Review of Linear Equations in Two Variables

Concept #1 Graphing Linear Equations.

Definition of a Linear Equation in Two Variables
Let a, b, and c be real numbers where a and b are not both zero. Then an equation that can be written in the form:

$$ax + by = c$$

is called a **linear equation in two variables**.

There are two ways we can graph a linear equation. One way is make a table of values and plotting the points. The second way is by solving the equation for y and using the slope and y-intercept to generate the graph. Let's review both techniques.

Sketch the graph of the following:

Ex. 1 $3x + 2y = 6$

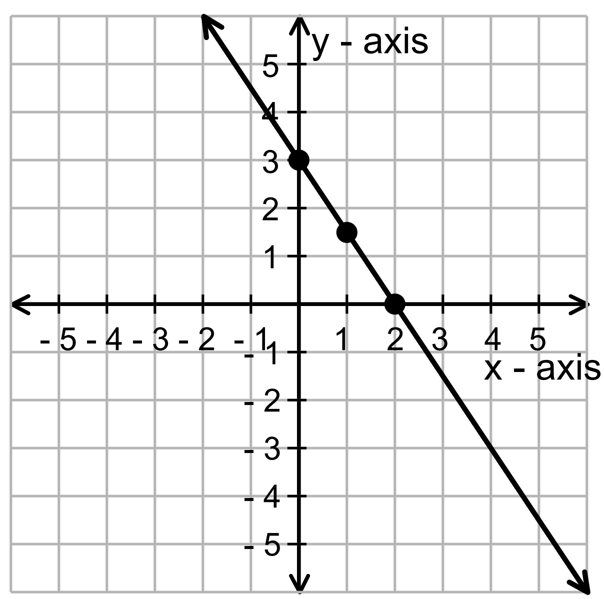
Solution:

First we need to find three solutions. Let us just pick some numbers for x and find the corresponding y values.

x = 0: $3(0) + 2y = 6$
$2y = 6$
$y = 3$

x = 1: $3(1) + 2y = 6$
$2y = 3$
$y = 1.5$

x = 2: $3(2) + 2y = 6$
$2y = 0$
$y = 0$



Now organize this in a table and plot the points:

x	y
0	3
1	1.5
2	0

The point where the line intersects the y - axis is called the **y-intercept**. The x-coordinate of the y-intercept is always zero. So, for our equation, (0, 3) is the y-intercept. The **x-intercept** is the point where the line intersects the x-axis. The y-coordinate of the x-intercept is always zero.
In this example, the x-intercept is (2, 0).

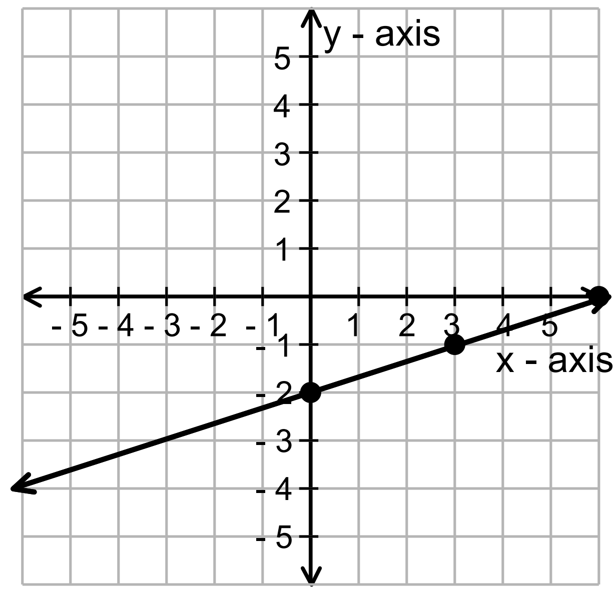
Ex. 2 $2x - 6y = 12$

<u>Solution:</u>

First solve the equation for y:

$$2x - 6y = 12$$
$$\underline{-2x \qquad = -2x}$$
$$\frac{-6y}{-6} = \frac{-2x + 12}{-6}$$
$$y = \frac{1}{3}x - 2$$

The slope is $\frac{1}{3} = \frac{\text{"rise"}}{\text{"run"}}$ and the y-intercept is (0, – 2). Plot the point (0, – 2). Then from that point rise 1 unit and run 3 units to get another point. From that new point, rise another 1 unit and run 3 more units to get the third point. Now, draw the graph.



