Sect 3.5 - Point-Slope Formula

Concept #1 Writing the Equation of the Line Using the Point-Slope Formula

To see where the point-slope form comes from, let us examine the formula for calculating the slope:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (rewrite the formula)

$\frac{y_2 - y_1}{x_2 - x_1} = m$ (replace (x_2, y_2) with the variables x and y)

$\frac{y - y_1}{x - x_1} = m$ (clear fractions by multiply both sides by $(x - x_1)$)

$\frac{y - y_1}{x - x_1}(x - x_1) = m(x - x_1)$

$\frac{y - y_1}{x - x_1}\frac{(x - x_1)}{1} = m(x - x_1)$ (reduce)

$y - y_1 = m(x - x_1)$ This is a point-slope formula.

A linear equation in two variables is said to be written in **point-slope form** if it is in the form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is any known point on the line.

Given the following information, find and write the equation in a) point-slope form, b) slope-intercept form, and c) standard form.

Ex. 1 A line with slope of 6 passing through the point (9, 5).

Solution:

$m = 6$; $(x_1, y_1) = (9, 5)$

$y - y_1 = m(x - x_1)$

a) $y - 5 = 6(x - 9)$ pt -slope

$y - 5 = 6x - 54$

$+ 5 = \quad + 5$

b) $y = 6x - 49$ slope-int

Ex. 2 A line with slope of $-\frac{2}{3}$ passing through the point (– 8, 4).

Solution:

$m = -\frac{2}{3}$; $(x_1, y_1) = (-8, 4)$

$y - y_1 = m(x - x_1)$

a) $y - 4 = -\frac{2}{3}(x - (-8))$ pt-slope

$y - 4 = -\frac{2}{3}(x + 8)$

$y - 4 = -\frac{2}{3}x - \frac{16}{3}$

$+ 4 = \quad + \frac{12}{3}$

b) $y = -\frac{2}{3}x - \frac{4}{3}$ slope-int

$y = 6x - 49$

$-6x = -6x$

$-6x + y = -49$

$-1 \quad -1$

c) $6x - y = 49$ std form

$3(y) = 3\left(-\frac{2}{3}x\right) - 3\left(\frac{4}{3}\right)$

$3y = -2x - 4$

$+2x = +2x$

c) $2x + 3y = -4$ std form

Concept #2 Writing the Equation of the Line Through Two Points

Ex. 3 A line that passes through the points (2, – 3) and (– 5, – 1).

Solution:

First, find the slope:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-3)}{-5-2}$

$= \frac{2}{-7} = -\frac{2}{7}$.

Now, pick one of the points.

$m = -\frac{2}{7}$; $(x_1, y_1) = (2, -3)$

$y - y_1 = m(x - x_1)$

a) $y - (-3) = -\frac{2}{7}(x - 2)$

$y + 3 = -\frac{2}{7}x + \frac{4}{7}$

$-3 = \quad -\frac{21}{7}$

b) $y = -\frac{2}{7}x - \frac{17}{7}$

$7y = 7\left(-\frac{2}{7}x\right) - 7\left(\frac{17}{7}\right)$

$7y = -2x - 17$

$+2x = +2x$

c) $2x + 7y = -17$

Ex. 4 A line that passes through the points (7, – 10) and (5, – 14).

Solution:

First, find the slope:

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-14-(-10)}{5-7}$

$= \frac{-4}{-2} = 2$.

Now, pick one of the points.

$m = 2$; $(x_1, y_1) = (7, -10)$

$y - y_1 = m(x - x_1)$

a) $y - (-10) = 2(x - 7)$

$y + 10 = 2x - 14$

$-10 = \quad -10$

b) $y = 2x - 24$

$-2x = -2x$

$-2x + y = -24$

$-1 \quad -1$

c) $2x - y = 24$

Concept #3 Writing an Equation of a Line Parallel or Perpendicular to Another Line.

<u>Write the equation of the line in slope-intercept form:</u>

Ex. 5 Find the equation of the line passing through (– 2, 6)

a) parallel to $y = 3x - 2$.

b) perpendicular to $y = 3x - 2$.

Solution:

a) Parallel lines have the same slope, so $m_{||} = 3$.

$y - y_1 = m(x - x_1)$
$y - 6 = 3(x - (-2)) = 3(x + 2) = 3x + 6$
$y - 6 = 3x + 6$
$\underline{\ \ +6 = \quad +6\ }$
$y = 3x + 12$

b) The slope of the line perpendicular is the negative reciprocal of 3. So, $m_{\perp} = -\frac{1}{3}$.

$y - y_1 = m(x - x_1)$
$y - 6 = -\frac{1}{3}(x - (-2)) = -\frac{1}{3}(x + 2) = -\frac{1}{3}x - \frac{2}{3}$
$y - 6 = -\frac{1}{3}x - \frac{2}{3}$
$\underline{\ \ +6 = \qquad + \frac{18}{3}\ }$
$y = -\frac{1}{3}x + \frac{16}{3}$

Ex. 6 Find the equation of the line passing through (4, 0)

a) parallel to $y = \frac{1}{6}x - 3$.

b) perpendicular to $y = \frac{1}{6}x - 3$.

