Sect 6.6 - General Factoring Summary

Concept #1 Factoring Strategy

The flow chart on the previous page gives us a visual picture of how to attack a factoring problem. We first start at the top and work our way down the chart:

1. Factor out the G.C.F. (top of the flow chart).
2. Determine whether the polynomial has two, three, or four or more terms.
3. If the polynomial has four or more terms, factor by grouping (left side of the chart).
4. If the polynomial has three terms, first check to see if it is a perfect square trinomial. If not, then factor it either using the trial-and-error method or the AC-method (middle part of the chart).
5. If the polynomial has two terms, first check to see if it is a difference of squares (remember the sum of squares is prime). If not, check to see if it is a sum or difference of cubes (right side of the chart).
6. Always check to see if everything is completely factored (bottom of the flow chart). If not, go through the flow chart again.

Concept #2 Mixed Practice

Factor the Following Completely:

Ex. 1 $64x^3y + 112x^2y^2 - 32xy^3$

Solution:

1. The G.C.F. is 16xy. Factoring out 16xy, we get:
$64x^3y + 112x^2y^2 - 32xy^3$
$= 16xy(4x^2 + 7xy - 2y^2)$

2. The polynomial inside the parentheses has 3 terms.

4. Since $-2y^2$ is not a perfect square, then the trinomial is not a perfect square trinomial. We will factor it using trial-and-error:

$16xy(4x^2 + 7xy - 2y^2)$

$x \bullet 4x$ $-y \bullet 2y$

$2x \bullet 2x$

$16xy(2x - y)(2x + 2y)$
No, G.C.F. ≠ 1

$16xy(x - y)(4x + 2y)$
No, G.C.F. ≠ 1

$16xy(x + 2y)(4x - y)$
O. $-xy$
I. $8xy$
$7xy$ Yes

6. Everything is factored, so the answer is $16xy(x + 2y)(4x - y)$.

Ex. 2 $54m^2 - 24z^2$

Solution:

1. The G.C.F. is 6. Factoring out 6, we get:
$54m^2 - 24z^2$
$= 6(9m^2 - 4z^2)$

2. The polynomial inside the parentheses has 2 terms.

5. $9m^2 - 4z^2$ is a difference of squares, so we get:
$6(9m^2 - 4z^2)$ $F^2 - L^2 = (F - L)(F + L)$; $F = 3m$, $L = 2z$
$= 6(3m - 2z)(3m + 2z)$

6. Everything is factored, so the answer is $6(3m - 2z)(3m + 2z)$.

Ex. 3 $x^5 - 36x^3 - x^2 + 36$

Solution:

1. The G.C.F. is 1.

2. The polynomial inside the parentheses has 4 terms.

3. We will group the first two and the last two terms:
$x^5 - 36x^3 - x^2 + 36 = (x^5 - 36x^3) + (- x^2 + 36)$ (factor out x^3 & 1)
$= x^3(x^2 - 36) - 1(x^2 - 36)$ (factor out $x^2 - 36$)
$= (x^2 - 36)(x^3 - 1)$

6. We can break down both factors:
5) $x^2 - 36$ has two terms and it is a difference of squares:
$x^2 - 36$ $F^2 - L^2 = (F - L)(F + L)$; $F = x$, $L = 6$
$= (x - 6)(x + 6)$
5) $x^3 - 1$ has two terms and it is a difference of cubes:
$x^3 - 1$ $F^3 - L^3 = (F - L)(F^2 + FL + L^2)$; $F = x$, $L = 1$
$= (x - 1)(x^2 + x + 1)$
Thus, $(x^2 - 36)(x^3 - 1) = (x - 6)(x + 6)(x - 1)(x^2 + x + 1)$
6) Since $x^2 + x + 1$ has degree of 2, it is prime. Thus, everything is completely factored.
Our answer is $(x - 6)(x + 6)(x - 1)(x^2 + x + 1)$.

Ex. 4 $125m^4n^3 - 400m^3n^3 + 200m^3n^4$

Solution:

1. The G.C.F. is $25m^3n^3$. Factoring out $25m^3n^3$, we get:
$125m^4n^3 - 400m^3n^3 + 200m^3n^4$
$= 25m^3n^3(5m - 16 + 8n)$

2. The polynomial inside the parentheses has 3 terms.

4. Since 5m is not a perfect square, then the trinomial is not a perfect square trinomial. Also, since the degree of the trinomial is one, it cannot be factored using the trial-&-error or the AC method. So, our answer is $25m^3n^3(5m - 16 + 8n)$.

Ex. 5 $405r^5 - 80rs^4$

Solution:

1. The G.C.F. is 5r. Factoring out 5r, we get:
$405r^5 - 80rs^4$
$= 5r(81r^4 - 16s^4)$

2. The polynomial inside the parentheses has 2 terms.

5. $81r^4 - 16s^4$ is a difference of squares, so we get:
$5r(81r^4 - 16s^4)$ $F^2 - L^2 = (F - L)(F + L)$; $F = 9r^2$, $L = 4s^2$
$= 5r(9r^2 - 4s^2)(9r^2 + 4s^2)$

6. We can break down the first binomial:

5) $9r^2 - 4s^2$ has two terms and it is a difference of squares:
$9r^2 - 4s^2$ $F^2 - L^2 = (F - L)(F + L)$; $F = 3r$, $L = 2s$
$= (3r - 2s)(3r + 2s)$

5) $9r^2 + 4s^2$ has two terms, but it is a sum of squares so it is prime.

