Rest of Sect 6.4 - Factoring Trinomials Using the AC-Method

Concept #1 Factoring Trinomials: AC-Method

An alternate method to factoring trinomials is called the AC method. With this method, you "uncombine" the middle term and then factor the problem by grouping. Here is the procedure:

Product-Sum Method

1) Multiply the coefficient of the first and last term (a•c).
2) List all pairs of factors of that result until you find a pair whose sum is the coefficient of the middle term (b).
3) Rewrite the middle term as the sum of two terms using the factors found in step #2 as the coefficients.
4) Factor by grouping.

Factor:

Ex. 1 $3x^2 - 5x - 12$

Solution:

The G.C.F. = 1. Let's go through our steps:

1) The coefficients of the first and last terms are 3 and – 12. Their product is – 36.
2)

Product = – 36	**Sum = – 5**
– 1•36	– 1 + 36 = 35 No
– 2•18	– 2 + 18 = 16 No
– 3•12	– 3 + 12 = 9 No
– 4•9	– 4 + 9 = 5 Yes, change the signs
– 6•6	

3) Thus, $-5x = 4x - 9x$, so $3x^2 - 5x - 12$
$= 3x^2 + 4x - 9x - 12$

4) Factor by grouping:
$= (3x^2 + 4x) + (-9x - 12)$ (factor out an x and a – 3)
$= x(3x + 4) - 3(3x + 4)$ (factor out (3x + 4))
$= (3x + 4)(x - 3)$

Ex. 2 $14x^2 + 31x + 15$

Solution:

The G.C.F. = 1. Let's go through our steps:

1) The coefficients of the first and last terms are 14 and 15. Their product is 210.

2)

Product = 210	**Sum = 31**
1•210	1 + 210 = 211 No
2•105	2 + 105 = 107 No
3•70	3 + 70 = 73 No
5•42	5 + 42 = 47 No
6•35	6 + 35 = 41 No
7•30	7 + 30 = 37 No
10•21	10 + 21 = 31 Yes

3) Thus, $31x = 10x + 21x$, so $14x^2 + 31x + 15$
$= 14x^2 + 10x + 21x + 15$

4) Factor by grouping:
$= (14x^2 + 10x) + (21x + 15)$ (factor out 2x and 3)
$=2x(7x + 5) + 3(7x + 5)$ (factor out (7x + 5))
$= (7x + 5)(2x + 3)$

Ex. 3 $-4a^3b - 5a^2b^2 + 30ab^3$

Solution:

G.C.F. = – ab, so $-4a^3b - 5a^2b^2 + 30ab^3 = -ab(4a^2 + 5ab - 30b^2)$

Let's go through our steps:

1) The coefficients of the first and last terms are 4 and – 30. Their product is – 120.

2)

Product = – 120	**Sum = 5**
– 1•120	– 1 + 120 = 119 No
– 2•60	– 2 + 60 = 58 No
– 3•40	– 3 + 40 = 37 No
– 4•30	– 4 + 30 = 26 No
– 5•24	– 5 + 24 = 19 No
– 6•20	– 6 + 20 = 14 No
– 8•15	– 8 + 15 = 7 No
– 10•12	– 10 + 12 = 2 No

Since there are no pairs that yield a sum of 5, the trinomial is prime. Thus, our answer is $-ab(4a^2 + 5ab - 30b^2)$.

Ex. 4 $-135x^2 + 9xy + 54y^2$

Solution:

The G.C.F. = – 9, so $-135x^2 + 9xy + 54y^2 = -9(15x^2 - xy - 6y^2)$.
Let's go through our steps:

1) The coefficients of the first and last terms are 15 and – 6. Their product is – 90.

2)

Product = – 90	**Sum = – 1**
– 1•90	– 1 + 90 = 89 No
– 2•45	– 2 + 45 = 43 No
– 3•30	– 3 + 30 = 27 No
– 5•18	– 5 + 18 = 13 No
– 6•15	– 6 + 15 = 9 No
– 9•10	– 9 + 10 = 1 Almost, change the signs

3) Thus, $-xy = 9xy - 10xy$, so $-9(15x^2 - xy - 6y^2)$
$= -9(15x^2 + 9xy - 10xy - 6y^2)$

4) Factor by grouping:
$= -9([15x^2 + 9xy] + [-10xy - 6y^2])$ (factor out $3x$ and $-2y$)
$= -9(3x[5x + 3y] - 2y[5x + 3y])$ (factor out $[5x + 3y]$)
$= -9(5x + 3y)(3x - 2y)$

Ex. 5 $12x^3 - 43x^2 + 35x$

Solution:

The G.C.F. = x, so $12x^3 - 43x^2 + 35 = x(12x^2 - 43x + 35)$.

Let's go through our steps:

1) The coefficients of the first and last terms are 12 and 35. Their product is 420.

2)

Product = 420	**Sum = – 43**
1• 420	1 + 420 = 421
2• 210	2 + 210 = 212
3• 140	3 + 140 = 143
4• 105	4 + 105 = 109
5• 84	5 + 84 = 89
6• 70	6 + 70 = 76
7•60	7 + 60 = 67
10•42	10 + 42 = 52
12•35	12 + 35 = 47
14•30	14 + 30 = 44
15•28	15 + 28 = 43 Almost, change the signs
20•21	20 + 21 = 41

3) Thus, $-43x = -15x - 28x$, so $x(12x^2 - 43x + 35)$
$= x(12x^2 - 15x - 28x + 35)$

4) Factor by grouping:
$= x([12x^2 - 15x] + [-28x + 35])$ (factor out 3x and – 7)
$= x(3x[4x - 5] - 7[4x - 5])$ (factor out [4x – 5])
$= x(4x - 5)(3x - 7)$