Sect 1.7 - Exponents, Square Roots and the Order of Operations

Objective a: Understanding and evaluating exponents.

We have seen that multiplication is a shortcut for repeated addition and division is a shortcut for repeated subtraction. Exponential notation is a shortcut for repeated multiplication. Consider the follow:

$$5\bullet5\bullet5\bullet5\bullet5\bullet5$$

Here, we are multiplying six factors of five. We will call the 5 our base and the 6 the exponent or power. We will rewrite this as 5 to the 6^{th} power:

$$5\bullet5\bullet5\bullet5\bullet5\bullet5 = 5^6$$

The number that is being multiplied is the base and the number of factors of that number is the power. Let's try some examples:

Write the following in exponential form:

Ex. 1a $8\bullet8\bullet8\bullet8$

Solution:

Since there are four factors of 8, we write 8^4.

Ex. 1b $7\bullet7\bullet7\bullet7\bullet7$

Solution:

Since there are five factors of 7, we write 7^5.

Simplify the following:

Ex. 2a 3^4

Solution:

Write 3^4 in expanded form and multiply:

$3^4 = 3\bullet3\bullet3\bullet3 = 9\bullet3\bullet3 = 27\bullet3 = 81$

Ex. 2b 7^3

Solution:

Write 7^3 in expanded form and multiply:

$7^3 = 7\bullet7\bullet7 = 49\bullet7 = 343$

Ex. 3a 1^5

Solution:

Write 1^5 in expanded form and multiply:

$1^5 = 1\bullet1\bullet1\bullet1\bullet1 = 1\bullet1\bullet1\bullet1$

$= 1\bullet1\bullet1 = 1\bullet1 = 1$

Ex. 3b $2^3\bullet6^2$

Solution:

Write $2^3\bullet6^2$ in expanded form and multiply:

$2^3\bullet6^2 = (2\bullet2\bullet2)\bullet(6\bullet6)$

$= 8\bullet36 = 288$

Ex. 4a 10^1

Solution:

$10^1 = 10$

Ex. 4b 10^2

Solution:

$10^2 = 10\bullet10 = 100$

Ex. 4c 10^3

Solution:

$10^3 = 10\bullet10\bullet10 = 100\bullet10 = 1000$

Notice with powers of ten, we get a one followed by the numbers of zeros equal to the exponent.

Ex. 4d 10^6

Solution:

This will be equal to 1 followed by 6 zeros: 1,000,000

Ex. 4e 10^{100}

Solution:

This will be equal to 1 followed by 100 zeros:

10................0

|-100 zeros-|

The number 10^{100} is called a "googol." The mathematician that was first playing around with this number asked his nine-year old nephew to give it a name. There are even larger numbers like 10^{googol}. This is a 1 followed by a googol number of zeros. If you were able to write 1 followed a googol number of zeros on a piece of paper, that paper could not be stuffed in the known universe!

Objective b: Understanding and applying square roots.

The square of a whole number is called a **perfect square**. So, 1, 4, 9, 16, 25 are perfect squares since $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, and $25 = 5^2$. We use the idea of perfect squares to simplify square roots. The square root of a number a asks what number times itself is equal to a. For example, the square root of 25 is 5 since 5 times 5 is 25.

The **square root** of a number a, denoted $\sqrt{a}$, is a number whose square is a. So, $\sqrt{25} = 5$.

Simplify the following:

Ex. 5a $\sqrt{49}$

Ex. 5b $\sqrt{144}$

Ex. 5c $\sqrt{0}$

Ex. 5d $\sqrt{625}$

Solution:

a) $\sqrt{49} = 7$ since $7^2 = 49$.

b) $\sqrt{144} = 12$ since $12^2 = 144$.

c) $\sqrt{0} = 0$ since $0^2 = 0$.
d) $\sqrt{625} = 25$ since $25^2 = 625$.

Objective c: Understanding and applying the Order of Operations

If you ever have done some cooking, you know how important it is to follow the directions to a recipe. An angel food cake will not come out right if you just mix all the ingredients and bake it in a pan. Without separating the egg whites from the egg yolks and whipping the eggs whites and so forth, you will end up with a mess. The same is true in mathematics; if you just mix the operations up without following the order of operations, you will have a mess. Unlike directions for making a cake that differ from recipe to recipe, the order of operations always stays the same. The order of operations are:

Order of Operations

1) Parentheses - Do operations inside of Parentheses ( ), [ ], { }, $| \ |$
2) Exponents including square roots.
3) Multiplication or Division as they appear from left to right.
4) Addition or Subtraction as they appear from left to right.

A common phrase people like to use is:

Please
Excuse
My **D**ear
Aunt **S**ally

(Be careful with the My Dear and the Aunt Sally part. Multiplication does not precede Division and Division does not precede multiplication; they are done as they appear from left to right. The same is true for addition and subtraction.)

