Sect 8.1 - Introduction to Relations

Concept #1: Domain and Range of a Relation.

In many instances, there is a relationship between two quantities in real life. For example, the distance an object falls is related the time the object has been falling; the longer an object falls, the further distance it falls. The table below shows the correspondence between the time an object has been falling and the distance the object has fallen. We can represent each data point as an ordered pair.

Ex. 1

The length of time in seconds x	**The distance fallen in meters y**	**Written as an Ordered Pair (x, y)**
0.8	3.4	(0.8, 3.4)
1.9	17.7	(1.9, 17.7)
2.7	35.7	(2.7, 35.7)
3.8	70.8	(3.8, 70.8)
5.2	132.5	(5.2, 132.5)

The set of ordered pairs {(0.8, 3.4), (1.9, 17.7), (2.7, 35.7), (3.8, 70.8), (5.2, 132.5)} defines a **relation** between the time and the distance fallen. The first component, x, of each ordered pair is the independent variable while the second component, y, of each ordered pair is the dependent variable. The set of all first components is called the **domain** of the relation and the set of all second components is called the **range** of the relation. Thus, the domain of the relation in example #1 is {0.8, 1.9, 2.7, 3.8, 5.2} and the range of the relation is {3.4, 17.7, 35.7, 70.8, 132.5}. This leads us to the following definition:

Definition of a Relation in x and y

A **relation in x and y** is any set of ordered pairs (x, y).

The **domain of the relation** is the set of all first components in the ordered pairs.

The **range of the relation** is the set of all second components in the ordered pairs.

Ex. 2 The number of customers buying ice cream from an ice cream parlor is related to the high temperature of the day and is listed in the table below:

High Temperature x	Number of Customers y
85˚	135
97˚	236
92˚	185
94˚	201
97˚	247

a) Write the relation as a set of ordered pairs.
b) What is the domain of the relation?
c) What is the range of the relation?

Solution:

a) The relation is defined as:
{(85˚, 135), (97˚, 236), (92˚,185), (94˚, 201), (97˚, 247)}

b) The domain is the set of all first components in the relation. Even though 97˚ occurs twice in the relation, we only list it once when we state the domain.
Domain: {85˚, 97˚, 92˚, 94˚}

c) The range is the set of all second components in the relation.
Range: {135, 236, 185, 201, 247}

Ex. 3 The number of electoral votes of the top six states is listed in the table below:

State x	Number of Electoral Votes y
California	55
Texas	34
New York	31
Florida	27
Illinois	21
Pennsylvania	21

a) Write the relation as a set of ordered pairs.
b) What is the domain of the relation?
c) What is the range of the relation?

Solution:

a) The relation is defined as:
{(California, 55), (Texas, 34), (New York, 31), (Florida, 27), (Illinois, 21), (Pennsylvania, 21)}

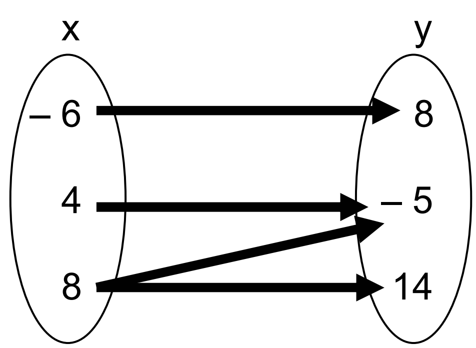
b) The domain is the set of all first components in the relation.
Domain: {California, Texas, New York, Florida, Illinois, Pennsylvania}

c) The range is the set of all second components in the relation. Even though 21 occurs twice in the relation, we only list it once when we state the range.
Range: {55, 34, 31, 27, 21}

Aside from using a set of ordered pairs, we can represent a relation with a graph, with a correspondence between domain and range, or with an equation. Let's look at some other ways to define a relation:

Find the domain and range of the following relation:

Ex. 4



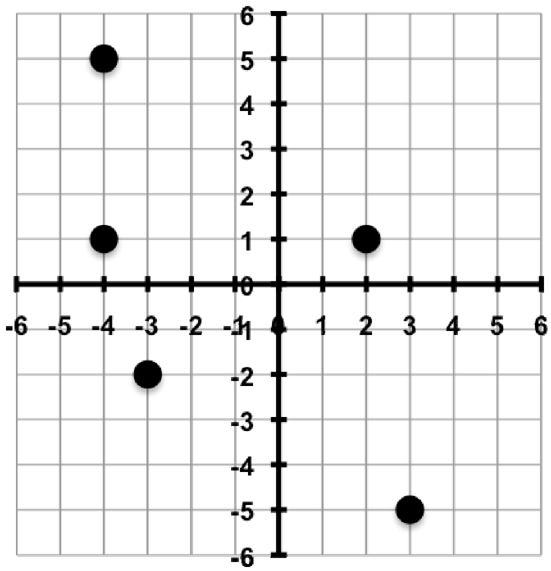
Solution:

We can write this correspondence as a set of ordered pairs:

