Sect 1.4 - Subtraction of Real Numbers

Concept #1: Subtraction of Real Numbers

Let's consider the following examples:

Simplify the following:

Ex. 1 a) $7.2 - 3.9$ b) $11 - 3$ c) $\frac{13}{9} - \frac{8}{9}$

Solution:

a) $7.2 - 3.9 = 3.3$ b) $11 - 3 = 8$ c) $\frac{13}{9} - \frac{8}{9} = \frac{5}{9}$

Ex. 2 a) $7.2 + (-3.9)$ b) $11 + (-3)$ c) $\frac{13}{9} + (-\frac{8}{9})$

Solution:

a) $7.2 + (-3.9) = 3.3$ (Diff. signs - Difference - Sign of "larger" #)

b) $11 + (-3) = 8$ (Different signs - Difference - Sign of "larger" #)

c) $\frac{13}{9} + (-\frac{8}{9}) = \frac{5}{9}$ (Diff. signs - Difference - Sign of "larger" #)

Now, compare examples #1 and #2. Note: $7.2 - 3.9 = 3.3 = 7.2 + (-3.9)$, $11 - 3 = 8 = 11 + (-3)$ and $\frac{13}{9} - \frac{8}{9} = \frac{5}{9} = \frac{13}{9} + (-\frac{8}{9})$. This says that we can rewrite any subtraction problem as an addition problem so long as we add the opposite of the number to the right of the operation. Then, we just follow the rules for adding real numbers.

Subtracting Real Numbers:

When subtracting, change the operation of subtraction to the operation of addition and change the sign of the number to the right of the operation.

$a - b = a + (-b)$ and $a - (-b) = a + b$

Simplify the following:

Ex. 3a $12 - 7$ Ex. 3b $-12 - 7$

Ex. 3c $12 - (-7)$ Ex. 3d $-12 - (-7)$

Solution:

a) $12 - 7 = 12$ **+ (– 7)** $= 5$

b) $-12 - 7 = -12$ **+ (– 7)** $= -19$

Notice in part a & b, subtracting 7 is the same as adding – 7.

c) $12 - (-7) = 12$ **+ (+ 7)** $= 19$

d) $-12 - (-7) = -12$ **+ (+ 7)** $= -5$

Notice in part c & d, subtracting – 7 is the same as adding 7.

Simplify the following:

Ex. 4a $-1.98 - 2.4$ Ex. 4b $9.87 - (-3.2)$

Ex. 4c $-\frac{5}{7} - (-\frac{13}{9})$ Ex. 4d $45 - 97$

Solution:

a) $-1.98 - 2.4 = -1.98$ **+ (– 2.4)** $= -4.38$

b) $9.87 - (-3.2) = 9.87$ **+ (+ 3.2)** $= 13.07$

c) $-\frac{5}{7} - (-\frac{13}{9}) = -\frac{5}{7}$ **+ (+** $\frac{13}{9}$**)** $= -\frac{45}{63} + \frac{91}{63} = \frac{46}{63}$

d) $45 - 97 = 45$ **+ (– 97)** $= -52$

Concept #2 Translations

Translate the following and then simplify:

Ex. 5a – 52 less than 25.

Ex. 5b The difference of – 5.67 and – 8.91.

Ex. 5c The total of – 32 and 48 subtracted from 3.

Ex. 5d $-\frac{5}{6}$ decreased by $\frac{9}{8}$.

Solution:

a) – 52 less than 25: $25 - (-52)$

$25 - (-52)$

$= 25$ **+ (+ 52)** $= 77$

b) The difference of – 5.67 and – 8.91: $-5.67 - (-8.91)$

$-5.67 - (-8.91)$

$= -5.67$ **+ (+ 8.91)** $= 3.24$

c) The total of – 32 and 48: $(-32 + 48)$

The total of – 32 and 48 subtracted from 3: $3 - (-32 + 48)$

$3 - (-32 + 48)$ (Do not change addition to subtraction)

$= 3 - (16)$

$= 3$ **+ (– 16)** $= -13$

d) $-\frac{5}{6}$ decreased by $\frac{9}{8}$: $-\frac{5}{6} - \frac{9}{8}$

$-\frac{5}{6} - \frac{9}{8}$

$= -\frac{5}{6}$ **+** $\left(-\frac{9}{8}\right)$ (Find the LCD)

$= -\frac{20}{24} + \left(-\frac{27}{24}\right) = -\frac{47}{24}$

Concept #3 Applications Involving Subtraction

Write as an expression involving real numbers and then simplify:

Ex. 6 At the beginning of January, Jon had – $3421.67 balance on his credit card. During the month, he had three charges of $34.87, $657.84, and $231.62 and made a payment of $110. If the interest during the month was $80.69, what was he credit card balance at the end of the month?

