Sect 10.5 – Order of Operations with Real Numbers

Objective a: Applying the Order of Operations to Real Numbers.

Recall the order of operations:

Order of Operations
1)   Parentheses - Do operations inside of Parentheses ( ), [ ], { }, $|\;|$
2)   Exponents including square roots.
3)   Multiplication or Division as they appear from left to right.
4)   Addition or Subtraction as they appear from left to right.

Simplify the following:

Ex. 1 $(-2)^3 \div \sqrt{16}\,(-2) + 3$

Solution:

$(-2)^3 \div \sqrt{16}\,(-2) + 3$ (#2-exponents)

$= -8 \div 4(-2) + 3$ (#3-division)

$= -2(-2) + 3$ (#3-multiplication)

$= 4 + 3$ (#4-addition)

$= 7$

Ex. 2 $3|0.5 - 0.3(4)|^2 \div (-0.4 - 0.3)$

Solution:

$3|0.5 - 0.3(4)|^2 \div (-0.4 - 0.3)$ (#1-parentheses, #3-multiplication)

$= 3|0.5 - 1.2|^2 \div (-0.4 - 0.3)$ (change to addition & change the sign to the right)

$= 3|0.5 + (-1.2)|^2 \div (-0.4 + (-0.3))$ (#1-parentheses, #3-addition)

$= 3|-0.7|^2 \div (-0.7)$ (absolute value of -0.7 is 0.7)

$= 3(0.7)^2 \div (-0.7)$ (#2-exponents)

$= 3(0.49) \div (-0.7)$ (#3-multiplication)

$= 1.47 \div (-0.7)$ (#3-division)

$= -2.1$

Ex. 3 $\dfrac{3\bullet(-3)^2 - 5\left(\frac{27}{3} - 2\right)}{-4 + 4(\sqrt{9}\,\bullet 5 \bullet 1) + (-6 \bullet 9)}$

Solution:

Let's first work out the numerator:

$3 \bullet (-3)^2 - 5\left(\frac{27}{3} - 2\right)$ (#1-parentheses, #3-division)

$= 3 \bullet (-3)^2 - 5\left(9 - 2\right)$ (#1-parentheses, #3-subtraction)

$= 3 \bullet (-3)^2 - 5(7)$ (#2-exponents)

$= 3 \bullet (9) - 5(7)$ (#3-multiplication)

$= 27 - 5(7)$ (#3-multiplication)

$= 27 - 35$ (change to addition, change the sign to the right)

$= 27 + (-35)$ (#4-addition)

$= -8$

Now, let's work the denominator:

$-4 + 4(3 \bullet 5 \bullet 1) + (-6 \bullet 9)$ (#1-parentheses, #3-multiplication)

$= -4 + 4(15 \bullet 1) + (-6 \bullet 9)$ (#1-parentheses, #3-multiplication)

$= -4 + 4(15) + (-6 \bullet 9)$ (#1-parentheses, #3-multiplication)

$= -4 + 4(15) + (-54)$ (#3-multiplication)

$= -4 + 60 + (-54)$ (#4-addition)

$= 56 + (-54)$ (#4-addition)

$= 2$

Thus, $\dfrac{3 \bullet (-3)^2 - 5\left(\frac{27}{3} - 2\right)}{-4 + 4(\sqrt{9} \bullet 5 \bullet 1) + (-6 \bullet 9)} = \dfrac{-8}{2} = -4$

Ex. 4 $-4.2(2.4) \div \left(-\frac{7}{10}\right)\left(\frac{3}{10}\right) - 18 \bullet 3 \div 6$

<u>Solution:</u>

Since $\frac{7}{10} = 0.7$ and $\frac{3}{10} = 0.3$, replace the fractions by their decimal equivalents:

$-4.2(2.4) \div \left(-\frac{7}{10}\right)\left(\frac{3}{10}\right) - 18 \bullet 3 \div 6$

$= -4.2(2.4) \div (-0.7)(0.3) - 18 \bullet 3 \div 6$ (#3-multiplication)

$= -10.08 \div (-0.7)(0.3) - 18 \bullet 3 \div 6$ (#3-division)

$= 14.4(0.3) - 18 \bullet 3 \div 6$ (#3-multiplication)

$= 4.32 - 18 \bullet 3 \div 6$ (#3-multiplication)

$= 4.32 - 54 \div 6$ (#3-division)

$= 4.32 - 9$ (change to addition, change the sign to the right)

$= 4.32 + (-9)$ (#4-addition)

$= -4.68$

Ex. 5 $-\left(-\frac{2}{3}\right)^2 - \left[2\frac{1}{3} - 3\frac{5}{6}\right] - \frac{-5}{18} - 1.5$

Solution:

$-\left(-\frac{2}{3}\right)^2 - \left[2\frac{1}{3} - 3\frac{5}{6}\right] - \frac{-5}{18} - 1.5$ (change to an improper fraction)

$= -\left(-\frac{2}{3}\right)^2 - \left[\frac{7\cdot 2}{3\cdot 2} - \frac{23}{6}\right] - \frac{-5}{18} - 1.5$ (L.C.D. = 6, build fractions)

$= -\left(-\frac{2}{3}\right)^2 - \left[\frac{14}{6} - \frac{23}{6}\right] - \frac{-5}{18} - 1.5$ (change to addition, change the sign to the right)

$= -\left(-\frac{2}{3}\right)^2 - \left[\frac{14}{6} + \left(-\frac{23}{6}\right)\right] - \frac{-5}{18} - 1.5$ (#1-parentheses, #4-add.)

