Sect 10.4 – Multiplying and Dividing Real Numbers

Objective a: Understanding how to multiply two real numbers.

To see how multiplying and dividing a negative and a positive number works, let's look at some examples.

Solve:

Ex. 1 A tank is losing 3 gallons of water a day. How many gallons will it lose in 5 days?

Solution:

To find the answer, we multiply three by five. Thus, 3(5) = 15. So, the tank will lose 15 gallons.

Ex. 2 On a savings account, a bank accidentally charges a service fee of $3 per month for four months. How much did the bank have to adjust the balance to correct the error?

Solution:

To find the answer, we need to multiply 3 and 4: 3(4) = 12. The bank had to add $12 to the account to correct the error.

Now, let's solve these problems using real numbers:

Losing 3 gallons is written as – 3, 5 days is written as 5, and a 15 gallon lost can be written as – 15 gallon. So, the problem can be written as – 3(5) = – 15.

A monthly service fee of $3 can be written as – $3, deleting four transactions can be written as – 4, and adding $12 to the account can be written as 12. So, the problem becomes – 3(– 4) = 12.

Example one shows us that a positive number times a negative number gives a negative answer. Also, since multiplication is commutative, a negative number times a positive number will also yield a negative answer. Example two that a negative number times a negative number is a positive number. So, let's write down the rules:

Multiplying Real Numbers:

The product of two nonzero real numbers with the same signs is positive. (+ #)•(+ #) = + Ans. (– #)•(– #) = + Ans.

The product of two nonzero real numbers with the different signs is negative. (– #)•(+ #) = – Ans. (+ #)•(– #) = – Ans.

Note: Zero times any real number is zero.

Simplify the following:

Ex. 3a $16 \bullet (-8)$ Ex. 3d $-0.7(0.03)$

Ex. 3b $-3\frac{3}{4}\left(-3\frac{3}{8}\right)$ Ex. 3e $-0.7 + 0.03$

Ex. 3c $-3\frac{3}{4} + \left(-3\frac{3}{8}\right)$ Ex. 3f $3(13)$

Solution:

a) $16 \bullet (-8) = -128$ {(– #)•(+ #) = – Ans.}

b) $-3\frac{3}{4}\left(-3\frac{3}{8}\right) = -\frac{15}{4}\left(-\frac{27}{8}\right) = \frac{405}{32} = 12\frac{21}{32}$
{(– #)•(– #) = + Ans.}

c) $-3\frac{3}{4} + \left(-3\frac{3}{8}\right) = -3\frac{6}{8} + \left(-3\frac{3}{8}\right) = -6\frac{9}{8} = -7\frac{1}{8}$
{Same signs - Sum - Same Sign}

d) $-0.7(0.03) = -0.021$ {(– #)•(+ #) = – Ans.}

e) $-0.7 + 0.03 = -0.67$ {Different signs - Difference - Sign of "larger" #}

f) $3(13) = 39$ {(+ #)•(+ #) = + Ans.}

Ex. 4a $-5(8)$ Ex. 4d $-33 - 11$

Ex. 4b $-5 + 8$ Ex. 4e $-33(-11)$

Ex. 4c $-3(0)$ Ex. 4f $-33 + 11$

Solution:

a) $-5(8) = -40$ {(– #)•(+ #) = – Ans.}

b) $-5 + 8 = 3$ {Different signs - Difference - Sign of "larger" #}

c) $-3(0) = 0$ {any number times zero is zero}

d) $-33 - 11 = -33 + (-11)$ {change to addition, change sign}
$= -44$ {Same Signs – Sum – Same Sign}

e) $-33(-11) = 363$ {(– #)•(– #) = + Ans.}

f) $-33 + 11 = -22$ {Different signs - Difference - Sign of "larger" #}

Ex. 5a – 17 subtract – 3 Ex. 5c The product of $-\frac{2}{5}$ & $\frac{4}{3}$.

Ex. 5b – 17 times – 3 Ex. 5d $-\frac{2}{5}$ plus $\frac{4}{3}$

Solution:

a) – 17 subtract – 3 = – 17 – (– 3) = – 17 + 3 = – 14.

b) – 17 times – 3 = – 17(– 3) = 51

c) The product of $-\frac{2}{5}$ & $\frac{4}{3} = -\frac{2}{5} \bullet \frac{4}{3} = -\frac{8}{15}$

d) $-\frac{2}{5}$ plus $\frac{4}{3} = -\frac{2}{5} + \frac{4}{3} = -\frac{2\bullet3}{5\bullet3} + \frac{4\bullet5}{3\bullet5} = -\frac{6}{15} + \frac{20}{15} = \frac{14}{15}$

Ex. 6a $(-3)(-3)$ Ex. 6c $(-3)(-3)(-3)(-3)$

Ex. 6a $(-3)(-3)(-3)$ Ex. 6c $(-3)(-3)(-3)(-3)(-3)$

Solution:

a) $(-3)(-3) = +9$

b) $(-3)(-3)(-3) = +9(-3) = -27$

c) $(-3)(-3)(-3)(-3) = +9(-3)(-3) = -27(-3) = 81$

d) $(-3)(-3)(-3)(-3)(-3) = +9(-3)(-3)(-3) = -27(-3)(-3)$

$= 81(-3) = -243$

Objective b: Multiplying Many Factors.

