Sect 6.2 & The Rest of Sect 6.3 - Factoring Trinomials Using Trial-and-Error

Section 6.2 Concept #1 and Section 6.3 Concepts #1 & 3:
Factoring Trinomials Using Trial and Error.

Recall a problem we have worked previously:

Simplify the following:

Ex. 1 (3x – 5)(2x + 7)

Solution:

F. O. I. L.

$(3x - 5)(2x + 7) = 6x^2 + 21x - 10x - 35 = 6x^2 + 11x - 35.$

In this section, we want to go backwards. Notice that the first term in the trinomial is the product of the first terms of the two binomials that we foiled. Also, the last term in the trinomial is the product of the last terms of two binomials that we foiled. If we are factoring a trinomial, we can look at the factors of the first term and that will give us the possible first terms for the two binomials and we can look at the factors of the last term and that we give us the possible last terms for the two binomials. Then, just check the outer and the inner part of FOIL to see if we get the correct middle term. If so, then we are done. If not, then we will need to try a different combination. So, let's try factoring $6x^2 + 11x - 35$.

Factor the following:

Ex. 2 $6x^2 + 11x - 35$

Solution:

The possible factors of $6x^2$ are x•6x and 2x•3x and the possible factors of – 35 are – 1•35 and – 5•7.

$6x^2 + 11x - 35$

$6x^2$	-35
x•6x	– 1•35
2x•3x	– 5•7

The possible combinations are (x – 1)(6x + 35), (x + 35)(6x – 1), (x – 5)(6x + 7), (x + 7)(6x – 5), (2x – 1)(3x + 35), (2x + 35)(3x – 1), (2x – 5)(3x + 7), & (2x + 7)(3x – 5)

In terms of where we start, there is a higher probability that the correct combination will involve the factors that are closest to each other in value. We should therefore try the combinations involving 2x•3x and – 5•7. To find the correct combination, compute the O. and I. part of F.O.I.L. to see which one yields 11x as a middle term.

(2x – 5)(3x + 7)	(2x + 7)(3x – 5)
O. 14x	O. – 10x
I. – 15x	I. 21x
– x No	11x Yes

Therefore, $6x^2 + 11x - 35 = (2x + 7)(3x - 5)$.

You might ask how do we know which number to give the negative sign to. Actually, it does not matter. If we had put the negative sign on the 7 instead of the five, we would have computed – 11x as our middle term on the second trial. If we get the correct middle term, but with the wrong sign, we just change each sign.

Ex. 2 $y^2 + 11y + 24$

Solution:

$y^2 + 11y + 24$

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y•y 1•24
2•12
3•8
4•6

The factors of y^2 = y•y. The factors of 24 = 1•24, 2•12, 3•8, and 4•6. Try 4 and 6 first, then 3 and 8 and continue until you get the correct middle term.

(y + 4)(y + 6)	(y + 3)(y + 8)
O. 6y	O. 8y
I. 4y	I. 3x
10y No	11y Yes

So, $y^2 + 11y + 24 = (y + 3)(y + 8)$.

You may have notice that if we add the factors of the last term, we get the coefficient of the middle term. That works if the coefficient of the first or last term is one. We will use that to help us find the combination quickly, but we will still verify it using the O. and I. part of F.O.I.L.

Ex. 3 $x^2 - 13xy + 36y^2$

Solution:

$x^2 - 13xy + 36y^2$

Λ Λ

x•x y•36y
2y•18y
3y•12y
4y•9y
6y•6y

Since 4 + 9 = 13, then 4y, 9y is the combination we want to try;

(x + 4y)(x + 9y)
O. 9xy
I. 4xy
13xy

We have the wrong sign so change the signs.

Thus, $x^2 - 13xy + 36y^2 = (x - 4y)(x - 9y)$

Ex. 4 $25x^2 - 225x + 75$

<u>Solution:</u>

G.C.F. = 25, so $25x^2 - 225x + 75 = 25(x^2 - 9x + 3)$

$25(x^2 - 9x + 3)$

x•x 1•3

There is only one combination to try, but $1 + 3 = 4 \neq -9$

So $(x + 3)(x + 1)$ will yield a middle term of 4x. This means that $x^2 - 9x + 3$ is prime.

