Sect 1.3 - Addition of Real Numbers

Concepts #1 & 2 Addition of Real Numbers and the Number Line.

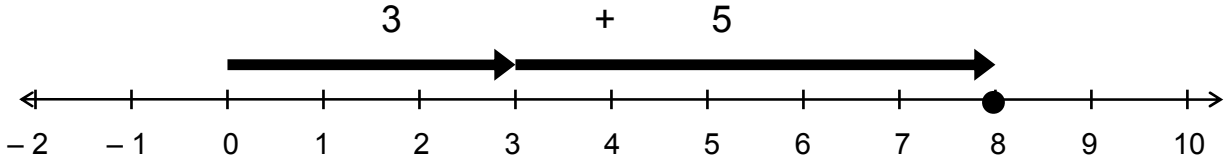
Addition of numbers can be done on a number line. A positive number like 5 can be represented as an arrow moving five ticks to the right on a number line. A negative number like – 3 can be represented as an arrow moving three ticks to the left on a number line. Using this idea, we can represent the addition of two real numbers as two successive arrows on a number. Let's look at some examples:

Add using a number line:

Ex. 1 3 + 5

Solution:

We all know that the answer will be eight, but let's do it on a number line. First draw an arrow moving three ticks to the right of zero to 3 and then draw another arrow starting at three and moving 5 more ticks to the right:

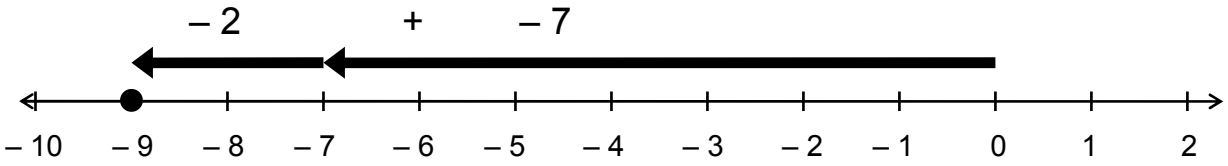


Notice that we landed on 8, just what we expected.

Ex. 2 – 7 + (– 2)

Solution:

First draw an arrow moving seven ticks to the left of zero to – 7 and then draw another arrow starting at – 7 and moving 2 more ticks to the left:



Notice that we landed on – 9, so – 7 + (– 2) = – 9.

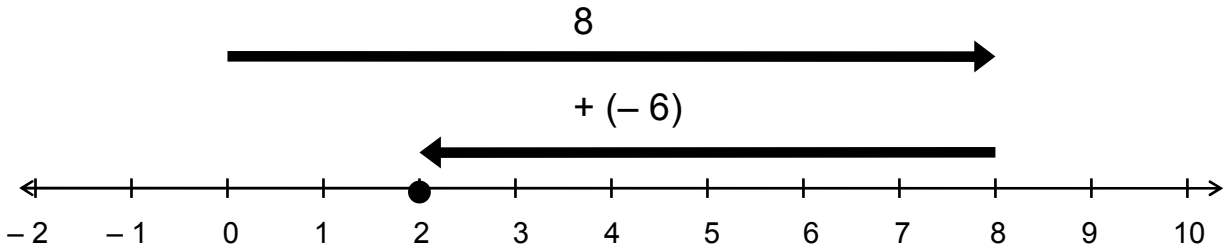
When adding two real numbers with the same signs, add their absolute values and apply the common sign in the sum. It is like a checking account; if you make two deposits, you add them together and your balance goes up. If, on the other hand, you write two checks, your balance will go down by the total of the two checks.

Add using a number line:

Ex. 3 $8 + (-6)$

Solution:

First draw an arrow moving eight ticks to the right of zero to 8 and then draw another arrow starting at eight and moving 6 ticks to the left:

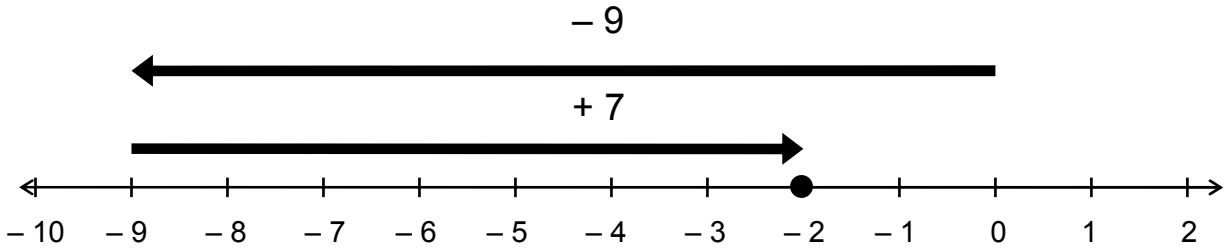


Notice that we landed on 2, so $8 + (-6) = 2$.

