Sect 1.6 - Division of Whole Numbers

Objective a: Understanding Division.

Let's start with an example:

Solve the following:

Ex. 1 Leroy wants to make dress shirts. Each shirt requires 6 feet of material. If he has 24 feet of material, how many shirts can he make?

Solution:

We can do this by repeatedly subtracting 6 feet for every shirt created:

24	feet of material	
− 6	← first shirt	So, since 6 gets subtracted four times, then Leroy can make four shirts. This is like asking how many times does six go into 24 or 24 ÷ 6. This means that division can be thought of as a shortcut for repeated subtraction.
18		
− 6	← second shirt	
12		
− 6	← third shirt	
6		
− 6	← fourth shirt	
0		

There are several ways to write this problem:

$24 \div 6 = 4$ "Twenty-four divided by 6 equals 4."

$\frac{24}{6} = 4$ "The quotient of 24 and 6 is 4."

$6\overline{)24}$ $\quad 4$ "6 divided into 24 is 4."

The number 6 that we are dividing by is called the **divisor**. The number 24 that we are divided into is called the **dividend** and the answer 4 is called the **quotient**.

Objective b: Properties involving division.

Is Division commutative and/or associative? To determine this, let's try some examples.

Find how much donut each person gets:

Ex. 2a 10 donuts ÷ 5 people

Solution:

10 ÷ 5 = 2, so each person gets 2 donuts.

Ex. 2b 5 donuts ÷ 10 people

Solution:

In this problem, you have to cut each donut in half to divide the 5 donuts among 10 people. So, each person gets a half of a donut.

Clearly, the results in example 2a and 2b are not the same so division is not commutative. Let's see if division is associative.

Find the following:

Ex. 3a (12 ÷ 6) ÷ 2

Solution:

(12 ÷ 6) ÷ 2

= (2) ÷ 2 = 1

Ex. 3b 12 ÷ (6 ÷ 2)

Solution:

12 ÷ (6 ÷ 2)

= 12 ÷ (3) = 4

Since the results are not the same, division is not associative.

Objective c: Dividing Whole Numbers

Find the following:

Ex. 4a 14 ÷ 7

Solution

7 goes into 14 twice so,

14 ÷ 7 = 2.

Ex. 4b 31 ÷ 1

Solution:

1 goes into 31 thirty-one times so, 31 ÷ 1 = 31.

Ex. 4c 19 ÷ 19

Solution:

19 goes into 19 one time so,

19 ÷ 19 = 1.

Ex. 5a 24 ÷ 3

Solution

3 goes into 24 eight times so,

24 ÷ 3 = 8.

Ex. 5b 0 ÷ 7

Solution:

7 goes into 0 zero times so,

0 ÷ 7 = 0.

Ex. 5c 6 ÷ 0

Solution:

0 does not go into 6 so,

6 ÷ 0 is **Undefined**.

Let's examine example five more carefully to be sure we understand the difference between 5b and 5c. Any division question can be rephrased as a multiplication question. In example 5a, asking 24 ÷ ***3*** = ? is the same as asking "what do you have to multiply 3 by to get 24:" ***3*** • ? = 24. The answer is eight. In example 5b, asking 0 ÷ ***7*** = ? is the same as asking "what do you have to multiply 7 by to get 0:" ***7*** • ? = 0. The answer is zero. In example 5c, asking 6 ÷ ***0*** = ? is the same as asking "what do you have to multiply 0 by to get 6:" ***0*** • ? = 6. There is no number that works since 0 times any number is 0. The problem is **undefined**.

Objective d: Long division

We begin by dividing the divisor into the leftmost digit(s) of the dividend being careful enough to ensure we are using enough digits in the dividend to do the division. We write down the number of times the divisor goes into the left part of the dividend above the last digit used in the dividend. Multiply this number with the divisor and subtract it from the digits used in the dividend. Bring down the next digit of the dividend and repeat the process. Continue until all the digits of the dividend are used. If you have anything left over, that will be the remainder and we write "R" and that number to the right of the quotient. Here are some key words that impliy division:

7 divided into 21	$21 \div 7 = \frac{21}{7}$ or $7\overline{)21}$
7 divided by 21	$7 \div 21 = \frac{7}{21}$ or $21\overline{)7}$
the quotient of 15 and 3	$(15 \div 3) = \frac{15}{3}$ or $3\overline{)15}$
the ratio of 4 to 11	$4 \div 11 = \frac{4}{11}$
8 split into 4 equal parts	$8 \div 4 = \frac{8}{4}$ or $4\overline{)8}$

<u>Find the following:</u>

Ex. 6 73168 ÷ 8

<u>Solution:</u>

```
    9
  ------
8 | 73168
  - 72
  ----
    11
```

8 does not go into 7, but it does go into 73 nine times. We write 9 above the 3 and multiply 8 and 9 to get 72. Subtract 72 from 73 and bring down the 1. Now, repeat the process.

```
   91
8 | 73168
– 72
   11
 –  8
    36
```