So, $25(x^2 - 9x + 3)$ is our answer.

Ex. 5 $y^2 - 11yz - 42z^2$

<u>Solution:</u>

When we list the factors of $-42z^2$, we will need to make one of them negative.

$y^2 - 11yz - 42z^2$

y•y – z•42z

– 2z•21z

– 3z•14z

– 6z•7z

Since – 3yz + 14yz = 11yz, that is the combination we want except we will need to change the signs.

$(y + 3z)(y - 14z)$

O. – 14yz

I. 3yz

– 11yz Yes

Thus, $y^2 - 11yz - 42z^2 = (y + 3z)(y - 14z)$

Ex. 6 $x^4 - 5x^2 + 4$

<u>Solution:</u>

$x^4 - 5x^2 + 4$

$x^2 \bullet x^2$ 1•4

2•2

Since 1 + 4 = 5, that is the combination we want except we will have to change the signs: $(x^2 - 4)(x^2 - 1)$

O. $-x^2$

I. $-4x^2$

$-5x^2$ Yes

Thus, $x^4 - 5x^2 + 4 = (x^2 - 4)(x^2 - 1)$

But, $x^2 - 1$ and $x^2 - 4$ are both a difference of squares, so

$x^2 - 1 = (x - 1)(x + 1)$ and $x^2 - 4 = (x - 2)(x + 2)$.

Hence, our answer is $(x - 1)(x + 1)(x - 2)(x + 2)$.

Ex. 7 $10x^2(3x - 2) + 10x(3x - 2) - 120(3x - 2)$

<u>Solution:</u>

G.C.F. = 10(3x – 2), so

$10x^2(3x - 2) + 10x(3x - 2) - 120(3x - 2)$

$= 10(3x - 2)(x^2 + x - 12)$

$= 10(3x - 2)(x^2 + x - 12)$

$x \bullet x$ $-x \bullet 12x$

$-2x \bullet 6x$

$-3x \bullet 4x$

Since $-3x + 4x = x$, that is the combination we want

$(x - 3)(x + 4)$

O. $-3x$

I. $4x$

x Yes

Thus, $10(3x - 2)(x^2 + x - 12) = 10(3x - 2)(x - 3)(x + 4)$

Ex. 8 $30m^2 - 11m + 1$

Solution:

$30m^2 - 11m + 1$

$m \bullet 30m$ $1 \bullet 1$

$2m \bullet 15m$

$3m \bullet 10m$

$5m \bullet 6m$

Since $5m + 6m = 11m$, that is the combination we want except we will need to change the signs.

$(5m - 1)(6m - 1)$

O. $-5m$

I. $-6m$

$-11m$ Yes

Thus, $30m^2 - 11m + 1 = (5m - 1)(6m - 1)$.

Ex. 9 $9x^2 + 9xy - 10y^2$

Solution:

Since neither the coefficients of first nor the coefficient of the last term are one, we cannot just see which factors add up to 9xy. We will have to try out combinations to see which one gives us the correct middle term:

$9x^2 + 9xy - 10y^2$

$x \bullet 9x$ $-y \bullet 10y$

$3x \bullet 3x$ $-2y \bullet 5y$

Since the last term is negative, we will need to make one of the factors negative. Now, try the factors that are closest together.

$(3x - 2y)(3x + 5y)$

O. $15xy$

I. $-6xy$

$9xy$ Yes

So, $9x^2 + 9xy - 10y^2 = (3x - 2y)(3x + 5y)$

Ex. 10 $20xy^2 - 130xy + 60x$

Solution:

G.C.F. = 10x, so $20xy^2 - 130xy + 60x = 10x(2y^2 - 13y + 6)$

$= 10x(2y^2 – 13y + 6)$

y•2y 1•6

2•3

Since the G.C.F. of $2y^2 – 13y + 6$ is 1, we can eliminate the combinations that have a G.C.F. not equal to 1.

