Sect 1.6 - Properties of Real Numbers and Simplifying Expressions

Concept #1 Commutative Properties of Real Numbers

Simplify:

Ex. 1a $-9.34 + 2.5$ Ex. 1b $2.5 + (-9.34)$
Ex. 1c $-6.3(4.2)$ Ex. 1d $4.2(-6.3)$

Solution:

a) $-9.34 + 2.5 = -6.84$ b) $2.5 + (-9.34) = -6.84$

Notice that if we add two numbers in either order, we get the same results. This property is called the **Commutative Property of Addition.**

c) $-6.3(4.2) = -26.46$ d) $4.2(-6.3) = -26.46$

Notice that if we multiply two numbers in either order, we get the same results. This property is known as **Commutative Property of Multiplication.**

Commutative Properties of Real Numbers

If a and b are real numbers, then

1. $a + b = b + a$ **Commutative Property of Addition**

2. $a \bullet b = b \bullet a$ **Commutative Property of Multiplication**

Note that subtraction and division are not commutative ($7 - 10 = -3$, but $10 - 7 = +3$ and $5 \div 10 = 0.5$, but $10 \div 5 = 2$), but if we first rewrite a subtraction problem as an addition problem and a division problem as a multiplication problem, then we can use the appropriate property.

Use the appropriate commutative property to rewrite each expression:

Ex. 2a $7x^3 + 11x^4$ Ex. 2b $5.2y - 4.3x$
Ex. 2c $x^2(-19)$ Ex. 2d $x \div \frac{2}{3}$

Solution:

a) We need to use the commutative property of addition:

$7x^3 + 11x^4$
$= 11x^4 + 7x^3$

b) First, rewrite as an addition problem and then use the commutative property of addition:

$5.2y - 4.3x$
$= 5.2y + (-4.3x)$
$= -4.3x + 5.2y$

c) We need to use the commutative property of multiplication:

$x^2(-19)$

$= -19x^2$

d) First, rewrite as a multiplication problem and then use the commutative property of multiplication:

$x \div \frac{2}{3}$

$= x \bullet \frac{3}{2}$

$= \frac{3}{2}x$

Concept #2 Associative Properties of Real Numbers

<u>Simplify:</u>

Ex. 3a $-18 + (3 + 9)$ Ex. 3b $(-18 + 3) + 9$

Ex. 3c $-5(4 \bullet 8)$ Ex. 3d $(-5 \bullet 4) \bullet 8$

<u>Solution:</u>

a) $-18 + (3 + 9) = -18 + (12) = -6$

b) $(-18 + 3) + 9 = -15 + 9 = -6$

Notice it does not matter how we group the numbers, we get the same result. This property is called the **Associative Property of Addition**:

c) $-5(4 \bullet 8) = -5(32) = -160$

d) $(-5 \bullet 4) \bullet 8 = (-20) \bullet 8 = -160$

Notice it does not matter how we group the numbers, we get the same result. This property is called the **Associative Property of Multiplication**:

Associative Properties of Real Numbers

If a, b and c are real numbers, then

1. $(a + b) + c = a + (b + c)$ **Associative Property of Addition**

2. $(a \bullet b)c = a(b \bullet c)$ **Associative Property of Multiplication**

Note that subtraction and division are not associative ($(9 - 6) - 3 = 3 - 3 = 0$, but $9 - (6 - 3) = 9 - 3 = 6$ and $(12 \div 4) \div 2 = 3 \div 2 = 1.5$, but $12 \div (4 \div 2) = 12 \div 2 = 6$), but if we first rewrite a subtraction problem as an addition problem and a division problem as a multiplication problem, then we can use the appropriate property.

<u>Use the appropriate associative property to rewrite each expression:</u>

Ex. 4a $5 + (-5 + x)$ Ex. 4b $(9r)r$

Ex. 4c $-\frac{13}{9}\left(-\frac{9}{13}g\right)$ Ex. 4d $(x + 11) + 5$

Solution:

a) Use the associative property of addition:
$5 + (-5 + x) = (5 + (-5)) + x = 0 + x = x$

b) Use the associative property of multiplication:
$(9r)r = 9(r \bullet r) = 9r^2$

c) Use the associative property of multiplication:
$-\frac{13}{9}\left(-\frac{9}{13}g\right) = \left[\left(-\frac{13}{9}\right)\left(-\frac{9}{13}\right)\right]g = 1 \bullet g = g$

d) Use the associative property of addition:
$(x + 11) + 5 = x + (11 + 5) = x + 16$

Concept #3 Identity and Inverse Properties of Real Numbers

Simplify:

Ex. 5a $-\frac{13}{9} + 0$

Ex. 5b $-\frac{7}{8} \bullet 1$

Solution:

a) $-\frac{13}{9} + 0 = -\frac{13}{9}$

b) $-\frac{7}{8} \bullet 1 = -\frac{7}{8}$

If you add zero to any number, you get the same number. Also, a number times one will yield the same number. These properties are called the **Identity Property of Addition** and **Identity Property of Multiplication** respectively:

Identity Properties of Real Numbers

If a is a real number, then

1. $a + 0 = a$ & $0 + a = a$ **Identity Property of Addition**

2. $a \bullet 1 = a$ & $1 \bullet a = a$ **Identity Property of Multiplication**

Simplify:

Ex. 6a $-\frac{5}{7} + \frac{5}{7}$

Ex. 6b $\frac{5}{7} \bullet \frac{7}{5}$

Solution:

a) $-\frac{5}{7} + \frac{5}{7} = 0$

b) $\frac{5}{7} \bullet \frac{7}{5} = 1$

A number plus its opposite or additive inverse is zero and a number times its reciprocal or multiplicative inverse is one. These properties are known as the Inverse Property of Addition and the Inverse Property of Multiplication respectively:

Inverse Properties of Real Numbers
If a and b are real numbers and b ≠ 0, then

1. $a + (-a) = 0$ & $-a + a = 0$ **Inverse Property of Addition**

2. $b \bullet \frac{1}{b} = 1$ & $\frac{1}{b} \bullet b = 1$ **Inverse Property of Multiplication**

Concept #4 The Distributive Property of Multiplication over Addition

Let's look at an example that illustrates the distributive property.

Find the following:

Ex. 7 7(3 + 5)

Solution:
Normally, we would add the three and the five together first and then multiply:
7(**3 + 5**) = 7(8) = 56
But, the distributive property says that we can multiply the 3 and 5 both by seven first and then add the results:
7(3 + 5) = **7•**3 + **7•**5 = 21 + 35 = 56.

Notice that we get the same result either way. This is an extremely important property in mathematics.

Distributive Property
If a, b, and c are real numbers, then
a(b + c) = **a•**b + **a•**c and (b + c)**a** = **a•**b + **a•**c
Using this property is sometimes called "clearing parentheses."

Let's see how this applied in algebra.

Simplify:

Ex. 8 7(3x + 5)

Solution:
Here, we apply the distributive property:
7(3x + 5) = **7•**3x + **7•**5 = 21x + 35.

Ex. 9 – 2.2(3q – 4)

Solution:
– 2.2(3q – 4) = **– 2.2**•3q – (**– 2.2**)•4 = – 6.6q – (– 8.8) = – 6.6q + 8.8.

Ex. 10 (2x – 4y + 5)(5)

Solution:
(2x – 4y + 5)**(5)** = 2x**(5)** – 4y**(5)** + 5**(5)** = 10x – 20y + 25

Ex. 11 $-4(-3x + 6 - 2y)$

Solution:

$\mathbf{-4}(-3x + 6 - 2y) = \mathbf{-4}(-3x) + (\mathbf{-4})\bullet 6 - (\mathbf{-4})\bullet 2y$
$= 12x + (-24) + 8y$, but, in algebra, we want to use the least number of symbols possible to express our answer, so we will rewrite the answer as $12x - 24 + 8y$.

Ex. 12 $r(3r - 4)$

Solution:

$\mathbf{r}(3r - 4) = \mathbf{r}\bullet 3r - r\bullet 4 = 3r^2 - 4r$

Ex. 13 $-\frac{4}{7}(28x^2 - 35x)$

Solution:

$-\frac{4}{7}(28x^2 - 35x) = \left(-\frac{4}{7}\right)(28x^2) - \left(-\frac{4}{7}\right)(35x)$

$= -\frac{4}{7}\left(\frac{28x^2}{1}\right) - \left(-\frac{4}{7}\right)\left(\frac{35x}{1}\right) = -4(4x^2) - (-4)(5x) = -16x^2 + 20x$

Ex. 14 $-(-11 + 3p - p^2)$

Solution:

$-(-11 + 3p - p^2) = \mathbf{-1}(-11 + 3p - p^2)$
$= \mathbf{(-1)}(-11) + \mathbf{(-1)}(3p) - \mathbf{(-1)}(p^2) = 11 + (-3p) - (-p^2)$
$= 11 - 3p + p^2$

Concept #5 Simplifying Algebraic Expressions

We say a **term** is a number, a letter, or the product and/or quotient of numbers and letters. Notice that there is no mention of addition or subtraction in this definition. When we look at an expression, we can look for addition and subtraction to determine where one term ends and another one begins.