– 6 corresponds to 8: (– 6, 8)
4 corresponds to – 5: (4, – 5)
8 corresponds to – 5: (8, – 5)
8 corresponds to 14: (8, 14)

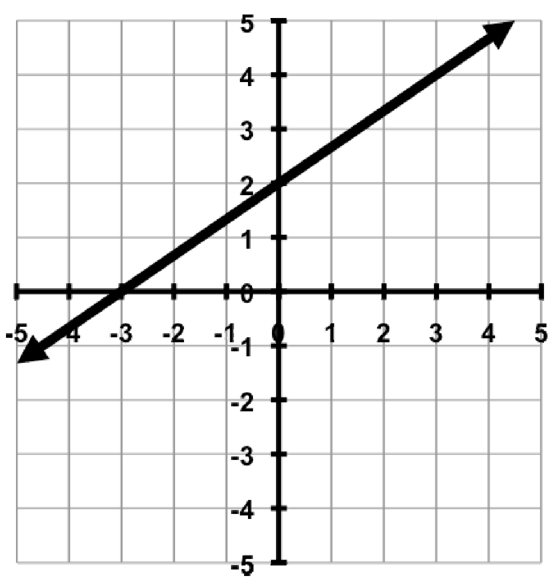
Thus, is relation is {(– 6, 8), (4, – 5), (8, – 5), (8, 14)}
The domain is { – 6, 4, 8}
The range is {8, – 5, 14}

Ex. 5



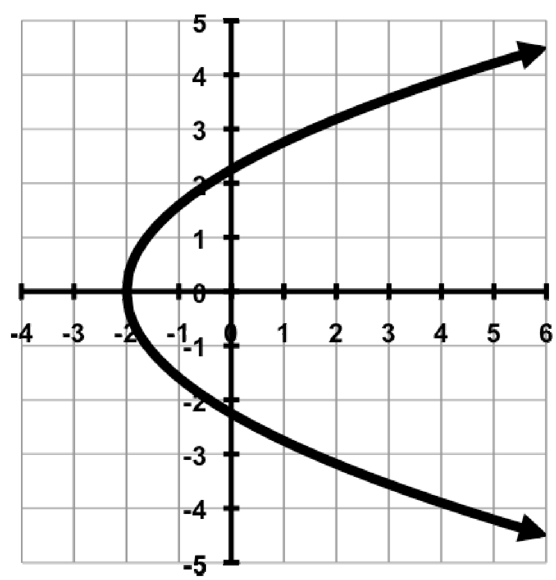
Solution:
The ordered pairs are (– 4, 5), (– 4, 1), (– 3, – 2), (2, 1), & (3, – 5).
Thus, our relation is {(– 4, 5), (– 4, 1), (– 3, – 2), (2, 1), (3, – 5)}.
The domain is {– 4, – 3, 2, 3} and the range is {5, 1, – 2, – 5}

Ex. 6



Solution:
The relation can be defined as the infinite set of ordered pairs on the line $y = \frac{2}{3}x + 2$ or

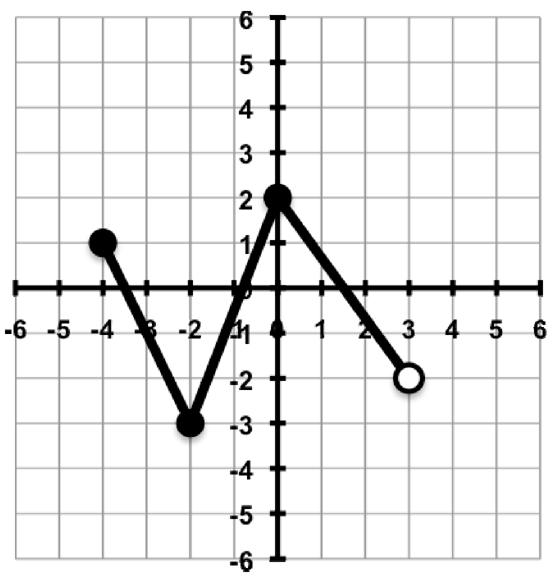
Ex. 7



Solution:
The relation can be defined as the infinite set of ordered pairs on the curve $x = \frac{32}{81}y^2 - 2$ or

{(x, y) | y = $\frac{2}{3}$x + 2}. The line extends forever in both direction, so the domain is (– ∞, ∞) and the range is (– ∞, ∞).