8 goes into eleven one time. We write 1 above the 1 and multiply 8 and 1 to get 8. Subtract 8 from 11 and bring down the 6. Now, repeat the process.

```
   914
8 | 73168
– 72
   11
 –  8
    36
  – 32
    48
```

8 goes into thirty-six four times. We write 4 above the 6 and multiply 8 and 4 to get 32. Subtract 32 from 36 and bring down the 8. Now, repeat the process.

```
   9146
8 | 73168
– 72
   11
 –  8
    36
  – 32
    48
  – 48
     0
```

8 goes into forty-eight six times. We write 6 above the 8 and multiply 8 and 6 to get 48. Subtract 48 from 48 to get 0. There is no remainder, so the problem is done.

The answer is 9,146.

Let's take a closer look at this problem to see what is actually happening. When we say that 8 goes into 73 nine times, we really mean that 8 goes into 73,000 nine thousand times. Likewise, when we say 8 goes into 11 one time, we really mean that 8 goes into 1100 one hundred times. The same goes for the subtraction:

```
    9146
  ------
8 | 73168
  - 72        ← Here, we are really subtracting 72,000 from 73,000.
    11            That is why the 9 is in the thousands place.
  -  8        ← Here, we are subtracting 800 from 1100.
    36            That is why the 1 is in the hundreds place.
  - 32        ← Here, we are subtracting 320 from 360.
    48            That is why the 4 is in the tens place.
  - 48        ← Here, we are subtracting 48 from 48.
     0            That is why the 6 is in the one place.
```

Check: 9146•8 = 73,168.

Ex. 7 145268 divided by 34

Solution:

```
     4272 R. 20
  ----------
34 | 145268      Take 145 divided by 34 to get 4.2...
  - 136         4•34 = 136, 145 – 136 = 9. Bring down the 2.
    92          Take 92 divided by 34 to get 2.7...
  - 68          2•34 = 68, 92 – 68 = 24. Bring down the 6.
    246         Take 246 divided by 34 to get 7.2...
  - 238         7•34 = 238, 246 – 238 = 8, Bring down the 8.
     88         Take 88 divided by 34 to get 2.5...
   - 68         2•34 = 68, 88 – 68 = 20. Remainder is 20.
     20
```

The answer is 4272 R. 20.

To check our answer, take the whole number part times the divisor plus the remainder. This should be equal to the dividend:

4272•34 + 20 = 145248 + 20 = 145268

Ex. 8 The quotient of 785264 and 193

Solution:

```
     4068 R. 140
  ----------
193 | 785264      Take 785 divided by 193 to get 4.0...
  - 772         4•193 = 772, 785 – 772 = 13. Bring down the 2.
    1326        Take 132 divided by 193 to get 0. Bring down the 6.
  - 1158        Take 1326 divided by 193 to get 6.8...
    1684        6•193 = 1158, 1326 – 1158 = 168. Bring down the 4.
  - 1544        Take 1684 divided by 193 to get 8.7...
     140        8•193 = 1544, 1684 – 1544 = 140. The remainder
                is 140. The answer is 4068 R. 140.
```

To check our answer, take the whole number part times the divisor plus the remainder. This should be equal to the dividend:

4068•193 + 140 = 785124 + 140 = 785264.

Ex. 9 In 2007, the average household credit card debt in the U.S. was $9840 (source: Mark Brinker's article "Credit Card Debt Statistics at www.hoffmanbrinker.com). If there were about three people per household, estimate what was the average credit card debt per person?

Solution:

Take $9840 divided by 3:

```
  3280
3 | 9840
– 9
  8
– 6
  24
– 24
   0
```

3 divided by 3 = 1. 3 – 3•1 = 0, bring down the 8.
8 divided by 3 = 2. 8 – 3•2 = 2, bring down the 4.
24 divided by 3 = 8. 24 – 3•8 = 0.
Since the four was in the tens place, we will need to write a zero in the ones place. So, the average credit card debt was $3280 per person in 2007.

Ex. 10 In February 2009, the Federal government passed a $787 billion stimulus plan. According to the Census Bureau, there were 305,685,508 people in the United States as of February 2009 (www.census.gov),. Estimate how much will the stimulus plan cost per person by rounding each number to the nearest million?

Solution:

Since $787 billion = $787,000,**000,000** = $787,000 million and 305,685,508 ≈ 306,000,000 = 306 million, we can divide 787,000 by 306 to get the answer:

```
   2571
306 | 787000
– 612
  1750
– 1530
   2200
– 2142
    580
– 306
    274
```

787 divided by 306 = 2. 787 – 306•2 = 175. Bring down the zero.
1750 divided by 306 = 5. 1750 – 5•306 = 220. Bring down the zero.
2200 divided by 306 = 7. 2200 – 7•306 = 58. Bring down the zero.
580 divided by 306 = 1. 580 – 1•306 = 274. Our remainder is 274.

So, it will cost each person over $2571.