Solution:

a) Parallel lines have the same slope, so $m_{||} = \frac{1}{6}$.

$y - y_1 = m(x - x_1)$
$y - 0 = \frac{1}{6}(x - 4) = \frac{1}{6}x - \frac{4}{6} = \frac{1}{6}x - \frac{2}{3}$
$y = \frac{1}{6}x - \frac{2}{3}$

b) The slope of the line perpendicular is the negative reciprocal of $\frac{1}{6}$. So, $m_{\perp} = -6$.

$y - y_1 = m(x - x_1)$
$y - 0 = -6(x - 4) = -6x + 24$
$y = -6x + 24$

Ex. 7 Find the equation of the line passing through (8, – 3)

a) parallel to x = 5.

b) perpendicular to x = 5.

Solution:

a) A line parallel to x = 5 is a vertical line and it passes through (8, – 3). So, the equation is x = 8.

b) A line perpendicular to x = 5 is a horizontal line and it passes through (8, – 3). So, the equation is y = – 3.

Solve the following:

Ex. 8 In 1999, there were about 2.8 million high speed lines in the U.S. By 2003, that number had risen to 28.2 million. (source: www.fcc.gov). a) Write the equation of the line in slope-intercept form with x corresponding to the number of years after 1999 and y being the number of high speed lines measured in millions. b) Use your result to predict the number of high speed lines in 2005.

Solution:

a) The slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{28.2 - 2.8}{2003 - 1999} = \frac{25.4}{4} = 6.35$ million per year.

Since x is the number of years after 1999, then 2.8 million is the y-coordinate of the y-intercept. Thus, m = 6.35 and b = 2.8. So, our equation is y = 6.35x + 2.8

b) Since 2005 corresponds to x = 6, evaluate y = 6.35x + 2.8 for x = 6: y = 6.35(6) + 2.8 = 38.1 + 2.8 = 40.9

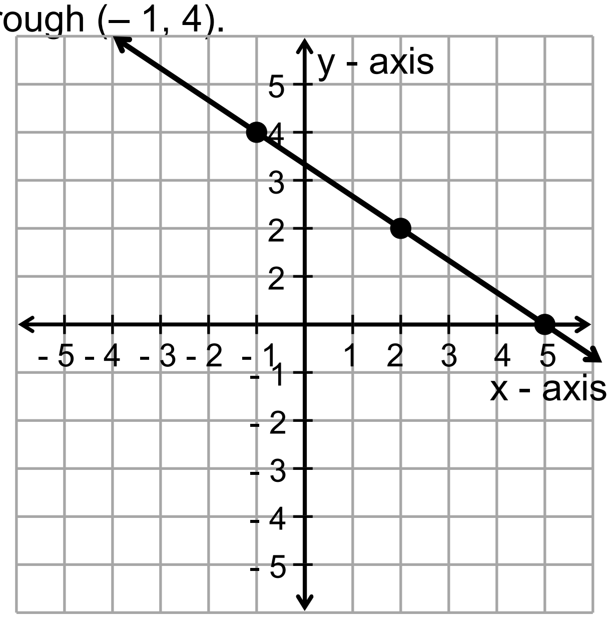
So, the number of high speed lines in 2005 should be 40.9 million.

Use the information given to sketch the graph:

Ex. 9 $m = -\frac{2}{3}$ and the line passes through (– 1, 4).

Solution:

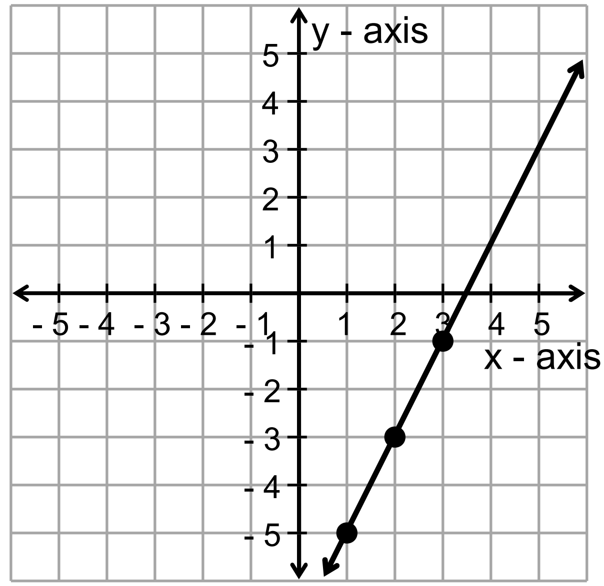
Plot the point (– 1, 4). Then from that point fall 2 units and run 3 units to get another point. From that new point fall another 2 units and run 3 more units to get the third point. Now, draw the graph.



Ex. 10 m = 2 and the line passes through (1, – 5).