Simplify the following:

Ex. 6 99 – 12 + 3 – 14 + 5

Solution:
We need to add and subtract as they appear from left to right:
99 – 12 + 3 – 14 + 5 (#4-subtraction)
= 87 + 3 – 14 + 5 (#4-addition)
= 90 – 14 + 5 (#4-subtraction)
= 76 + 5 (#4-addition)
= 81

Ex. 7 $72 \div 9(4) \div \sqrt{4}\,(5)$

<u>Solution:</u>

$72 \div 9(4) \div \sqrt{4}\,(5)$ (#2-exponents)

We need to multiply and divide as they appear from left to right:

$= 72 \div 9(4) \div 2(5)$ (#3-division)

$= 8(4) \div 2(5)$ (#3-multiply)

$= 32 \div 2(5)$ (#3-division)

$= 16(5)$ (#3-multiply)

$= 80$

Ex. 8 $8 \bullet 3 + 4 \bullet 2$

<u>Solution:</u>

Since there are no parentheses or exponents, we start with step #3, multiply or divide as they appear from left to right:

$8 \bullet 3 + 4 \bullet 2$ (#3-multiplication)

$= 24 + 8$ (#4-addition)

$= 32$

Ex. 9 $18 \div 3^2 \bullet \sqrt{16} + 4 \bullet 5^2$

<u>Solution:</u>

Since there are no parentheses, we start with step #2, exponents:

$18 \div 3^2 \bullet \sqrt{16} + 4 \bullet 5^2$ (#2-exponents)

$= 18 \div 9 \bullet 4 + 4 \bullet 25$ (#3-division)

$= 2 \bullet 4 + 4 \bullet 25$ (#3-multiplication)

$= 8 + 4 \bullet 25$ (#3-multiplication)

$= 8 + 100$ (#4-addition)

$= 108$

Ex. 10 $(\sqrt{81} - 8)^3 + 3 \bullet 2^4 + 0 \bullet 5^2$

<u>Solution:</u>

$(\sqrt{81} - 8)^3 + 3 \bullet 2^4 + 0 \bullet 5^2$ (#1-parentheses, #2-exponents)

$= (9 - 8)^3 + 3 \bullet 2^4 + 0 \bullet 5^2$ (#1-parentheses, #4-subtraction)

$= (1)^3 + 3 \bullet 2^4 + 0 \bullet 5^2$ (#2-exponents)

$= 1 + 3 \bullet 16 + 0 \bullet 25$ (#3-multiplication)

$= 1 + 48 + 0 \bullet 25$ (#3-multiplication)

$= 1 + 48 + 0$ (#4-addition)

$= 49$

Ex. 11 $9^2 - (24 - 12 \div 3) + 3\bullet(5 - 2)^2$

Solution:

$9^2 - (24 - 12 \div 3) + 3\bullet(5 - 2)^2$ (#1-parentheses, #3-division)

$= 9^2 - (24 - 4) + 3\bullet(5 - 2)^2$ (#1-parentheses, #4-subtraction)

$= 9^2 - 20 + 3\bullet(3)^2$ (#2-exponents)

$= 81 - 20 + 3\bullet9$ (#3-multiplication)

$= 81 - 20 + 27$ (#4-subtraction)

$= 61 + 27$ (#4-addition)

$= 88$

Ex. 12 $9^1 + 6(7 - 2) \div 3 - \{8 - [3^2 - \sqrt{16}\,]\}$

Solution:

When there is a grouping symbol inside of another grouping symbol, work out the innermost set. So, we will work out $[3^2 - \sqrt{16}\,]$ first:

$9^1 + 6(7 - 2) \div 3 - \{8 - [3^2 - \sqrt{16}\,]\}$ (#1-parent., #1-parent., #2-exp.)

$= 9^1 + 6(7 - 2) \div 3 - \{8 - [9 - 4]\}$ (#1-parent., #1-parent., #4-subt.)

$= 9^1 + 6(7 - 2) \div 3 - \{8 - [5]\}$

Notice that we can drop the [ ] since they are not needed.

$= 9^1 + 6(7 - 2) \div 3 - \{8 - 5\}$ (#1-parent., #4-subt.)

$= 9^1 + 6(5) \div 3 - \{3\}$ We can drop {} but not the ().

$= 9^1 + 6(5) \div 3 - 3$ (#2-exponents)

$= 9 + 6(5) \div 3 - 3$ (#3-multiplication)

$= 9 + 30 \div 3 - 3$ (#3-division)

$= 9 + 10 - 3$ (#4-addition)

$= 19 - 3$ (#4-subtraction)

$= 16$

Objective d: Computing the mean (average).

To find the average or mean of a set a numbers, we first add the numbers and then divided by the number of numbers.

Ex. 13 Find the average of 79, 83, 91, 78, and 64.

Solution:

To find the average of a set of numbers, we add the numbers and then divide by the number of numbers:

$(79 + 83 + 91 + 78 + 64) \div 5 = (395) \div 5 = 79$

So, the average is 79.