Solution:

Beginning Balance – Three New Charges + Payment – Interest:

– $3421.67 – $34.87 – $657.84 – $231.62 + $110 – $80.69

= – 3421.67 + (– 34.87) + (– 657.84) + (– 231.62) + 110 + (– 80.69)

The first four numbers have the same signs so add:

= – 3421.67 + (– 34.87) + (– 657.84) + (– 231.62) + 110 + (– 80.69)

= – 4346 + 110 + (– 80.69)

= – 4236 + (– 80.69)

= – 4316.69

He had a balance of – $4316.69 at the end of the month.

Ex. 7 On a winter day in Chicago, the temperature at 8 am was – 6˚ F. At 6 pm on the same day, the temperature was – 11˚ F. Find the change in temperature during the day.

Solution:

Take the temperature at 6 pm minus the temperature at 8 am:

– 11 – (– 6)

= – 11 + 6 = – 5˚

The change in temperature was – 5˚ or the temperature fell 5˚ during the day.

Concept #4 Applying the Order of Operations

Simplify the following:

Ex. 8a $\{-7-[25-(-3)]\}+(-12)$

Ex. 8b $\frac{3}{4}-\left(\frac{5}{7}-\frac{13}{14}\right)+\left(-2+\frac{1}{2}\right)$

Ex. 8c $0.2\sqrt{0.09-(-0.16)}-0.5$

Ex. 8d $-11+\left|6-15\right|+[8+(-5)]^2$

Solution:

a) $\{-7 - \mathbf{[25 - (-3)]}\} + (-12)$ (write as addition in [])

$= \{-7 - \mathbf{[25 + (+3)]}\} + (-12)$ (#1-par., #1-par., #4 - add.)

$= \mathbf{\{-7 - [28]\}} + (-12)$ (write as addition in {})

$= \mathbf{\{-7 + [-28]\}} + (-12)$ (#1-parenthesis, #4 - add.)

$= \{-35\} + (-12)$ (#4 - addition)

$= -47$

b) $\frac{3}{4} - \left(\frac{5}{7} - \frac{13}{14}\right) + \left(-2 + \frac{1}{2}\right)$ (write – 2 over 1)

$= \frac{3}{4} - \left(\frac{5}{7} - \frac{13}{14}\right) + \left(-\frac{2}{1} + \frac{1}{2}\right)$ (write as addition in ())

$= \frac{3}{4} - \left(\frac{5}{7} + \left(-\frac{13}{14}\right)\right) + \left(-\frac{2}{1} + \frac{1}{2}\right)$ (LCD in ())

$= \frac{3}{4} - \left(\frac{10}{14} + \left(-\frac{13}{14}\right)\right) + \left(-\frac{4}{2} + \frac{1}{2}\right)$ (#1-parent., #4 - add.)

$= \frac{3}{4} - \left(-\frac{3}{14}\right) + \left(-\frac{3}{2}\right)$ (write as addition)

$= \frac{3}{4} + \left(+\frac{3}{14}\right) + \left(-\frac{3}{2}\right)$ (LCD)

$= \frac{21}{28} + \left(+\frac{6}{28}\right) + \left(-\frac{42}{28}\right)$ (#4-addition)

$= \frac{27}{28} + \left(-\frac{42}{28}\right)$ (#4-addition)

$= -\frac{15}{28}$

c) $0.2\sqrt{0.09-(-0.16)} - 0.5$ (write as addition under $\sqrt{\ }$)

$= 0.2\sqrt{0.09+(+0.16)} - 0.5$ (#1-radical, #4-addition)

$= 0.2\sqrt{0.25} - 0.5$ (#2-exponents)

$= 0.2(0.5) - 0.5$ (#3-multiplication)

$= 0.1 - 0.5$ (write as addition)

$= 0.1 + (-0.5)$ (#4-addition)

$= -0.4$

d) $-11 + |6 - 15| + [8 + (-5)]^2$ (write as addition in $|\ |$)

$= -11 + |6 + (-15)| + [8 + (-5)]^2$ (#1-parent., #4 - add.)

$= -11 + |-9| + [3]^2$ (absolute value)

$= -11 + 9 + [3]^2$ (#2-exponents)

$= -11 + 9 + 9$ (#4-addition)

$= -2 + 9$ (#4-addition)

$= 7$