Thus, $5r(9r^2 - 4s^2)(9r^2 + 4s^2) = 5r(3r - 2s)(3r + 2s)(9r^2 + 4s^2)$

6) Everything is now completely factored.

Our answer is $5r(3r - 2s)(3r + 2s)(9r^2 + 4s^2)$.

Ex. 6 $ax^2 - 3ax - 4a + 6x^2 - 18x - 24$

Solution:

1. The G.C.F. is 1.

2. The polynomial has more than 4 terms.

3. We will group the first three and the last three terms:
$ax^2 - 3ax - 4a + 6x^2 - 18x - 24$
$= (ax^2 - 3ax - 4a) + (6x^2 - 18x - 24)$ (factor out a and 6)
$= a(x^2 - 3x - 4) + 6(x^2 - 3x - 4)$ (factor out $x^2 - 3x - 4$)
$= (x^2 - 3x - 4)(a + 6)$

6. We can break the trinomial:

4) Since – 4 is not a perfect square, then $x^2 - 3x - 4$ is not a perfect square trinomial. Using trial-&-error:

$x^2 - 3x - 4$
⋀ ⋀
$x \bullet x$ $-1 \bullet 4$
$-2 \bullet 2$

We quickly see that we want to use – 1 and 4 though we need to switch the signs.

$x^2 - 3x - 4 = (x + 1)(x - 4)$

Thus, $(x^2 - 3x - 4)(a + 6) = (x + 1)(x - 4)(a + 6)$

6) Everything is completely factored.

Our answer is $(x + 1)(x - 4)(a + 6)$.

Ex. 7 $3000a^4b + 81ab^4$

Solution:

1. The G.C.F. is 3ab. Factoring out 3ab, we get:
$3000a^4b + 81ab^4$
$= 3ab(1000a^3 + 27b^3)$

2. The polynomial inside the parentheses has 2 terms.

5. $1000a^3 + 27b^3$ is not a difference of squares, but is a sum of cubes: $1000a^3 + 27b^3$
$= (10a)^3 + (3b)^3$ $F^3 + L^3 = (F + L)(F^2 - FL + L^2)$
$= (10a + 3b)(100a^2 - 30ab + 9b^2)$
Thus, $3ab(1000a^3 + 27b^3)$
$= 3ab(10a + 3b)(100a^2 - 30ab + 9b^2)$

6. Since $100a^2 - 30ab + 9b^2$ is degree 2, it is prime. Everything is completely factored.
Our answer is $3ab(10a + 3b)(100a^2 - 30ab + 9b^2)$.

Ex. 8 $6w^2(4w^2 + 49) + 6w(4w^2 + 49) - 72(4w^2 + 49)$

Solution:

1. The G.C.F. is $6(4w^2 + 49)$. Factoring out $6(4w^2 + 49)$, we get:
$6w^2(4w^2 + 49) + 6w(4w^2 + 49) - 72(4w^2 + 49)$
$= 6(4w^2 + 49)(w^2 + w - 12)$

2. The polynomial $4w^2 + 49$ has 2 terms. The polynomial $w^2 + w - 12$ is a trinomial.

4. Since – 12 is not a perfect square, then the trinomial is not a perfect square trinomial. We will factor it using trial-and-error:

$w^2 + w - 12$

$w \bullet w$	$-1 \bullet 12$	We quickly see that we want
	$-2 \bullet 6$	to use – 3 and 4
	$-3 \bullet 4$	

Thus, $w^2 + w - 12 = (w - 3)(w + 4)$
So, $6(4w^2 + 49)(w^2 + w - 12) = 6(4w^2 + 49)(w - 3)(w + 4)$

5. $4w^2 + 49$ is not a difference of squares, but is a sum of squares and hence it is prime.

6. Everything is completely factored.
Our answer is $6(4w^2 + 49)(w - 3)(w + 4)$.

Ex. 9 $162x^2y - 243x^2 - 252xy^2 + 378xy + 98y^3 - 147y^2$

Solution:

1. The G.C.F. is 1.

2. The polynomial has more than 4 terms.

3. We will group the first two, the middle two and the last two terms:

$162x^2y - 243x^2 - 252xy^2 + 378xy + 98y^3 - 147y^2$

$= (162x^2y - 243x^2) + (- 252xy^2 + 378xy) + (98y^3 - 147y^2)$

Factor out $81x^2$, $- 126xy$, and $49y^2$:

$= 81x^2(2y - 3) - 126xy(2y - 3) + 49y^2(2y - 3)$

Factor out $(2y - 3)$:

$= (2y - 3)(81x^2 - 126xy + 49y^2)$

4. Both $81x^2$ and $49y^2$ are perfect squares where F = 9x and L = 7y. We need to check the middle term:

$- 2FL = - 2(9x)(7y) = - 126xy$ which is a match.

Hence, the trinomial is a perfect square trinomial:

$81x^2 - 126xy + 49y^2$

$= (9x - 7y)^2 F^2 - 2FL + L^2 = (F - L)^2$; F = 9x, L = 7y

Thus, $(2y - 3)(81x^2 - 126xy + 49y^2)$

$= (2y - 3)(9x - 7y)^2$

5. $2y - 3$ is not a difference of squares nor is it a sum and difference of cubes, so it is prime.

6. Everything is completely factored.

Our answer is $= (2y - 3)(9x - 7y)^2$,