Determine the number of terms in the following expressions:

Ex. 15 $-0.3a^2 + 4.2b^2$

Solution:

$-0.3a^2$ is the first term and $4.2b^2$ is the second term, so $-0.3a^2 + 4.2b^2$ has two terms.

Ex. 16 $3x^2 - 6xy + 5$

Solution:

$3x^2$ is the first term, $-6xy$ is the second, and 5 is the third, so $3x^2 - 6xy + 5$ has three terms.

Terms that have variables (letters) as factors are called **variable terms** while terms that do not have variables (letters) as factors are called **constant terms**. In example #16, $3x^2$ and $-6xy$ are the variable terms while 5 is the constant term.

Ex. 17 $-\frac{5}{6}r^2s + \frac{2}{3}rs^2 - \frac{11}{9}$

Solution:

The terms are $-\frac{5}{6}r^2s$, $\frac{2}{3}rs^2$, and $-\frac{11}{9}$, so the expression has three terms. The variable terms are $-\frac{5}{6}r^2s$ and $\frac{2}{3}rs^2$ and the constant term is $-\frac{11}{9}$.

Ex. 18 $-\frac{7a^4b^5c}{8x^2y}$

Solution:

Since $-\frac{7a^4b^5c}{8x^2y}$ is the only term, then $-\frac{7a^4b^5c}{8x^2y}$ has one term which is a variable term. It has no constant term.

The **numerical coefficient** of a term is the numerical (constant) factor of a term. In – 6xy, the numerical coefficient is – 6. Let's try some examples:

Determine the numerical coefficient of the indicated term:

Ex. 19a $-0.3a^2 + 4.2b^2$; 1st term

Ex. 19b $-0.3a^2 + 4.2b^2$; 2nd term

Ex. 19c $14x - 17y + 6$; 2nd term

Ex. 19d $9x^2 - 4xy - y^3$; 3rd term

Ex. 19e $\frac{19}{6}y^2 + y - 7$; 2nd term

Ex. 19f $-\frac{7a^4b^5c}{8x^2y}$; 1st term

Ex. 19g $9x^2 - 7xy + 8$; 3rd term

Solution:

a) The numerical coefficient of $-0.3a^2$ is – 0.3.
b) The numerical coefficient of $4.2b^2$ is 4.2.
c) The numerical coefficient of – 17y is – 17.
d) The numerical coefficient of $-y^3$ is – 1.
e) The numerical coefficient of y is 1.
f) The numerical coefficient of $-\frac{7a^4b^5c}{8x^2y}$ is $-\frac{7}{8}$.
g) The numerical coefficient of 8 is 8.

In terms of simplifying variable expressions, we first need to examine what happens in the real world when you combine items:

Simplify the following:

Ex. 20a 3 apples + 4 apples

Ex. 20b 3 apples + 4 oranges

Solution:

a) 3 apples + 4 apples = 7 apples.

b) 3 apples + 4 oranges = 3 apples + 4 oranges.

In example 20a, we are able to add the items together since the units (apples) are the same whereas in example 20b, we cannot put the items together since the units (apples and oranges) are not the same. It would not make any sense to say 7 appleoranges for the answer to 20b since there is no such thing as an appleorange (or at least, scientists have not invented one yet!). This same idea applies to algebra:

Simplify the following:

Ex. 21a $3x^2 + 4x^2$ Ex. 21b $3x^2 + 4y$

Solution:

a) Instead of having apples, we have x^2's so $3x^2 + 4x^2 = 7x^2$.

b) This is exactly like adding apples and oranges so, $3x^2 + 4y = 3x^2 + 4y$. It cannot be simplified any further.

In example 21a, we are able to add the items together since the "units" (x^2) are the same whereas in example 21b, we cannot since the "units" (x^2 and y) are different. When terms have exactly the same "units", they are called **like terms**. Here is a more precise definition:

Terms with exactly the same variables with exactly the same corresponding exponents are called **like terms**. Only like terms can be combined by combining their numerical coefficients.

Simplify the following:

Ex. 22 $-3x + 5 + 7x - 3$

Solution:

We have two sets of like terms; the first pair is – 3x and 7x and the second pair is 5 and – 3. We will combine each pair by adding their numerical coefficients:

$\underline{-3x}\ \mathbf{+ 5}\ \underline{+7x}\ \mathbf{- 3} = \underline{-3x}\ \underline{+7x}\ \mathbf{+ 5 - 3} = 4x + 2$

Ex. 23 $0.8x^2 - 6xy + 1.2y^2 - 1.9x^2 - 7xy + 3.4y^2$

Solution:

First identify are sets of like terms and then combine:

$\mathbf{0.8x^2} - 6xy\ \underline{+1.2y^2}\ \mathbf{- 1.9x^2} - 7xy\ \underline{+3.4y^2}$

$= \mathbf{0.8x^2 - 1.9x^2} - 6xy - 7xy\ \underline{+1.2y^2 + 3.4y^2} = \mathbf{-1.1x^2} - 13xy\ \underline{+4.6y^2}$

$= -1.1x^2 - 13xy + 4.6y^2$.