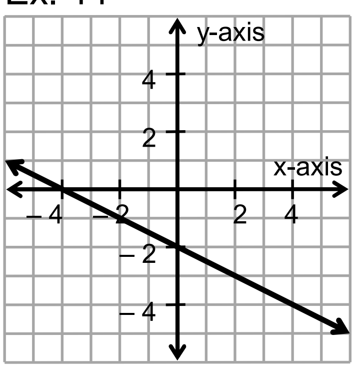
Solution:

Plot the point (1, – 5). Then from that point rise 2 units and run 1 unit to get another point. From that new point rise another 2 units and run 1 more unit to get the third point. Now, draw the graph.

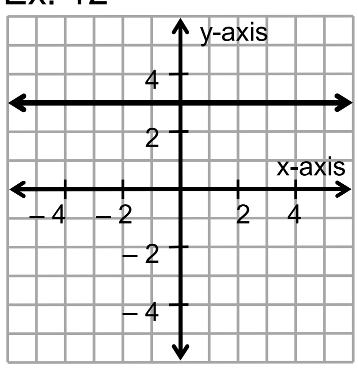


Given the graph below, find
a) the intercepts
b) the slope if applicable
c) the equation of the line if applicable:

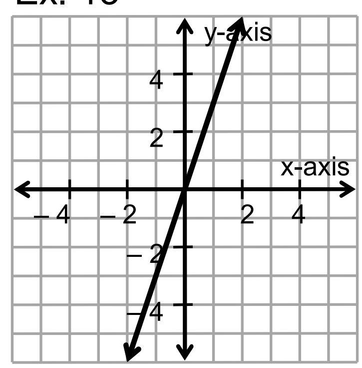
Ex. 11



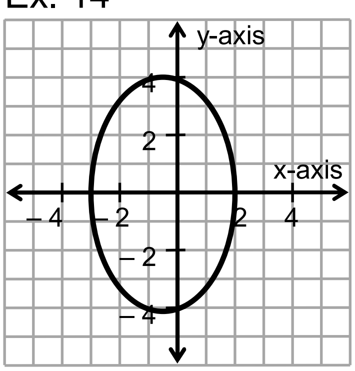
Ex. 12



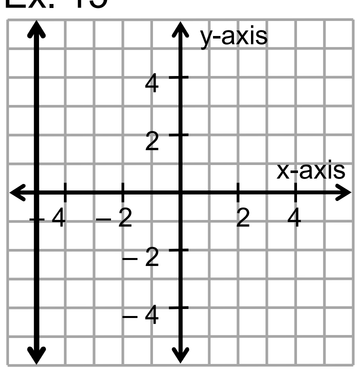
Ex. 13



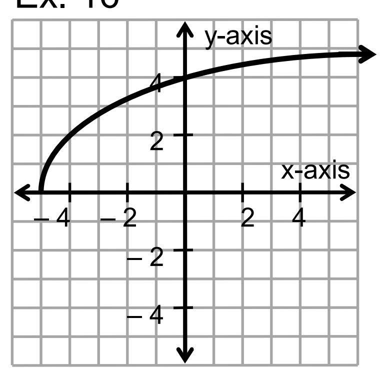
Ex. 14



Ex. 15



Ex. 16



Solution:

11a) x-int: (– 4, 0) y-int: (0, – 2)

11b) From (– 4, 0) to (0, – 2), fall 2 units and run 4 units. So, $m = \frac{\text{"rise"}}{\text{"run"}} = \frac{-2}{4} = -\frac{1}{2}$.

11c) $m = -\frac{1}{2}$ and b = – 2, so the equation is $y = -\frac{1}{2}x - 2$.

12a) x-int: None y-int: (0, 3)

12b) The slope of a horizontal line is 0, so m = 0.

12c) The equation of a horizontal line is in the form y = #, so the equation is y = 3.

13a) x-int: (0, 0) y-int: (0, 0)

13b) From (0, 0) to (1, 3), rise 3 units and run 1 unit. So, $m = \frac{\text{"rise"}}{\text{"run"}} = \frac{3}{1} = 3$.

13c) m = 3 and b = 0, so the equation is y = 3x.

14a) x-int: (– 3, 0) and (2, 0) y-int: (0, – 4) and (0, 4)

14b & c) N/A

15a) x-int: (– 5, 0) y-int: None

15b) The slope of a vertical line is undefined, so m is undefined.

15c) The equation of a vertical line is in the form x = #, so the equation is x = – 5.

16a) x-int: (– 5, 0) y-int: (0, 4)

16b & c) N/A

Concept #4 Different Forms of Linear Equations: A Summary

Form	**Example**	**Notes**
Standard Form Ax + By = C	5x – 3y = 17	A and B are integers & must not both be 0.
Horizontal Line y = constant	y = – 2.3	m = 0, y-int: (0, constant)
Vertical Line x = constant	x = 3	m is undefined x-int: (constant, 0)
Slope-Intercept Form y = mx + b	m = 3, (0, b) = (0, – 5) y = 3x – 5	m is the slope, (0, b) is the y-int.
Point-Slope Formula $y - y_1 = m(x - x_1)$	$m = -4$, $(x_1, y_1) = (2, 3)$ y – 3 = – 4(x – 2)	m is the slope, (x_1, y_1) is any point on the line.