Sect 10.2 – Adding Real Numbers

Objective a: Using a number line to understand how to add two integers

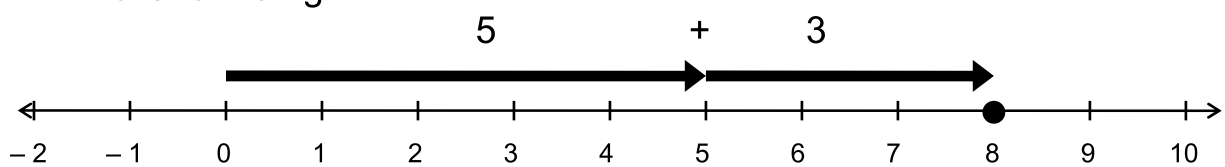
Addition of numbers can be done on a number line. A positive number like 5 can be represented as an arrow moving five ticks to the right on a number line. A negative number like – 3 can be represented as a arrow moving three ticks to the left on a number line. Using this idea, we can represent the addition of two real numbers as two successive arrows on a number. Let's look at some examples:

Add using a number line:

Ex. 1 5 + 3

Solution:

We all know that the answer will be eight, but let's do it on a number line. First draw an arrow moving five ticks to the right of zero to 5 and then draw another arrow starting at five and moving 3 more ticks to the right:

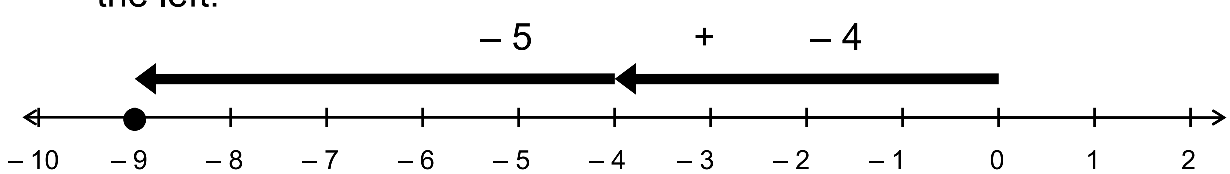


Notice that we landed on 8, just as we expected.

Ex. 2 – 4 + (– 5)

Solution:

First draw an arrow moving four ticks to the left of zero to – 4 and then draw another arrow starting at – 4 and moving 5 more ticks to the left:



Notice that we landed on – 9, so – 4 + (– 5) = – 9.

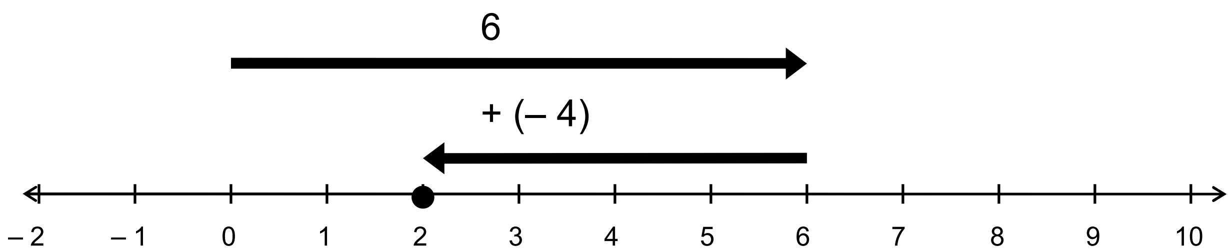
If we add two numbers with the same signs, we add their absolute values and use the common sign in the answer. It is like a checking account; if you make two deposits, you add them together and your balance goes up. If, on the other hand, you write two checks, your balance will go down by the total of the two checks.

Add using a number line:

Ex. 3 $6 + (-4)$

Solution:

First draw an arrow moving six ticks to the right of zero to 6 and then draw another arrow starting at six and moving 4 ticks to the left:

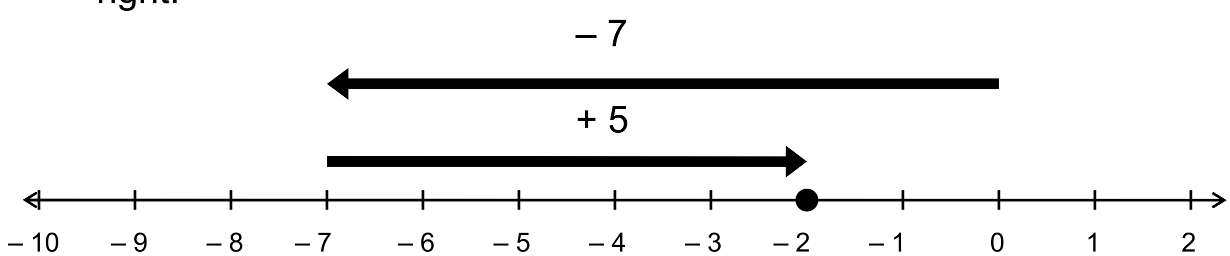


Notice that we landed on 2, so $6 + (-4) = 2$.

