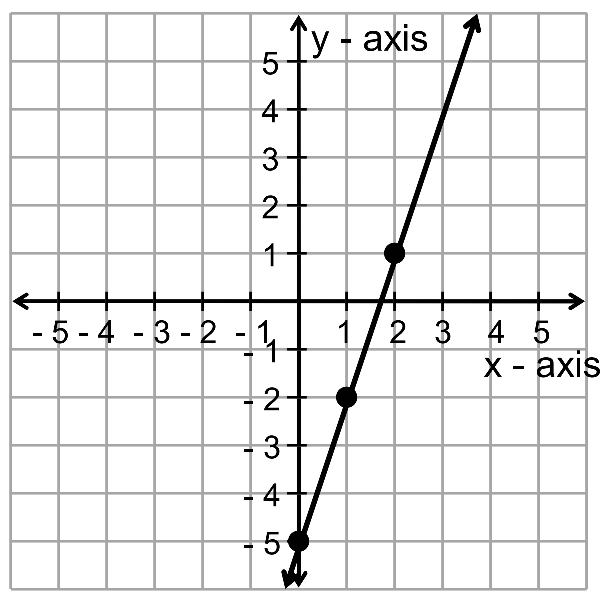
<u>Find the slope and y-intercept and sketch the graph:</u>

Ex. 7 $-3x + y = -5$

Solution:
First solve the equation for y:

$$\begin{array}{r} -3x + y = -5 \\ \underline{+3x \qquad = +3x} \\ y = 3x - 5 \end{array}$$

The slope is $3 = \frac{3}{1} = \frac{\text{"rise"}}{\text{"run"}}$ and the y-intercept is (0, – 5). Plot the point (0, – 5). Then from that point rise 3 units and run 1 unit to get another point. From that new point, rise another 3 units and run 1 more unit to get the third point. Now, draw the graph.



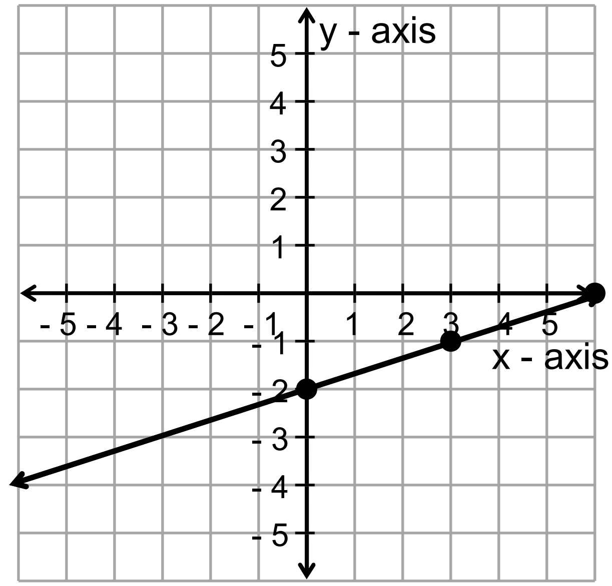
Ex. 8 $2x - 6y = 12$

<u>Solution:</u>

First solve the equation for y:

$$2x - 6y = 12$$
$$\underline{-2x \qquad = -2x}$$
$$\frac{-6y}{-6} = \frac{-2x + 12}{-6}$$
$$y = \frac{1}{3}x - 2$$

The slope is $\frac{1}{3} = \frac{\text{"rise"}}{\text{"run"}}$ and the y-intercept is (0, – 2). Plot the point (0, – 2). Then from that point rise 1 unit and run 3 units to get another point. From that new point, rise another 1 unit and run 3 more units to get the third point. Now, draw the graph.



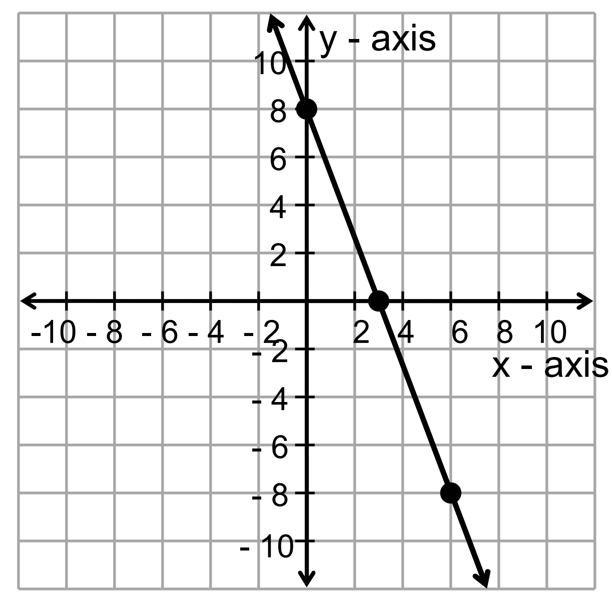
Ex. 9 $8x + 3y = 24$

<u>Solution:</u>

First solve the equation for y:

$$8x + 3y = 24$$
$$\underline{-8x \qquad = -8x}$$
$$\frac{3y}{3} = \frac{-8x + 24}{3}$$
$$y = -\frac{8}{3}x + 8$$

