Sect 1.7 - Exponents, Square Roots and the Order of Operations

Objective a: Understanding and evaluating exponents.

We have seen that multiplication is a shortcut for repeated addition and division is a shortcut for repeated subtraction. Exponential notation is a shortcut for repeated multiplication. Consider the follow:

\[ 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \]

Here, we are multiplying six factors of five. We will call the 5 our base and the 6 the exponent or power. We will rewrite this as 5 to the 6th power:

\[ 5^6 \]

The number that is being multiplied is the base and the number of factors of that number is the power. Let's try some examples:

Write the following in exponential form:

Ex. 1a 8 \cdot 8 \cdot 8 \cdot 8
Solution: Since there are four factors of 8, we write 8^4.
Ex. 1b 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7
Solution: Since there are five factors of 7, we write 7^5.

Simplify the following:

Ex. 2a 3^4
Solution: Write 3^4 in expanded form and multiply:
3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81
Ex. 2b 7^3
Solution: Write 7^3 in expanded form and multiply:
7^3 = 7 \cdot 7 \cdot 7 = 49 \cdot 7 = 343

Ex. 3a 1^5
Solution: Write 1^5 in expanded form and multiply:
1^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \cdot 1 \cdot 1 = 1
Ex. 3b 2^3 \cdot 6^2
Solution: Write 2^3 \cdot 6^2 in expanded form and multiply:
2^3 \cdot 6^2 = (2 \cdot 2 \cdot 2) \cdot (6 \cdot 6) = 8 \cdot 36 = 288

Ex. 4a 10^1
Solution: 10^1 = 10
Ex. 4b 10^2
Solution: 10^2 = 10 \cdot 10 = 100
Ex. 4c  $10^3$  
Solution:  
$$10^3 = 10 \cdot 10 \cdot 10 = 100 \cdot 10 = 1000$$  
Notice with powers of ten, we get a one followed by the numbers of zeros equal to the exponent.

Ex. 4d  $10^6$  
Ex. 4e  $10^{100}$  
Solution: Solution:  
This will be equal to This will be equal to  
1 followed by 6 zeros: 1 followed by 100 zeros:  
1,000,000 10...............0  
|-100 zeros-|  
The number $10^{100}$ is called a "googol." The mathematician that was first playing around with this number asked his nine-year old nephew to give it a name. There are even larger numbers like $10^{googol}$. This is a 1 followed by a googol number of zeros. If you were able to write 1 followed a googol number of zeros on a piece of paper, that paper could not be stuffed in the known universe!

Objective b: Understanding and applying square roots.

The square of a whole number is called a perfect square. So, 1, 4, 9, 16, 25 are perfect squares since $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, and $25 = 5^2$. We use the idea of perfect squares to simplify square roots. The square root of a number $a$ asks what number times itself is equal to $a$. For example, the square root of 25 is 5 since 5 times 5 is 25.

The square root of a number $a$, denoted $\sqrt{a}$, is a number whose square is a. So, $\sqrt{25} = 5$.

Simplify the following:  
Ex. 5a $\sqrt{49}$  
Ex. 5c $\sqrt{0}$  
Ex. 5b $\sqrt{144}$  
Ex. 5d $\sqrt{625}$  
Solution:  
a) $\sqrt{49} = 7$ since $7^2 = 49$.  
b) $\sqrt{144} = 12$ since $12^2 = 144$.  

c) \( \sqrt{0} = 0 \) since \( 0^2 = 0 \).

d) \( \sqrt{625} = 25 \) since \( 25^2 = 625 \).

Objective c: Understanding and applying the Order of Operations

If you ever have done some cooking, you know how important it is to follow the directions to a recipe. An angel food cake will not come out right if you just mix all the ingredients and bake it in a pan. Without separating the egg whites from the egg yolks and whipping the eggs whites and so forth, you will end up with a mess. The same is true in mathematics; if you just mix the operations up without following the order of operations, you will have a mess. Unlike directions for making a cake that differ from recipe to recipe, the order of operations always stays the same. The order of operations are:

**Order of Operations**

1) Parentheses - Do operations inside of Parentheses ( ), [ ], { }, |
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

A common phrase people like to use is:

Please (Be careful with the My Dear and the
Excuse Aunt Sally part. Multiplication does not
My Dear precede Division and Division does not
Aunt Sally precede multiplication; they are done
as they appear from left to right. The
same is true for addition and subtraction.)