$= -\left(-\frac{2}{3}\right)^2 - \left[-\frac{9}{6}\right] - \frac{-5}{18} - 1.5$ (reduce)

$= -\left(-\frac{2}{3}\right)^2 - \left[-\frac{3}{2}\right] - \frac{-5}{18} - 1.5$ (#2-exponents)

Since $\left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \frac{4}{9}$, then $-\left(-\frac{2}{3}\right)^2 = -\frac{4}{9}$

So, $= -\left(-\frac{2}{3}\right)^2 - \left[-\frac{3}{2}\right] - \frac{-5}{18} - 1.5 = -\frac{4}{9} - \left[-\frac{3}{2}\right] - \frac{-5}{18} - 1.5$

Now, change 1.5 into a fraction: $1.5 = 1\frac{5}{10} = 1\frac{1}{2} = \frac{3}{2}$

Thus, $-\frac{4}{9} - \left[-\frac{3}{2}\right] - \frac{-5}{18} - 1.5$

$= -\frac{4}{9} - \left[-\frac{3}{2}\right] - \frac{-5}{18} - \frac{3}{2}$ (change to addition, change the sign to the right)

$= -\frac{4\cdot 2}{9\cdot 2} + \frac{3\cdot 9}{2\cdot 9} + \frac{5}{18} + \left(-\frac{3\cdot 9}{2\cdot 9}\right)$ (L.C.D. = 18, build fractions)

$= -\frac{8}{18} + \frac{27}{18} + \frac{5}{18} + \left(-\frac{27}{18}\right)$ (#4-addition)

$= \frac{19}{18} + \frac{5}{18} + \left(-\frac{27}{18}\right)$ (#4-addition)

$= \frac{24}{18} + \left(-\frac{27}{18}\right)$ (#4-addition)

$= -\frac{3}{18}$ (reduce)

$= -\frac{1}{6}$

Ex. 6 $19 - 3 \div 0 + 4$

Solution:

Since division by zero is undefined, the problem is undefined.

Ex. 7 $\dfrac{-9.6 - 1.6 \div 8(-0.2) - (-0.82)^1}{(0.4 - \sqrt{5.29}\ )^2}$

Solution:

First, work out the expression on the top (numerator):

$-9.6 - 1.6 \div 8(-0.2) - (-0.82)^1$ (#2-exponents)

$= -9.6 - 1.6 \div 8(-0.2) - (-0.82)$ (#3-division)

$= -9.6 - 0.2(-0.2) - (-0.82)$ (#3-multiplication)

$= -9.6 - (-0.04) - (-0.82)$ (change to addition, change the sign to the right)

$= -9.6 + (0.04) + (0.82)$ (#4-addition)

$= -9.56 + 0.82$ (#4-addition)

$= -8.74$

Now, work out the expression on the bottom (denominator):

$(0.4 - \sqrt{5.29}\ )^2$ (#1-parentheses, #4-exponents)

$= (0.4 - 2.3)^2$ (change to addition, change the sign to the right)

$= (0.4 + [-2.3])^2$ (#1-parentheses, #4-addition)

$= (-1.9)^2$ (#2-exponents)

$= 3.61$

So, $\dfrac{-9.6 - 1.6 \div 8(-0.2) - (-0.82)^1}{(0.4 - \sqrt{5.29}\ )^2} = \dfrac{-8.74}{3.61} = -2.421052631....$

This is too messy to write as decimal so let's write it as a fraction.

To make $\dfrac{-8.74}{3.61}$ into a fraction, both the numerator and denominator need to be whole numbers. Slide the decimal point two places to the right and reduce: $\dfrac{-8.74}{3.61} = -\dfrac{874}{361} = -\dfrac{19 \bullet 46}{19 \bullet 19} = -\dfrac{46}{19} = -2\dfrac{8}{19}.$

Ex. 8 $-(-2) + 0 \div (-4.3) - 6 - [4.5 - 0.91(-2.3)](0)$

Solution:

Since anything times zero is zero, then the expression in brackets times zero is zero. So, $[4.5 - 0.91(-2.3)](0) = 0$. Also, 0 divided by any nonzero number is zero, so $0 \div (-4.3) = 0$. Thus, our problem becomes:

$-(-2) + 0 \div (-4.3) - 6 - [4.5 - 0.91(-2.3)](0) = -(-2) + 0 - 6 - 0$

$= -(-2) - 6$ (change to addition, change the sign to the right)

$= 2 + (-6)$ (#4-addition)

$= -4$