(y + 2)(2y + 3)

O. 3y

I. 4y

7y No

(y + 3)(2y + 2)

No, since the G.C.F. of 2y + 2 ≠ 1.

The combinations with 2 and 3 did not work so let's try 1 and 6.

(y + 1)(2y + 6)

No, since the G.C.F. of 2y + 6 ≠ 1.

(y + 6)(2y + 1)

O. y

I. 12y

13y Yes, wrong sign.

So, we need to change the signs.

Thus, $10x(2y^2 – 13y + 6) = 10x(y – 6)(2y – 1)$

Ex. 11 $8x^2 + 18xy – 35y^2$

Solution:

$8x^2 + 18xy – 35y^2$

x•8x – y•35y

2x•4x – 5y•7y

Since the G.C.F. of $8x^2 + 18xy – 35y^2$ is 1, we can eliminate the combinations that have a G.C.F. not equal to 1. Make one of the last factors negative.

(2x – 5y)(4x + 7y)

O. 14xy

I. – 20xy

– 6xy No

(2x + 7y)(4x – 5y)

O. – 10xy

I. 28xy

18xy Yes

So, $8x^2 + 18xy – 35y^2 = (2x + 7y)(4x – 5y)$.

Ex. 12 $40x^3y – 84x^2y^2 – 40xy^3$

Solution:

G.C.F. = 4xy, so $40x^3y – 84x^2y^2 – 40xy^3$

$= 4xy(10x^2 – 21xy – 10y^2)$

x•10x – y•10y

2x•5x – 2y•5y

Since the G.C.F. of $10x^2 – 21xy – 10y^2$ is 1, we can eliminate the combinations that have a G.C.F. not equal to 1. Make one of the last factors negative.

(2x – 2y)(5x + 5y)

No, G.C.F. ≠ 1.

(2x + 5y)(5x – 2y)

O. – 4xy

I. 25xy

21xy Yes, change signs.

So, $4xy(10x^2 – 21xy – 10y^2) = 4xy(2x + 5y)(5x – 2y)$

Ex. 13 $14x^2 + 41x + 15$

Solution:

$14x^2 + 41x + 15$

$x \bullet 14x$ $1 \bullet 15$

$2x \bullet 7x$ $3 \bullet 5$

Since the G.C.F. of $14x^2 + 41x + 15$ is 1, we can eliminate the combinations that have a G.C.F. not equal to 1.

$(2x + 3)(7x + 5)$	$(2x + 5)(7x + 3)$
O. 10x	O. 6x
I. 21x	I. 35x
31x No	41x Yes

So, $14x^2 + 41x + 15 = (2x + 5)(7x + 3)$.

Ex. 14 $-2xy^2 - 17xy + 24x$

Solution:

Since G.C.F. = – x, then $-2xy^2 - 17xy + 24x = -x(2y^2 + 17y - 24)$

$2y^2 + 17y - 24$

$y \bullet 2y$ $-1 \bullet 24$

$-2 \bullet 12$

$-3 \bullet 8$

$-4 \bullet 6$

Since the G.C.F. of $2y^2 + 17y - 24$ is 1, we can eliminate the combinations that have a G.C.F. not equal to 1. Make one of the last factors negative.

$(y - 4)(2y + 6)$	$(y + 6)(2y - 4)$
No, G.C.F. ≠ 1.	No, G.C.F. ≠ 1.

Try – 3 and 8:

$(y - 3)(2y + 8)$	$(y + 8)(2y - 3)$
No, G.C.F. ≠ 1.	O. – 3y
	I. 16y
	13y No

Try – 2 and 12:

$(y - 2)(2y + 12)$	$(y + 12)(2y - 2)$
No, G.C.F. ≠ 1.	No, G.C.F. ≠ 1.

Try – 1 and 24:

$(y - 1)(2y + 24)$	$(y + 24)(2y - 1)$
No, G.C.F. ≠ 1.	O. 48y
	I. – y
	47y No

Thus, $2y^2 + 17y - 24$ is prime. So, our answer is $-x(2y^2 + 17y - 24)$. Notice by using the G.C.F. argument, we eliminated six combinations.