Notice in the last example, when there is an even number of negative numbers in the product, the answer is positive, but if there is an odd number of negative numbers in the product, then answer is negative.

Simplify:

Ex. 7a $-8(3)(-2)(-4)$ Ex. 7b $6(-3)(-2)(-1)(-4)$

Solution:

a) Since there are an odd number of negative numbers, the answer is negative: $-8(3)(-2)(-4) = -192$

b) Since there are an even number of negative numbers, the answer is positive: $6(-3)(-2)(-1)(-4) = 144$

Objective c: Understanding exponents

Recall this type of problem from chapter 1:

Simplify the following:

Ex. 8 $25 - 4^2$

Solution:

$25 - 4^2$ (#2-Exponents)

$= 25 - 16$ (#4-subtraction)

$= 9.$

Now, let's try it using the techniques developed in this chapter.

$25 - 4^2 =$ (add the opposite)
$= 25 + -4^2$ (#2-Exponents) Keep in mind our results have to consistent with our prior results, so $-4^2 = -16$)
$= 25 + -16$ (#4-Addition)
$= 9$

This says that $-4^2 = -16$. The negative does not get squared, only the number. If we want to write -4 the quantity squared, we will need to use parenthesis and write $(-4)^2$.
So, $-4^2 = -4 \bullet 4 = -16$ and $(-4)^2 = (-4)(-4) = 16$.

Simplify the following:

Ex. 9a $(-3)^2$ Ex. 9d $(-4)^3$
Ex. 9b -3^2 Ex. 9e $(-2)^4$
Ex. 9c -4^3 Ex. 9f -2^4

Solution:

a) $(-3)^2 = (-3)(-3) = 9$.
b) $-3^2 = -3 \bullet 3 = -9$.
c) $-4^3 = -4 \bullet 4 \bullet 4 = -16 \bullet 4 = -64$.
d) $(-4)^3 = (-4)(-4)(-4) = 16(-4) = -64$.
e) $(-2)^4 = (-2)(-2)(-2)(-2) = 4(-2)(-2) = -8(-2) = 16$.
f) $-2^4 = -2 \bullet 2 \bullet 2 \bullet 2 = -4 \bullet 2 \bullet 2 = -8 \bullet 2 = -16$.

Objective d: Dividing Real Numbers

We have seen with fractions that when we are dividing, we get the same answer if we multiply be the reciprocal of the number to the right of the operation. Thus, the rules for dividing two nonzero real numbers are the same as for multiplication.

Dividing Real Numbers:

The quotient of two nonzero real numbers with the same signs is positive: $(+\#) \div (+\#) = +$ Ans. $(-\#) \div (-\#) = +$ Ans.

The quotient of two nonzero real numbers with the different signs is negative. $(-\#) \div (+\#) = -$ Ans. $(+\#) \bullet (-\#) = -$ Ans.

Note: $0 \div$ nonzero real number $= 0$

nonzero real number $\div 0$ is undefined.

Simplify the following:

Ex. 10a $-16 \div (-8)$

Ex. 10b $33 \div (-11)$

Ex. 10c $-4.2 \div 0$

Ex. 10d $-3 \div (-13)$

Ex. 10e $\dfrac{-6\frac{1}{4}}{6\frac{2}{3}}$

Ex. 10f $0 \div (-9.45)$

Solution:

a) $-16 \div (-8) = 2$ {(– #) ÷ (– #) = + Ans.}

b) $33 \div (-11 = -3$ {(+ #) ÷ (– #) = – Ans.}

c) $-4.2 \div 0$ is undefined

d) $-3 \div (-13) = \frac{3}{13}$ {(– #) ÷ (– #) = + Ans.}

e) $\dfrac{-6\frac{1}{4}}{6\frac{2}{3}} = -6\frac{1}{4} \div 6\frac{2}{3} = -\frac{25}{4} \div \frac{20}{3} = -\frac{25}{4} \bullet \frac{3}{20} = -\frac{5}{4} \bullet \frac{3}{4} = -\frac{15}{16}$

f) $0 \div (-9.45) = 0$

Comment on fractions and real numbers.

The following are equivalent:

$-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4} = -\frac{-3}{-4}$ ($-\frac{3}{4}$ is the most simplified form)

But $\frac{-3}{-4} \neq -\frac{3}{4}$, since $\frac{-3}{-4} = \frac{3}{4}$.

Also, the following are equivalent:

$\frac{3}{4} = -\frac{-3}{4} = -\frac{3}{-4} = \frac{-3}{-4}$ ($\frac{3}{4}$ is the most simplified form)