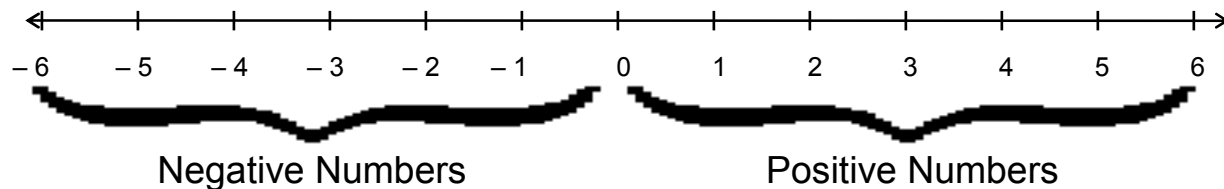


Sect 10.1 – Real Numbers and the Real Number Line

Objective a: Integers

In this chapter, we will significantly expand the number system that we have discussed thus far in the textbook. There are certain quantities in real life that cannot be properly expressed in our current number system. For example, in Alaska, the temperature might read as 20° degrees below zero. Another example would be a diver is at a depth of 500 ft below sea level. Or perhaps, a customer is overdrawn by \$25 in his or her checking account. All of these quantities occur in real life but we cannot adequately express them using our current number system. We need a way to express them so that they are different from 20° above zero, 500 ft above sea level, and having \$25 in an account. To do this, we will take a copy of our number line and paste it on the other side of zero. The numbers that are greater than zero we will call the positive numbers and the numbers that are less than zero we will call the negative numbers. To represent a positive number like four, we will write “4” or “+ 4”. To represent a negative number like negative four, we will write “– 4”. A number like -4 is four units below zero.



Recall that Whole Numbers were 0, 1, 2, 3, 4, ... We can now add negative numbers and define what are called integers:

Integers: ..., -4 , -3 , -2 , -1 , 0, 1, 2, 3, 4, ...

Represent the following as an integer:

- Ex. 1 a) 20° F below zero.
 b) 500 feet below sea level.
 c) \$25 overdrawn.

Solution:

- a) 20° F below zero = -20° F.
 b) 500 feet below sea level = -500 ft.
 c) \$25 overdrawn = $-\$25$.

Objective b: Rational and Irrational Numbers.

If we “add” fractions and decimals that terminate or repeat to the integers, we get the **Rational Numbers**. This set is too complicated to write as a list, so we will use the fact that every integer can be written as a fraction by putting the integer over 1 and that any repeating decimal can be expressed as the quotient of two integers. Thus, the rational numbers are defined as:

Rational Numbers: Any number that can be represented as the quotient

of two integers so long as the integer in the denominator is not zero.

Some examples of rational numbers are:

$$3, -4, \frac{2}{3}, \frac{-27}{11}, 0, -0.234, 5.\overline{68}$$

Numbers that are not rational are numbers that cannot be represented as the quotient of two integers. In other words, there are numbers that are decimals that never terminate and never repeat. We call these numbers the **Irrational Numbers**.

Irrational Numbers: Any number that is a non-terminating, non-repeating decimal.

Some examples of irrational numbers are

$$\pi = 3.14159265358979323846\dots \text{ and}$$

$$\sqrt{2} = 1.4142135623730950488\dots$$

If we put the set of rational numbers together the set of irrational numbers, we get the **Real Numbers**.

Real Numbers: A number that is either rational or irrational.

To develop a better sense of order with real numbers, let’s work through an example of plotting various real numbers on a number line:

Graph each number on the same number line:

Ex. 2 $2\frac{1}{3}, -1.25, 4, -5\frac{7}{8}, 0.85$ and -3

Solution:

$2\frac{1}{3}$ is between 2 and 3, but it is closer to 2.

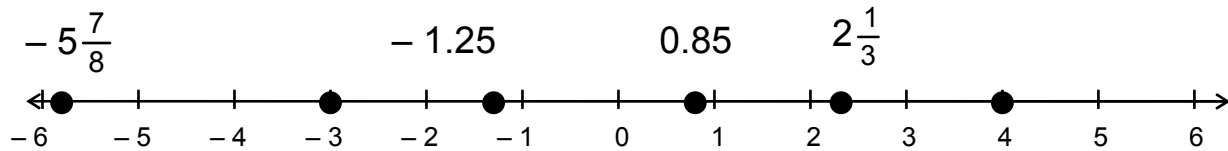
-1.25 is between -1 and -2 , but it is closer to -1 .

Four is marked at 4.

$-5\frac{7}{8}$ is between -6 and -5 , but is closer to -6 .

0.85 is between 0 and 1, but closer to 1.

Finally, -3 is marked at -3 .



Objective c Comparing Real Numbers.