Ex. 16 $(25x^2 – 70x + 49) – (4x^2 – 4x + 1)$

Solution:

There are at least two ways to work this problem:

Method One:

$(25x^2 – 70x + 49) – (4x^2 – 4x + 1)$ (distribute)

$= 25x^2 – 70x + 49 – 4x^2 + 4x – 1$ (combine like terms)

$= 21x^2 – 66x + 48$ (factor out the G.C.F. of 3)

$= 3(7x^2 – 22x + 16)$ (trial and error)

x•7x 1•16 (x + 4)(7x + 4)

2•8 O. 4x

4•4 I. 28x

32x No

(x + 2)(7x + 8)

O. 8x

I. 14x

22x Yes, change the signs.

So, the answer is 3(x – 2)(7x – 8).

Method Two:

$(25x^2 – 70x + 49) – (4x^2 – 4x + 1)$ (perfect square trinomials)

$F_1 = 5x$ $L_1 = 7$ $F_2 = 2x$ $L_2 = 1$

– 2FL = – 2(5x)(7) – 2FL = – 2(2x)(1)

= – 70x match = – 4x match

Since $F^2 – 2FL + L^2 = (F – L)^2$, then

$(25x^2 – 70x + 49) – (4x^2 – 4x + 1)$

$= (5x – 7)^2 – (2x – 1)^2$ (difference of squares)

F = (5x – 7) L = (2x – 1)

Since $F^2 – L^2 = (F – L)(F + L)$, then

$(5x – 7)^2 – (2x – 1)^2$

= [(5x – 7) – (2x – 1)][(5x – 7) + (2x – 1)] (distribute)

= [5x – 7 – 2x + 1][5x – 7 + 2x – 1] (combine like terms)

= (3x – 6)(7x – 8)(factor out the G.C.F. of 3)

= 3(x – 2)(7x – 8)

So, the answer is 3(x – 2)(7x – 8).

Notice that we get the same answer though the problem was factored in vastly different ways.

Ex. 16 $6(x + 3y)^2 – (x + 3y) – 1$

Solution:

We could first try expanding and combining like terms:

$6(x + 3y)^2 – (x + 3y) – 1$

$= 6(x + 3y)(x + 3y) – (x + 3y) – 1$ (multiply)
$= 6(x^2 + 6xy + 9y^2) – (x + 3y) – 1$ (distribute)
$= 6x^2 + 36xy + 54y^2 – x – 3y – 1$

Unfortunately, there are no like terms so we have reached a dead end. Let's start again and see if we can work this in a different direction.

6**(x + 3y)**^{2} – **(x + 3y)** – 1

"Squared Term" "Variable Term" "Constant Term"

Notice that this is in the form of a second degree trinomial. To make it easier to work with, let's let **B** = (x + 3y). So, our problem becomes:

$6(x + 3y)^2 – (x + 3y) – 1$
$= 6B^2 – B – 1$ Now, use trial and error.

$6B^2$: B•6B, 2B•3B	1: – 1•1	(2B – 1)(3B + 1)
		O. 2B
		I. – 3B
		– B Yes

So, $6B^2 – B – 1 = (2B – 1)(3B + 1)$

Now, replace B by (x + 3y):

(2**B** – 1)(3**B** + 1) = (2**(x + 3y)** – 1)(3**(x + 3y)** + 1) (distribute)
= (2x + 6y – 1)(3x + 9y + 1)

Ex. 17 $x^2(x + 10) – 2x(x – 8)$

Solution:

We could first try expanding and combining like terms:

$x^2(x + 10) – 2x(x – 8)$ (distribute)
$= x^3 + 10x^2 – 2x^2 + 16x$ (combine like terms)
$= x^3 + 8x^2 + 16x$ (factor out the G.C.F. of x)
$= x(x^2 + 8x + 16)$ (perfect square trinomial)

F = x L = 4
2FL = 2(x)(4) = 8x match

Since $F^2 + 2FL + L^2 = (F + L)^2$, then
$x^2 + 8x + 16 = (x + 4)^2$.
Thus, $x(x^2 + 8x + 16) = x(x + 4)^2$.