The slope is $-\frac{8}{3} = \frac{-8}{3} = \frac{\text{"rise"}}{\text{"run"}}$ and the y-intercept is (0, 8). Plot the point (0, 8). Then from that point fall 8 units and run 3 units to get another point. From that new point fall another 8 units and run 3 more units to get the third point. Now, draw the graph.



Concept #3 Determining Whether Two Lines are Parallel, Perpendicular, or Neither.

Determine if the lines are parallel, perpendicular, or neither.

Ex. 10 $8x - 7y = 6$
$8y = -7x + 3$

Solution:
First, solve each equation for y to find the slope.

$8x - 7y = 6$
$\underline{-8x \quad = -8x}$
$\frac{-7y}{-7} = \frac{-8x + 6}{-7}$
$y = \frac{8}{7}x - \frac{6}{7}$
$m_1 = \frac{8}{7}$
$\frac{8y}{8} = \frac{-7x + 3}{8}$
$y = -\frac{7}{8}x + \frac{3}{8}$
$m_2 = -\frac{7}{8}$

Since $m_1 \bullet m_2 = \frac{8}{7}\left(-\frac{7}{8}\right)$ $= -1$, the lines are perpendicular.

Ex. 11 $4x - 2 = y$
$4x + y = 5$

Solution:
First, solve each equation for y to find the slope.

$4x - 2 = y$
$y = 4x - 2$
$m_1 = 4$
$4x + y = 5$
$\underline{-4x = -4x}$
$y = -4x + 5$
$m_2 = -4$

But, $m_1 \neq m_2$, so they are not parallel. Also, $m_1 \bullet m_2 = 4(-4)$ $= -16 \neq -1$, so they are not perpendicular. There the lines are neither.

Ex. 12 $\frac{2}{9}x - \frac{1}{6}y = 1$
$1.2x - 0.9y = 1.8$

Solution:
First, solve each equation for y to find the slope.

$\frac{18}{1}\left(\frac{2}{9}x\right) - \frac{18}{1}\left(\frac{1}{6}y\right) = 18(1)$
$\frac{2}{1}\left(\frac{2}{1}x\right) - \frac{3}{1}\left(\frac{1}{1}y\right) = 18(1)$
$4x - 3y = 18$
$\underline{-4x \quad = -4x}$
$\frac{-3y}{-3} = \frac{-4x + 18}{-3}$
$y = \frac{4}{3}x - 6$
$m_1 = \frac{4}{3}$

$10(1.2x) - 10(0.9y) = 10(1.8)$
$12x - 9y = 18$
$\underline{-12x \quad = -12x}$
$\frac{-9y}{-9} = \frac{-12x + 18}{-9}$
$y = \frac{4}{3}x - 2$
$m_2 = \frac{4}{3}$

Since $m_1 = m_2$, then the lines are parallel.

Concept #4 Writing the Equation of the Line Given Its Slope and y-Intercept

Use the given information to write the equation of the line:

Ex. 13 The slope of the line is $\frac{3}{8}$ and it passes through (0, – 2).

Solution:

m = $\frac{3}{8}$ and (0, b) = (0, – 2) which implies b = – 2.

Plugging into y = mx + b, we get:

$y = \frac{3}{8}x - 2$

So, the equation is $y = \frac{3}{8}x - 2$.

Ex. 14 The slope of the line is – 2 and it passes through (0, 1)

Solution:

m = – 2 and (0, b) = (0, 1) which implies b = 1.

Plugging into y = mx + b, we get:

y = – 2x + 1

So, the equation is y = – 2x + 1.

Ex. 15 The slope of the line is 0 and it passes through (– 2, 5).

Solution:

m = 0, but we do not have a y-intercept. A line with a slope of 0 is a horizontal line. Recall that a horizontal is in the form of y = constant. Since it has to pass through the point (– 2, 5), then y has to be 5.

So, the equation is y = 5.

Ex. 16 The slope of the line is undefined and it passes through (– 2, 5).

Solution:

m is undefined and we do not have a y-intercept. A line with a slope that is undefined is a vertical line. Recall that a vertical is in the form of x = constant. Since it has to pass through the point (– 2, 5), then x has to be – 2.

So, the equation is x = – 2.