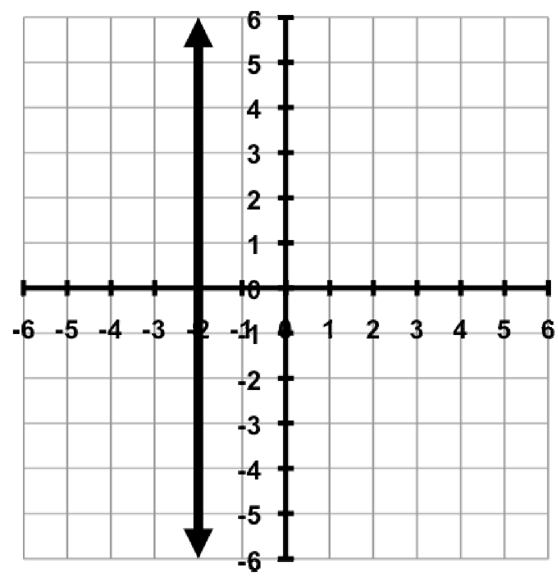
Ex. 8



Solution:
This relation is very difficult represent as anything, but a graph. The x-values extend from – 4, including – 4, up to 3, not including 3. So, the domain is [– 4, 3). The y-values extend from – 3, including – 3 to 2, including 2. So, the range is [– 3, 2].

{(x, y) | x = $\frac{32}{81}y^2$ – 2}. The curve extends up and to the right and down and to the left forever. It, however, stops at x = – 2 when moving to the left. So, the domain is [– 2, ∞) and the range is (– ∞, ∞).

Ex. 9

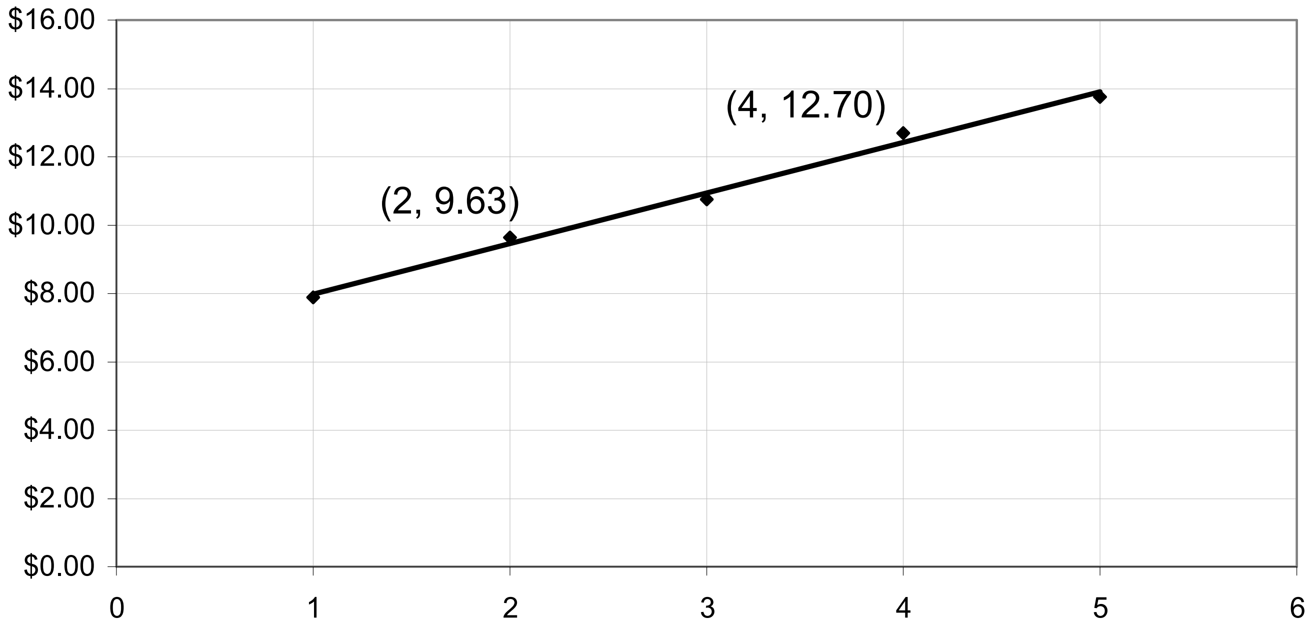


Solution:
This relation can be represented as the infinite set of points on the x = – 2. The only value of x is this relation is x = – 2. The y-values extend forever in both directions. Thus, the domain is {– 2}, the range is (– ∞, ∞).

Concept #2 Applications Involving Relations

Solve the following:

Ex. 10 The graph below shows the US natural gas price in dollars per thousand cubic feet beginning in the year 2002. (Source: www.eia.doe.gov)



Year (x = 0 corresponds to 2001)

a) What is the domain of the linear equation?
b) What is the range of the linear equation?
c) Define the relation.

Solution:

a) The x - values extend from 1 to 5, including 1 and 5. Thus, the domain of the equation is [1, 5].

b) The y - values extend from approximately \$8 to approximately \$14. So, the range is ≈ [\$8, \$14].

c) To define the relation, we will first find the equation of the line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12.7 - 9.63}{4 - 2} = \frac{3.07}{2} = \$1.535 \text{ per year.}$$

$y - y_1 = m(x - x_1)$

$y - 9.63 = 1.535(x - 2)$

$y - 9.63 = 1.535x - 3.07$

$y = 1.535x + 6.56$ where $1 \le x \le 5$

Thus, the relation is $\{ (x, y) \mid y = 1.535x + 6.56, 1 \le x \le 5\}$.