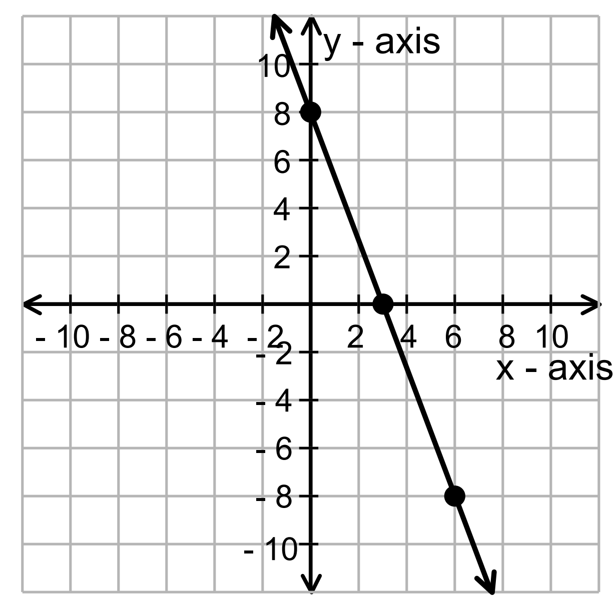
Ex. 3 $8x + 3y = 24$

<u>Solution:</u>

First solve the equation for y:

$$8x + 3y = 24$$
$$\underline{-8x \qquad = -8x}$$
$$\frac{3y}{3} = \frac{-8x + 24}{3}$$
$$y = -\frac{8}{3}x + 8$$

The slope is $-\frac{8}{3} = \frac{-8}{3} = \frac{\text{"rise"}}{\text{"run"}}$ and the y-intercept is (0, 8). Plot the point (0, 8). Then from that point fall 8 units and run 3 units to get another point. From that new point fall another 8 units and run 3 more units to get the third point. Now, draw the graph.



Calculate the slope of the line. The sketch the graph:

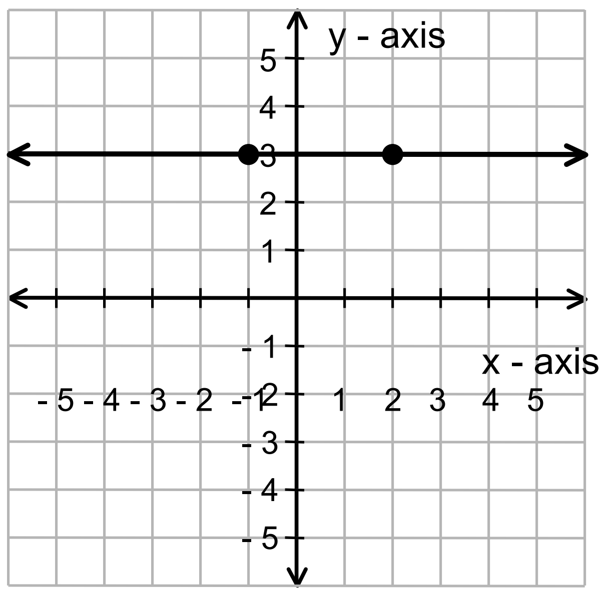
Ex. 4 $y = 3$

Solution:

Since there is no x term, we can make x anything and $y = 3$. So, two points on the line are $(-1, 3)$ and $(2, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{2-(-1)} = \frac{0}{3} = 0.$

Now, draw the graph:



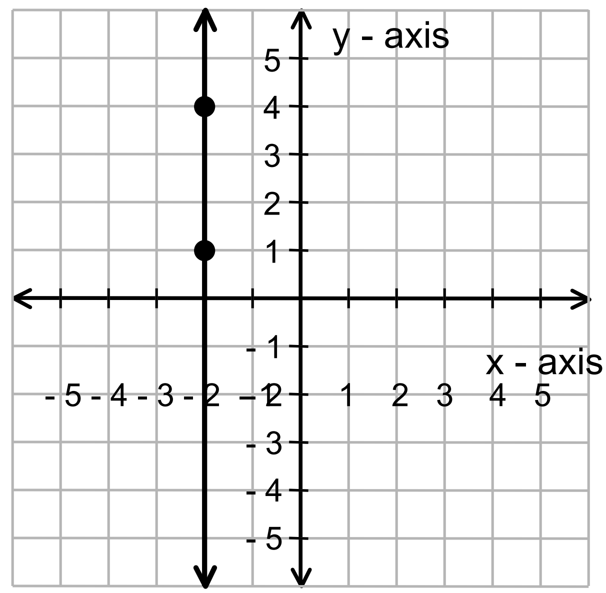
Ex. 5 $x = -2$

Solution:

Since there is no y term, we can make y anything and $x = -2$. So, two points on the line are $(-2, 1)$ & $(-2, 4)$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{-2-(-2)} = \frac{3}{0}$

which is undefined. Now, draw the graph:



In general, all horizontal lines have slope of zero. Think of a horizontal as being level ground; it has zero steepness. A vertical line however is so steep that you cannot measure it. It's slope is undefined.

