Sect 3.6 - Applications of Linear Equations

Concept #1 Interpreting a Linear Equation in Two Variables

As we have seen, a linear equation in two variables can be used to describe the relationship between two quantities in the real world. Usually, the variable y being predicted is called the **dependent variable** and the variable x that is used to make the prediction is called the **independent variable.** In example #8 of section 3.5, we found the equation: $y = 6.35x + 2.8$ where x was the number years after 1999 and y was the number of high speed lines in millions. The x variable was the independent variable since we plugged in values for x to predict how many high speed lines there were in a given year. The y variable was the dependent variable since we were trying to predict the number of high speed lines.

Solve the following:

Ex. 1 The cost, y, of a monthly cell phone bill (in dollars) is given by $y = 0.25x + 39.99$ where x is the number of minutes in excess of 400 minutes.

a) Which is the independent variable?
b) Which is the dependent variable?
c) What is the slope of the line?
d) Interpret the meaning of the slope?
e) Graph the line

Solution:

a) The independent variable is the number of minute in excess of 400 minutes represented by x.

b) The dependent variable is the cost of the monthly cell phone bill represented by y.

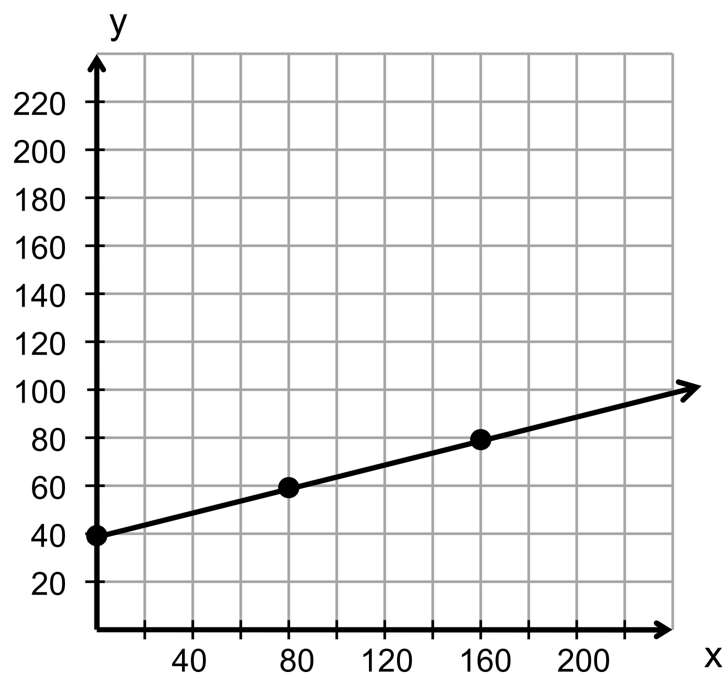
c) Since the equation is already in slope-intercept form, then the slope $m = 0.25$.

d) The slope = 0.25 or \$0.25 per minute which means that the consumer is charge \$0.25 per minute for every minute in excess of 400 minutes used.

e) For this application, we will only draw the first quadrant since y and x cannot be negative.

$m = 0.25 = \frac{25}{100} = \frac{1}{4}$

y-int.: (0, 39.99)



Number of minutes in excess of 400 minutes

Ex. 2 The cost c (in \$ per pound) of a particular brand of butter from 2003 through 2007 can be modeled by the equation $c = 0.23t + 1.61$ where t is the number of years after 2003.

a) Which is the independent variable?
b) Which is the dependent variable?
c) What is the slope of the line and interpret its meaning?
d) Use the equation to predict the cost of the butter in 2012.
e) Find the c-intercept.
f) What does this c-intercept mean?
g) Find the t-intercept.
h) What does this t-intercept mean?

<u>Solution:</u>

a) The independent variable is the number of years after 2003 represented by t.

b) The dependent variable is the cost in \$ per pound of a particular brand of butter represented by c.

c) Since the equation is already in slope-intercept form, then the slope m = 0.23. Since the slope = 0.23 or \$0.23 per pound which means that the price per pound of this particular brand of butter is increasing by 23¢ per pound per year.

d) Since 2012 is nine years after 2003, then t = 9. Plugging into our equation, we get: $c = 0.23t + 1.61$

$$c = 0.23(9) + 1.61 = 2.07 + 1.61 = 3.68$$

The butter will cost \$3.68 per pound in 2012 according to the model.

e) The c-intercept of $c = 0.23t + 1.61$ is (0, 1.61).

f) Since t = 0 corresponds to the year 2003, this means that the butter cost \$1.61 per pound in 2003.

g) To find the t-intercept, let c = 0 and solve:

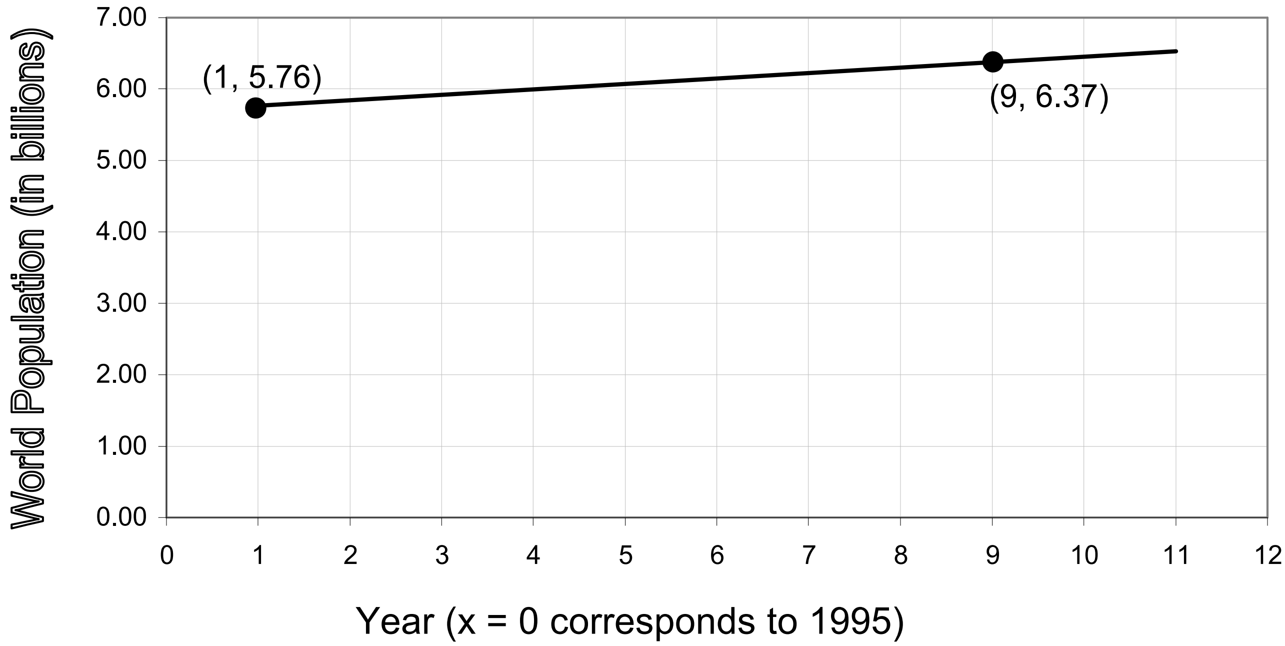
$$0 = 0.23t + 1.61$$
$$-1.61 = 0.23t$$
$$-7 = t$$

So, the t-intercept is (– 7, 0).

h) Since t = – 7 corresponds to 2003 – 7 = 1996, this means that the butter cost \$0 per pound in 1996. Obviously, this does not make any sense since the butter would have been free in 1996. This shows that the accuracy of the model is limited to the time period specified. Projecting outside of that time period can lead to wildly inaccurate results.

Concept #2 Writing a Linear Equation Using Observed Data Points

Ex. 3 The world population increased almost linearly from the years 1996 to 2006. Let x represent the number of years since 1995 and y represent the world population in billions of people. Use the following graph to answer the questions:



a) Find a linear equation that relates the world population versus the year.

b) Interpret what the slope of the line means.

c) Using the model found in part a, predict the world population in 2020.

<u>Solution:</u>

a) We have two points marked on the line: (1, 5.76) & (9, 6.37)
First, we need to calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.37 - 5.76}{9 - 1} = \frac{0.61}{8} = 0.07625$$

Next, we will use the point-slope formula to find the equation:

$$y - y_1 = m(x - x_1)$$
$$y - 5.76 = 0.07625(x - 1)$$
$$y - 5.76 = 0.07625x - 0.07625$$
$$y = 0.07625x + 5.68375$$

b) Since m = 0.07625, then this means that the world population is increasing by 0.07625 billion or 76,250,000 people per year.

c) Since 2020 is 25 years after 1995, then x = 25.
Substituting, we get:

$$y = 0.07625(25) + 5.68375 = 1.90625 + 5.68375 = 7.59$$

Thus, in 2020, the world population will be 7.59 billion people.

Concept #3 Writing the Linear Equation Given a Fixed Value and a Rate of Change.

If we examine the slope-intercept equation $y = mx + b$ more closely, the mx term is our variable term and b is our constant term. Since b is constant, it remains fixed despite the value of x. In the context of an application, our constant term b represents the fixed cost. Since mx is our variable term, then the value of that term varies as the value of x varies. This is why the slope is called the rate of change. We can use these ideas to set up an equation for an application where the fixed costs are given (b) and the rate of change or variable costs are also given (m).

Solve the following:

Ex. 4 The math department leases a copy machine. Each month, the department has to pay \$1500 plus \$0.02 per copy for each copy made in excess 30,000 copies.

a) Write a linear equation to compute the total monthly bill the math department has to pay if x is the number of copies made in excess of 30,000 copies.

b) Use your result from part a to compute the copy bill if the department makes 35,000 copies in a particular month.

Solution:

a) \$1500 is the constant or fixed cost. The variable cost is \$0.02 per copy, hence, our slope is m = \$0.02. Thus, our equation is:

$$y = mx + b$$
$$y = 0.02x + 1500$$

b) If the department makes 35,000 copies, that would be 5000 copies in excess of 30,000, so x = 5000. Substituting, we get:

$$y = 0.02(5000) + 1500$$
$$y = 100 + 1500 = \$1600$$

The department's bill for that month would be \$1600.