Some things to notice with the graph: $0.85 < 2\frac{1}{3}$, $-1.25 < 0.85$,

$-3 < -1.25$, and $-5\frac{7}{8} < -3$ since, in each case, the number to the left is less than the number on the right. This seems to contradict to what we have done before. When dealing with the negative numbers, if we think what happens in the real world, it makes sense. Since it is worse to owe \$10 ($-\10) than to owe \$5 ($-\5), then -10 is less than -5 . Of course, a positive number is always bigger than a negative number. In fact, a positive is greater than zero and a negative number is smaller than zero.

Let's work some inequalities.

Compare using $>$, $<$, or $=$:

Ex. 3a	3	5	Ex. 3f	-2	-10
Ex. 3b	-3	-5	Ex. 3g	4	-8
Ex. 3c	-8	1	Ex. 3h	$-\frac{7}{8}$	-0.88
Ex. 3d	0	6.2	Ex. 3i	$-\frac{1}{3}$	0.1
Ex. 3e	0	-6.2	Ex. 3j	-0.1	-0.08

Solution:

- Since 3 is smaller than 5, then $3 < 5$.
- Since -3 is to the right of -5 , then $-3 > -5$.
- Since $a - \#$ is always less than $a + \#$, then $-8 < 1$.
- Since $a + \#$ is always larger than 0, then $0 < 6.2$.
- Since $a - \#$ is always smaller than 0, then $0 > -6.2$.
- Since -2 is to the right of -10 , then $-2 > -10$.
- Since $a + \#$ is always larger than $a - \#$, then $4 > -8$.
- $\frac{7}{8} = 7 \div 8 = 0.875$. Since $0.875 < 0.880 = 0.88$,
then $-0.875 > -0.88$. So, $-\frac{7}{8} > -0.88$.
- Since $a - \#$ is always less than $a + \#$, then $-\frac{1}{3} < 0.1$.
- Since $0.1 = 0.10 > 0.08$, then $-0.1 < -0.08$.

Objective d: Understanding the absolute value of a number.

The **Absolute Value** of a number is the distance that number is from zero. The absolute value is denoted with two vertical lines so if we want to say the absolute value of 4, we write: $|4|$ or if we want to say the absolute value of -9 , we write: $|-9|$.

Evaluate the following:

Ex. 4a) $|4|$

Solution:

The absolute value of 4 is 4,
so $|4| = 4$.

Ex. 4b) $|-9|$

Solution:

The absolute value of -9 is 9, so $|-9| = 9$.

Objective e: Understanding the additive inverse or opposite of a number.

The **Additive Inverse** or the **Opposite** of a number is the same number, but with a different sign. If we want to say the opposite of 3, we write: $-(3)$ and if we want to say the opposite of -7.23 , we write: $-(-7.23)$.

Evaluate the following:

Ex. 5a) $-(3)$

Solution:

The opposite of 3 is -3 ,
so $-(3) = -3$.

Ex. 5b) $-(-7.23)$

Solution:

The opposite of -7.23 is 7.23, so $-(-7.23) = 7.23$.

Ex. 6a) $-(-\frac{6}{17})$

Solution:

The opposite of $-\frac{6}{17}$ is $\frac{6}{17}$,
so $-(-\frac{6}{17}) = \frac{6}{17}$.

Ex. 6b) $-(9)$

Solution:

The opposite of 9 is 9,
so $-(9) = -9$.

Remember: Additive Inverses or Opposites - we switch signs.

Absolute Value - we make the number inside the two lines positive.

Evaluate the following:

Ex. 7a) $-(78)$

Ex. 7b) $|-78|$

Ex. 7c) (-78)

Ex. 7d) $-|78|$

Ex. 7e) $-|-78|$

Ex. 7f) $-(-78)$

Solution:

- a) The opposite of 78 is -78 , so $-(78) = -78$.
- b) The absolute value of -78 is 78, so $|-78| = 78$.
- c) Negative seventy-eight is -78 , so $(-78) = -78$.
- d) The opposite of the absolute value of 78 is equal to the opposite of 78 which is -78 , so $-|78| = -(78) = -78$.
- e) The opposite of the absolute value of -78 is equal to the opposite of 78 which is -78 , so $-|-78| = -(78) = -78$.
- f) The opposite of -78 is 78, so $-(-78) = 78$.

Compare using $>$, $<$, or $=$:

Ex. 8a $-(-2.3)$ $-|-2.3|$

Solution:

Since $-(-2.3) = 2.3$ and
 $-|-2.3| = -(2.3) = -2.3$
 and $2.3 > -2.3$, then
 $-(-2.3) > -|-2.3|$.

Ex. 8b $-\left(\frac{7}{11}\right)$ $-|\frac{7}{11}|$

Solution:

Since $-\left(\frac{7}{11}\right) = -\frac{7}{11}$ and
 $-|\frac{7}{11}| = -\left(\frac{7}{11}\right) = -\frac{7}{11}$, and
 $-\frac{7}{11} = -\frac{7}{11}$, then
 $-\left(\frac{7}{11}\right) = -|\frac{7}{11}|$.