Concept #2 Parallel and Perpendicular Lines

Two distinct lines are **parallel** if and only if they have the same slope. We can denote the slope of a parallel line by $m_{||}$.

Two distinct lines that are neither vertical nor horizontal lines are **perpendicular** if and only if the product of their slopes is -1. In other words, the slope of one line is the negative reciprocal of the slope of the line perpendicular to it so long as they are not vertical or horizontal lines. Every vertical line is perpendicular to every horizontal line and every horizontal line is perpendicular to every vertical line. We can denote the slope of a perpendicular line by $m_{\perp}$.

Determine if the lines are parallel, perpendicular, or neither.

Ex. 6 $8x - 7y = 6$
$8y = -7x + 3$

Solution:
First, solve each equation for y to find the slope.

$$8x - 7y = 6$$
$$-8x \quad = -8x$$
$$\frac{-7y}{-7} = \frac{-8x + 6}{-7}$$
$$y = \frac{8}{7}x - \frac{6}{7}$$
$$m_1 = \frac{8}{7}$$
$$\frac{8y}{8} = \frac{-7x + 3}{8}$$
$$y = -\frac{7}{8}x + \frac{3}{8}$$
$$m_2 = -\frac{7}{8}$$

Since $m_1 \bullet m_2 = \frac{8}{7}\left(-\frac{7}{8}\right) = -1$, the lines are perpendicular.

Ex. 7 $4x - 2 = y$
$4x + y = 5$

Solution:
First, solve each equation for y to find the slope.

$$4x - 2 = y$$
$$y = 4x - 2$$
$$m_1 = 4$$
$$4x + y = 5$$
$$-4x = -4x$$
$$y = -4x + 5$$
$$m_2 = -4$$

But, $m_1 \neq m_2$, so they are not parallel. Also, $m_1 \bullet m_2 = 4(-4) = -16 \neq -1$, so they are not perpendicular. There the lines are neither.

Ex. 8 $\frac{2}{9}x - \frac{1}{6}y = 1$
$1.2x - 0.9y = 1.8$

Solution:
First, solve each equation for y to find the slope.

$$\frac{18}{1}\left(\frac{2}{9}x\right) - \frac{18}{1}\left(\frac{1}{6}y\right) = 18(1)$$
$$\frac{2}{1}\left(\frac{2}{1}x\right) - \frac{3}{1}\left(\frac{1}{1}y\right) = 18(1)$$
$$4x - 3y = 18$$
$$-4x \quad = -4x$$
$$\frac{-3y}{-3} = \frac{-4x + 18}{-3}$$
$$y = \frac{4}{3}x - 6$$
$$m_1 = \frac{4}{3}$$

$$10(1.2x) - 10(0.9y) = 10(1.8)$$
$$12x - 9y = 18$$
$$-12x \quad = -12x$$
$$\frac{-9y}{-9} = \frac{-12x + 18}{-9}$$
$$y = \frac{4}{3}x - 2$$
$$m_2 = \frac{4}{3}$$

Since $m_1 = m_2$, then the lines are parallel.

Concept #3 Writing the Equation of the Line Using the Point-Slope Formula

A linear equation in two variables is said to be written in **point-slope form** if it is in the form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is any known point on the line.

Write the equation of the line in slope-intercept form:

Ex. 9 Find the equation of the line passing through (– 2, 6)

a) parallel to y = 3x – 2.
b) perpendicular to y = 3x – 2.

Solution:

a) Parallel lines have the same slope, so $m_{||} = 3$.

$$y - y_1 = m(x - x_1)$$
$$y - 6 = 3(x - (-2)) = 3(x + 2) = 3x + 6$$
$$y - 6 = 3x + 6$$
$$+6 = \quad +6$$
$$y = 3x + 12$$

b) The slope of the line perpendicular is the negative reciprocal of 3. So, $m_{\perp} = -\frac{1}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - 6 = -\frac{1}{3}(x - (-2)) = -\frac{1}{3}(x + 2) = -\frac{1}{3}x - \frac{2}{3}$$
$$y - 6 = -\frac{1}{3}x - \frac{2}{3}$$
$$+6 = \quad + \frac{18}{3}$$
$$y = -\frac{1}{3}x + \frac{16}{3}$$

Ex. 10 Find the equation of the line passing through (8, – 3)

a) parallel to x = 5.
b) perpendicular to x = 5.

Solution:

a) A line parallel to x = 5 is a vertical line and it passes through (8, – 3). So, the equation is x = 8.
b) A line perpendicular to x = 5 is a horizontal line and it passes through (8, – 3). So, the equation is y = – 3.