Sect 4.3 - Solving Systems of Equations by Addition (Elimination)

Concept #1 Solving System of Linear Equations in Two Variables by Addition (Elimination).

A variant of the Addition Property of Equality that if you have two equations, you can add the left sides and then add the right sides of the equations to produce an equivalent equation. In other words, If A = B and C = D, then A + C = B + D. In this section, we will manipulate one or both equations until the coefficients of one of the variable terms are opposites. This is done by finding the L.C.M. of the coefficients of the term we are trying to eliminate. We will then add the equations together, eliminating the terms with opposite coefficients. This will give us one linear equation with one variable. We solve that equation and that will yield the value for one of the coordinates of our solution. We take that answer, substitute it into one of the two original equations and solve to find the value for the other coordinate of our solution.

Solve by addition:

Ex. 1 $3x - 2y = 6$
$x + 2y = 2$

Solution:
Since the coefficients of the y-terms are already opposites, we add the left sides and the right sides.

$$\begin{array}{r} 3x - 2y = 6 \\ \underline{x + 2y = 2} \\ 4x = 8 \end{array} \quad \text{(solve)}$$

$x = 2$

Substitute into equation #2:

$x + 2y = 2$
$2 + 2y = 2$
$2y = 0$
$y = 0$

The solution is (2, 0).

Ex. 2 $6x + 5y = 7$
$x - y = 4$

Solution:

We need to manipulate the equations so that the coefficients of one of the variable terms are opposites. Let's work on getting rid of y, the easiest way to do this is to multiply the second equation by 5:

$x - y = 4 \quad \Rightarrow \quad 5x - 5y = 20$

Now, the coefficients of the y-terms are opposites, so add the left sides and the right sides.

$$\begin{array}{r} 6x + 5y = 7 \\ \underline{5x - 5y = 20} \\ 11x = 27 \end{array} \qquad \text{(solve)}$$

$$x = \frac{27}{11}$$

Substitute into equation #2:

$$x - y = 4$$

$$\frac{27}{11} - y = 4 \quad \Rightarrow \quad \frac{27}{11} - y = \frac{44}{11} \quad \Rightarrow \quad -y = \frac{17}{11}$$

So, $y = -\frac{17}{11}$. Thus, the solution is $\left(\frac{27}{11}, -\frac{17}{11}\right)$.

Ex. 3

$$-\frac{1}{2}x - \frac{1}{7}y = 2$$

$$\frac{1}{4}x + \frac{1}{14}y = 5$$

<u>Solution:</u>

It is easiest to clear fractions first. The L.C.D. of equation #1 is 14 and the L.C.D. of equation #2 is 28.

1) $-\frac{1}{2}x - \frac{1}{7}y = 2$ (multiply both sides by 14)

$$14\left(-\frac{1}{2}x\right) - 14\left(\frac{1}{7}y\right) = 14(2) \quad \Rightarrow \quad -7x - 2y = 28$$

2) $\frac{1}{4}x + \frac{1}{14}y = 5$ (multiply both sides by 28)

$$28\left(\frac{1}{4}x\right) + 28\left(\frac{1}{14}y\right) = 28(5) \quad \Rightarrow \quad 7x + 2y = 140$$

Now, the coefficients of the x-terms are opposites, so add the left sides and the right sides.

$$\begin{array}{r} -7x - 2y = 28 \\ \underline{7x + 2y = 140} \\ 0 = 168 \end{array}$$

There is no solution.

Ex. 4

$$-0.2x + 0.3y = 4$$

$$0.3x + 0.2y = 0.5$$

Solution:

Since the signs of the x-terms are opposite, let's get rid of the x-terms. The L.C.M. (least common multiple) of 0.2 and 0.3 is 0.6. Therefore, we need to multiply both sides of equation #1 by 3 and both sides of equation #2 by 2:

$-0.2x + 0.3y = 4$ (multiply by 3) $\Rightarrow$ $-0.6x + 0.9y = 12$

$0.3x + 0.2y = 0.5$ (multiply by 2) $\Rightarrow$ $0.6x + 0.4y = 1$

Now, the coefficients of the x-terms are opposites, so add the left sides and the right sides.

$$\begin{array}{r} -0.6x + 0.9y = 12 \\ \underline{0.6x + 0.4y = 1} \\ 1.3y = 13 \\ y = 10 \end{array}$$

Substitute into equation #2:

$0.3x + 0.2y = 0.5$

$0.3x + 0.2(10) = 0.5$

$0.3x + 2 = 0.5$

$0.3x = -1.5$

$x = -5$

The solution is $(-5, 10)$.