**Simplify the following:**

Ex. 6 \( 99 - 12 + 3 - 14 + 5 \)

Solution:
We need to add and subtract as they appear from left to right:
\( 99 - 12 + 3 - 14 + 5 \) (#4-subtraction)
= \( 87 + 3 - 14 + 5 \) (#4-addition)
= \( 90 - 14 + 5 \) (#4-subtraction)
= \( 76 + 5 \) (#4-addition)
= 81
Ex. 7 \[72 ÷ 9(4) ÷ \sqrt{4} (5)\]

Solution:
\[72 ÷ 9(4) ÷ \sqrt{4} (5) \quad (#2-exponents)\]
We need to multiply and divide as they appear from left to right:
= \[72 ÷ 9(4) ÷ 2(5) \quad (#3-division)\]
= \[8(4) ÷ 2(5) \quad (#3-multiply)\]
= \[32 ÷ 2(5) \quad (#3-division)\]
= \[16(5) \quad (#3-multiply)\]
= 80

Ex. 8 \[8 \cdot 3 + 4 \cdot 2\]

Solution:
Since there are no parentheses or exponents, we start with step #3, multiply or divide as they appear from left to right:
\[8 \cdot 3 + 4 \cdot 2 \quad (#3-multiplication)\]
\[= 24 + 8 \quad (#4-addition)\]
\[= 32\]

Ex. 9 \[18 ÷ 3^2 \cdot \sqrt{16} + 4 \cdot 5^2\]

Solution:
Since there are no parentheses, we start with step #2, exponents:
\[18 ÷ 3^2 \cdot \sqrt{16} + 4 \cdot 5^2 \quad (#2-exponents)\]
\[= 18 ÷ 9 \cdot 4 + 4 \cdot 25 \quad (#3-division)\]
\[= 2 \cdot 4 + 4 \cdot 25 \quad (#3-multiplication)\]
\[= 8 + 4 \cdot 25 \quad (#3-multiplication)\]
\[= 8 + 100 \quad (#4-addition)\]
\[= 108\]

Ex. 10 \[(\sqrt{81} – 8)^3 + 3 \cdot 2^4 + 0 \cdot 5^2\]

Solution:
\[(\sqrt{81} – 8)^3 + 3 \cdot 2^4 + 0 \cdot 5^2 \quad (#1-parentheses, #2-exponents)\]
\[= (9 – 8)^3 + 3 \cdot 2^4 + 0 \cdot 5^2 \quad (#1-parentheses, #4-subtraction)\]
\[= (1)^3 + 3 \cdot 2^4 + 0 \cdot 5^2 \quad (#2-exponents)\]
\[= 1 + 3 \cdot 16 + 0 \cdot 25 \quad (#3-multiplication)\]
\[= 1 + 48 + 0 \cdot 25 \quad (#3-multiplication)\]
\[= 1 + 48 + 0 \quad (#4-addition)\]
\[= 49\]
Ex. 11 \[9^2 - (24 - 12 \div 3) + 3\cdot(5 - 2)^2\]

Solution:
\[9^2 - (24 - 12 \div 3) + 3\cdot(5 - 2)^2 \quad (#1\text{-parentheses}, \#3\text{-division})\]
\[= 9^2 - (24 - 4) + 3\cdot(5 - 2)^2 \quad (#1\text{-parentheses}, \#4\text{-subtraction})\]
\[= 9^2 - 20 + 3\cdot3^2 \quad (#2\text{-exponents})\]
\[= 81 - 20 + 9 \quad (#3\text{-multiplication})\]
\[= 11 + 27 \quad (#4\text{-subtraction})\]
\[= 68 \quad (#4\text{-addition})\]

Ex. 12 \[9^1 + 6(7 - 2) \div 3 - \{8 - [3^2 - \sqrt{16}]\}\]

Solution:
When there is a grouping symbol inside of another grouping symbol, work out the innermost set. So, we will work out \([3^2 - \sqrt{16}]\) first:
\[9^1 + 6(7 - 2) \div 3 - \{8 - [9 - 4]\} \quad (#1\text{-parent.}, \#1\text{-parent.}, \#2\text{-exp.})\]
\[= 9^1 + 6(7 - 2) \div 3 - \{8 - [5]\} \quad (#1\text{-parent.}, \#1\text{-parent.}, \#4\text{-subt.})\]
Notice that we can drop the [ ] since they are not needed.
\[= 9^1 + 6(5) \div 3 - \{8 - 5\} \quad (#1\text{-parent.}, \#4\text{-subt.})\]
\[= 9^1 + 6(5) \div 3 - 3 \quad \text{We can drop \{\} but not the ()}.\]
\[= 9^1 + 6(5) \div 3 - 3 \quad (#2\text{-exponents})\]
\[= 9 + 6(5) \div 3 - 3 \quad (#3\text{-multiplication})\]
\[= 9 + 30 \div 3 - 3 \quad (#3\text{-division})\]
\[= 9 + 10 - 3 \quad (#4\text{-addition})\]
\[= 19 - 3 \quad (#4\text{-subtraction})\]
\[= 16\]

Objective d: Computing the mean (average).

To find the average or mean of a set of numbers, we first add the numbers and then divide by the number of numbers.

Ex. 13 Find the average of 79, 83, 91, 78, and 64.

Solution:
To find the average of a set of numbers, we add the numbers and then divide by the number of numbers:
\[(79 + 83 + 91 + 78 + 64) \div 5 = (395) \div 5 = 79\]
So, the average is 79.