Ex. 4 $-9 + 7$

Solution:

First draw an arrow moving nine ticks to the left of zero to – 9 and then draw another arrow starting at – 9 and moving 7 ticks to the right:



Notice that we landed on – 2, so $-9 + 7 = -2$.

When adding numbers with different signs, we subtract their absolute values and apply the sign of the number with the larger absolute value to the difference. Again, it is like a checking account; if you make a deposit and then write a check, those two items work against each other on your account balance. Which ever is larger, the check or the deposit, will have a bigger impact your balance.

Simplify the following:

Ex. 5a $-7 + (-11)$

Ex. 5b $-8 + 17$

Ex. 5c $2.8 + (-3.5)$

Ex. 5d $-9.1 + (-8.7)$

Ex. 5e $-7\frac{5}{9} + 10\frac{5}{12}$

Ex. 5f $-9\frac{2}{3} + (-4\frac{3}{4})$

Solution:

a) Since the signs are the same, add their absolute values: 7 + 11 = 18. But, both numbers are negative, so the answer is – 18.

b) Since the signs are different, subtract their absolute values: 17 – 8 = 9. But, 17 has the larger absolute value, so the answer is 9.

c) Since the signs are different, subtract their absolute values: 3.5 – 2.8 = 0.7. But, – 3.5 has the larger absolute value, so the answer is – 0.7.

d) Since the signs are the same, add their absolute values: 9.1 + 8.7 = 17.8. But, both numbers are negative, so the answer is – 17.8.

e) Since the signs are different, subtract their absolute values:
$10\frac{5}{12} - 7\frac{5}{9} = \frac{125}{12} - \frac{68}{9} = \frac{125 \bullet 3}{12 \bullet 3} - \frac{68 \bullet 4}{9 \bullet 4} = \frac{375}{36} - \frac{272}{36} = \frac{103}{36}$
or $2\frac{31}{36}$. But, $10\frac{5}{12}$ has the larger absolute value, so the answer is $2\frac{31}{36}$.

f) Since the signs are the same, add their absolute values:
$9\frac{2}{3} + 4\frac{3}{4} = \frac{29}{3} + \frac{19}{4} = \frac{29 \bullet 4}{3 \bullet 4} + \frac{19 \bullet 3}{4 \bullet 3} = \frac{116}{12} + \frac{57}{12} = \frac{173}{12}$ or
$14\frac{5}{12}$. But, the numbers are negative, so the answer is $-14\frac{5}{12}$.

Concept #3 Translations

Translate the following and then simplify:

Ex. 6a The total of – 18, 7, – 3, – 5, and 11.

Ex. 6b Negative four-ninths more than negative two-thirds.

Ex. 6c The sum of – 15 and the absolute value of – 7.

Solution:

a) The total of – 18, 7, – 3, – 5, and 11:
[– 18 + 7 + (– 3) + (– 5) + 11] (different signs)
= – 11 + (– 3) + (– 5) + 11 (same signs)
= – 14 + (– 5) + 11 (same signs)
= – 19 + 11 (different signs)
= – 8

b) Negative four-ninths more than negative two thirds:

$-\frac{4}{9} + \left(-\frac{2}{3}\right)$ (common denominator is 9)

$= -\frac{4}{9} + \left(-\frac{6}{9}\right)$ (same signs)

$= -\frac{10}{9}$

c) The sum of – 15 and the absolute value of – 7:

$[-15 + |-7|]$ (absolute value)

$= [-15 + 7]$ (different signs)

$= -8$

Concept #4 Applications

Solve the following:

Ex. 7 The following table from American Airlines 2006 Annual Report shows the operating and net incomes for American Airlines (Source: http://www.shareholder.com/aa/downloads/ar2006.pdf).

	Year ended December 31,				
(in millions)	2006	2005	2004	2003	2002
Operating income	$1,060	– $89	– $134	– $843	– $3,331
Net income	$231	– $857	– $751	– $1,227	– $3,512

Find the total operating income for the years 2002 through 2006.

Solution:

To find the total, we simply add the numbers in the Operating income

$1060 + (– $89) + (– $134) + (– $843) + (– $3,331)

= $971 + (– $134) + (– $843) + (– $3,331)

= $837 + (– $843) + (– $3,331)

= – $6 + (– $3,331) = – $3,337 million

The total operating income was a loss of $3,337,000,000.

Ex. 8 During the week of November 12 through November 16 of 2007, the Dow Jones Industrial Average posted changes of + 67.31 points, – 120.63 points, – 74.46 points, + 331.98 points, and – 51.61 points. Find the net change for the week.

Solution:

67.31 + (– 120.63) + (– 74.46) + 331.98 + (– 51.61)

= – 53.32 + (– 74.46) + 331.98 + (– 51.61)

= – 127.78 + 331.98 + (– 51.61) = 204.20 + (– 51.61)

= 152.59 points. The Dow gained 152.59 points for the week.