Ex. 24 $-\frac{4}{3}x + \frac{2}{3}xy^2 - \frac{13}{8}x^2 - \frac{5}{6}x + \frac{9}{7}x^2y$

Solution:

Since $-\frac{4}{3}x$ and $-\frac{5}{6}x$ are the only like terms, they are the only ones that can be combined. Note that $\frac{2}{3}xy^2$ and $\frac{9}{7}x^2y$ are not like terms since the corresponding exponents are not the same (the power of x in $\frac{2}{3}xy^2$ is one whereas the power of x in $\frac{9}{7}x^2y$ is two.

Since $-\frac{4}{3} - \frac{5}{6} = -\frac{8}{6} + (-\frac{5}{6}) = -\frac{13}{6}$, then

$\underline{-\frac{4}{3}\mathbf{x}} + \frac{2}{3}xy^2 - \frac{13}{8}x^2 \underline{-\frac{5}{6}\mathbf{x}} + \frac{9}{7}x^2y = \mathbf{-}\frac{13}{6}\mathbf{x} + \frac{2}{3}xy^2 - \frac{13}{8}x^2 + \frac{9}{7}x^2y.$

Ex. 25 $5x^2 - 6xy + y^2 + 5y$

Solution:

There are no like terms. This expression cannot be simplified any further. Our answer is $5x^2 - 6xy + y^2 + 5y$.

Concept #6 Clearing Parentheses and Combining Like Terms

Ex. 26 4.1(3x – 2) – 5.2(2x + 8)

Solution:

Distribute first and then combine like terms (multiplication comes before adding and subtracting):

4.1(3x – 2) **– 5.2**(2x + 8) = **4.1**(3x) – **4.1**(2) **– 5.2**(2x) + **– 5.2**(8)
= *12.3x* <u>– 8.2</u> – *10.4x* <u>– 41.6</u> = *12.3x* – *10.4x* <u>– 8.2 – 41.6</u>
= 1.9x – 49.8

Ex. 27 $y(3y - 8) - 2(y^2 + 6)$

Solution:

Distribute first and then combine like terms:

$y(3y - 8) - 2(y^2 + 6) = \mathbf{y}(3y - 8) \mathbf{- 2}(y^2 + 6)$
$= \mathbf{y}(3y) - \mathbf{y}(8) - \mathbf{2}(y^2) + (\mathbf{-2})(6) = \underline{3y^2} - 8y - \underline{2y^2} - 12$
$= 1y^2 - 8y - 12 = y^2 - 8y - 12$

Ex. 28 – (5t – 6) – (3.2t + 7) – 7t

Solution:

Distribute first and then combine like terms:

– (5t – 6) – (3.2t + 7) – 7t
= **– 1**(5t – 6) **– 1**(3.2t + 7) – 7t
= **– 1**(5t) – **(– 1)(**6) **– 1**(3.2t) + **(– 1)**(7) – 7t
= – 5t + 6 – 3.2t – 7 – 7t = – 15.2t – 1

Ex. 29 $\frac{2}{3}(5x - \frac{9}{2}) - \frac{3}{13}(39x + \frac{1}{7})$

<u>Solution:</u>

Distribute first and then combine like terms:

$$\frac{2}{3}(5x - \frac{9}{2}) - \frac{3}{13}(39x + \frac{1}{7}) = \frac{2}{3}\left(\frac{5x}{1}\right) - \frac{2}{3}\left(\frac{9}{2}\right) - \frac{3}{13}\left(\frac{39x}{1}\right) - \frac{3}{13}\left(\frac{1}{7}\right)$$

$$= \frac{10x}{3} - \frac{3}{1} - \frac{9x}{1} - \frac{3}{91} = \frac{10x}{3} - \frac{9x \bullet 3}{1 \bullet 3} - \frac{3 \bullet 91}{1 \bullet 91} - \frac{3}{91}$$

$$= \frac{10x}{3} - \frac{27x}{3} - \frac{273}{91} - \frac{3}{91} = -\frac{17x}{3} - \frac{276}{91} = -\frac{17}{3}x - \frac{276}{91}$$