Ex. 4 $-7 + 5$

Solution:

First draw an arrow moving seven ticks to the left of zero to – 7 and then draw another arrow starting at – 7 and moving 5 ticks to the right:



Notice that we landed on – 2, so $-7 + 5 = -2$.

When adding numbers with different signs, we subtract their absolute values and keep the sign of the number with the larger absolute value. Again, it is like a checking account; if you make a deposit and then write a check, those two items work against each other on your account balance. Which ever is larger, the check or the deposit, will have a bigger impact your balance.

When we say add two Real numbers, we are using the word "add" in a much broader sense. The actual operation that we perform depends on the signs of the numbers that we are adding.

Objective b: Adding Real numbers

Adding Real Numbers:

1) When adding two numbers with the same signs, we add their absolute values and use the common sign in the answer.
   Same Signs – Sum – Same Sign

2) When adding numbers with different signs, we subtract their absolute values and keep the sign of the number with the larger absolute value.
   Different Signs – Difference – Sign of the "Larger" #

Simplify the following:

Ex. 5a $-27 + 23$

Ex. 5b $-\frac{8}{13} + \left(-\frac{4}{13}\right)$

Ex. 5c $0.95 + (-1.15)$

Ex. 5d $12\frac{7}{8} + \left(-5\frac{3}{10}\right)$

Ex. 5e $6 + 8$

Ex. 5f $\left(-\frac{9}{5}\right) + \frac{6}{5} + \left(-\frac{7}{5}\right)$

Solution:

a) Since the signs are different, subtract their absolute values: $27 - 23 = 4$. But, -27 has the larger absolute value, so the answer is -4.

b) Since the signs are the same, add their absolute values: $\frac{8}{13} + \frac{4}{13} = \frac{12}{13}$. But, both numbers are negative, so the answer is $-\frac{12}{13}$.

c) Since the signs are different, subtract their absolute values: $1.15 - 0.95 = 0.2$. But, -1.15 has the larger absolute value, so the answer is -0.2.

d) Since the signs are different, subtract their absolute values: The L.C.D. = 40, so $12\frac{7}{8} = 12\frac{35}{40}$ and $5\frac{3}{10} = 5\frac{12}{40}$. Thus, the problem becomes $12\frac{35}{40} - 5\frac{12}{40} = 7\frac{23}{40}$. Since $12\frac{7}{8} = 12\frac{35}{40}$ has the larger absolute value, the answer is $7\frac{23}{40}$.

e) Since the signs are the same, add their absolute values: 6 + 8 = 14. But, the numbers are positive, so the answer is 14.

f) Since $-\frac{9}{5}$ and $\frac{6}{5}$ have different signs, subtract their absolute values: $\frac{9}{5} - \frac{6}{5} = \frac{3}{5}$. But, $-\frac{9}{5}$ has the larger absolute value, so the result is $-\frac{3}{5}$. Thus, $\left(-\frac{9}{5}\right) + \frac{6}{5} + \left(-\frac{7}{5}\right)$ $= -\frac{3}{5} + \left(-\frac{7}{5}\right)$. Now, the numbers have the same signs, so add their absolute values: $\frac{3}{5} + \frac{7}{5} = \frac{10}{5} = 2$. Since both numbers are negative, our final answer is – 2.

Objective c: Translations

<u>Solve the following:</u>

Ex. 6 Find the sum of – 2.3, 1.6, – 12.4, 7, – 4.5, and – 1.8)

<u>Solution:</u>

Translate and work the problem from left to right:

– 2.3 + 1.6 + (– 12.4) + 7 + (– 4.5) + (– 1.8) (different signs)
= ***– 0.7 + (– 12.4)*** + 7 + (– 4.5) + (– 1.8) (same signs)
= ***– 13.1 + 7*** + (– 4.5) + (– 1.8) (different signs)
= ***– 6.1 + (– 4.5)*** + (– 1.8) (same signs)
= – 10.6 + (– 1.8) (same signs)
= – 12.4.

Ex. 7 The total of $-\frac{1}{7}, -\frac{5}{21}. \frac{3}{14}$, and $-\frac{2}{3}$.

<u>Solution:</u>

The L.C.D. = 42, so $\frac{1}{7} = \frac{6}{42}, \frac{5}{21} = \frac{10}{42}, \frac{3}{14} = \frac{9}{42}$, and $\frac{2}{3} = \frac{28}{42}$.

Thus, $\left(-\frac{1}{7}\right) + \left(-\frac{5}{21}\right) + \left(\frac{3}{14}\right) + \left(-\frac{2}{3}\right)$

$= \left(-\frac{6}{42}\right) + \left(-\frac{10}{42}\right) + \left(\frac{9}{42}\right) + \left(-\frac{28}{42}\right)$ (same signs)

$= \left(-\frac{16}{42}\right) + \left(\frac{9}{42}\right) + \left(-\frac{28}{42}\right)$ (different signs)

$= \left(-\frac{7}{42}\right) + \left(-\frac{28}{42}\right)$ (same signs)

$= -\frac{35}{42} = -\frac{5}{6}$. (reduce)