Ex. 5 $12x - 6y = -15$

$-4x = -2y + 5$

Solution:

Let's get rid of the x-terms. We will multiply equation #2 by 3:

$-4x = -2y + 5$ (multiply by 3) $\Rightarrow$ $-12x = -6y + 15$

Now, the coefficients of the x-terms are opposites, so add the left sides and the right sides. Be careful to align the equal signs:

$$\begin{array}{r} 12x - 6y = \quad -15 \\ \underline{-12x \quad = -6y + 15} \\ -6y = -6y \\ 0 = 0 \end{array}$$

$-6y = -6y$ (add 6y to both sides)

$0 = 0$ The equations are the same.

The solution is $\{ (x, y) \mid -4x = -2y + 5\}$

Ex. 6 $3x - 11 = 2(-y + 4)$

$7(3 - y) = 4(x - 5)$

Solution:

The first step is to simplify the equations and write them in standard form:

$3x - 11 = 2(-y + 4)$ $\qquad$ $7(3 - y) = 4(x - 5)$
$3x - 11 = -2y + 8$ $\qquad$ $21 - 7y = 4x - 20$
$3x + 2y - 11 = 8$ $\qquad$ $4x - 20 = 21 - 7y$
1) $3x + 2y = 19$ $\qquad$ $4x + 7y - 20 = 21$
2) $4x + 7y = 41$

Let's eliminate the x-terms. The LCM of 3 and 4 is 12.

1) $3x + 2y = 19$ (multiply by 4) $\Rightarrow 12x + 8y = 76$
2) $4x + 7y = 41$ (multiply by – 3) $\Rightarrow -12x - 21y = -123$

Now, the coefficients of the x-terms are opposites, so add the left sides and the right sides.

$$\begin{array}{r} 12x + 8y = 76 \\ \underline{-12x - 21y = -123} \\ -13y = -47 \\ y = \frac{47}{13} \end{array}$$

To find x, plug $y = \frac{47}{13}$ into equation #1:

$3x + 2y = 19$
$3x + 2(\frac{47}{13}) = 19$
$3x + \frac{94}{13} = 19$ (multiply both sides by 13 to clear fractions)
$39x + 94 = 247$
$39x = 153$
$x = \frac{153}{39} = \frac{51}{13}$

Hence, $\left(\frac{51}{13}, \frac{47}{13}\right)$ is the solution.

Ex. 8 $\quad 5a = -3b$
$-3b = 7 - 2a$

Solution:

Write the equations in standard form

$5a = -3b$ $\qquad$ $-3b = 7 - 2a$
1) $5a + 3b = 0$ $\qquad$ 2) $2a - 3b = 7$

Thus, our system is:

1) $5a + 3b = 0$ Just add since the x-terms are opposites.
2) $\underline{2a - 3b = 7}$

$7a = 7$
$a = 1$

Plug a = 1 into equation #1:

$5(1) + 3b = 0$

$3b = -5$

$b = -\frac{5}{3}$

Thus, $(1, -\frac{5}{3})$ is the solution.

Concept #2 Summary of Methods for Solving Linear Equations in Two Variables

If given a choice between using substitution or addition to solve a system of equations, we can follow some guidelines as to which method to use:

1. If one of the equations is already solved for one of the variables, then usually the substitution method is easier to use.

2. If both equations are in standard form and none of the coefficients of the variables terms are 1 or – 1, then usually the addition method is easier to use.

Otherwise, it is a toss up as to which method to use. Keep in mind that both methods will work on any system of two linear equations in two variables.