Section 2.2 – Prime Numbers and Factorization

Objective a: Factors and Factorizations.

If we look at a problem like 5•3 = 15, 15 is called the product and 5 and 3 are factors of 15. 5•3 is called a factorization of 15. Also, 15 is divisible by both 5 and 3. This leads us to make the following definition.

A **factor** of a number divides the number evenly with no remainder. In this discussion, we will consider only factors that are whole numbers greater than 0. A **factorization** of a particular number is a product of numbers that is equal to that particular number.

Find two different factorizations of the given number:

Ex. 1a 45

Ex. 1b 18

Solution:

a) 5•9 = 45 and 15•3 = 45, so 5•9 and 15•3 are two different factorizations of 45.

b) 3•6 = 18 and 2•3•3 = 18, so 3•6 and 2•3•3 are two different factorizations of 18.

Objective b: Divisibility Rules.

To determine if a whole number is divisible by another whole number, the following rules are helpful:

Divisibility Tests for Whole Numbers

A) **Two**: If a number is even (i.e., if the digit in the ones place is 0, 2, 4, 6, or 8), then the number is divisible by 2.
Examples: 46 and 272 since they are both even.

B) **Three**: If the sum of the digits of a number is divisible by three, then the number is divisible by 3.
Examples: 69 and 687 since 6 + 9 = 15 and 6 + 8 + 7 = 21.

C) **Five**: If the last digit (ones place) is 0 or 5, then the number is divisible by 5.
Examples: 85 and 190 since the last digits is 5 and 0.

D) **Ten:** If the last digit (ones place) is 0, then the number is divisible by 10.
Examples: 70 and 1230 since the last digit is 0.

<u>Determine if the given number is divisible by 2, 3, 5, and/or 10:</u>

Ex. 2 46,560

<u>Solution:</u>
Since 46,560 is even, it is divisible by 2.
Since 4 + 6 + 5 + 6 + 0 = 21, it is divisible by 3.
Since the last digit is 0, it is divisible by both 5 and 10.
So, 46,560 is divisible by 2, 3, 5, and 10.

Ex. 3 172,255

<u>Solution:</u>
Since 172,255 is not even, it is not divisible by 2.
Since 1 + 7 + 2 + 2 + 5 + 5 = 22, it is not divisible by 3.
Since the last digit is 5, it is divisible by 5, but not by 10.
So, 172,255 is divisible by 5.

Ex. 4 856,932

<u>Solution:</u>
Since 856,932 is even, it is divisible by 2.
Since 8 + 5 + 6 + 9 + 3 + 2 = 33, it is divisible by 3.
Since the last digit is 2, it is not divisible by either 5 or 10.
So, 856,932 is divisible by 2 and 3.

Objective c: Prime and Composite Numbers.

A **Prime Number** is a whole number larger than one with exactly two whole numbers as factors, 1 and itself.

A **Composite Number** is a whole number larger than one with more than two different whole numbers as factors.

The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...
The composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, ...

<u>Determine if the following are prime or composite numbers:</u>

Ex. 5a 63

<u>Solution:</u>
Since 9 is a factor of 63, 63 is a composite number.

Ex. 5b 31

<u>Solution:</u>
Since 1 and 31 are the only factors, 31 is a prime number.

Ex. 5c 110

<u>Solution:</u>
Since 2 is a factor of 110, 110 is a composite number.

Objective d: Finding the Prime Factorization of a number

Every composite number can be written as a product of prime numbers. To see how this works, let's look at the following examples:

<u>Write each number as a product of prime numbers:</u>

Ex. 6 63

<u>Solution:</u>

First, think of two number that you can multiply together to get 63.

63 = 9•7. Seven is prime, but nine can be broken down to 3•3.

So, 63 = ***9***•7 = ***3•3***•7 = 3^2•7.

Ex. 7 31

<u>Solution:</u>

Since 31 is prime, then 31 is the answer.

Ex. 8 110

<u>Solution:</u>

110 = 11•10 = 11***•10*** = 11***•2•5*** = 2•5•11. We always write the factors from smallest to largest in our final answer.

Ex. 10 504

<u>Solution:</u>

504 = 4***•126*** = 2•2***•126*** = 2•2•***<u>6</u>•21*** = 2•2•<u>2</u>•<u>3</u>•***21*** = 2•2•2•3•***3•7*** = 2^3•3^2•7.

Ex. 11 289

<u>Solution:</u>

None of our divisibility tests work on this number. So, we need to go through the primes numbers beyond five and see if any of them are factors of 289. Feel free to use a calculator to help:

289 ÷ 7 = 41.2... No, 289 ÷ 11 = 26.27... No,

289 ÷ 13 = 22.23... No, 289 ÷ 17 = 17 Yes

So, 289 = 17•17 = 17^2.

Objective e: Identifying all Factors of a Whole Number.

In finding all the factors of a number, it is important to be systematic and to write down factors are pairs. Let's do an example to illustrate.

<u>Find the factors of the following:</u>

Ex. 12 24

<u>Solution:</u>

Start with 1. Since 24 ÷ 1 = 24, then 1 and 24 are factors.

Next, go to 2. Since 24 ÷ 2 = 12, then 2 and 12 are factors.

Then, try 3. Since 24 ÷ 3 = 8, then 3 and 8 are factors.

Finally, try 4. Since 24 ÷ 4 = 6, then 4 and 6 are factors.

Five does not go into 24 evenly and we already have 6 down.
So, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Ex. 13 30

<u>Solution:</u>

30 ÷ 1 = 30, so 1, 30 are factors.

30 ÷ 2 = 15, so 2, 15 are factors.

30 ÷ 3 = 10, so 3, 10 are factors.

30 ÷ 4 does not go evenly.

30 ÷ 5 = 6, so 5, 6 are factors.

So, the factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

Ex. 14 51

<u>Solution:</u>

51 ÷ 1 = 51, so 1 and 51 are factors.
51 is not even so it is not divisible by 2.
5 + 1 = 6 which is divisible by 3, so 51 is divisible by 3. This means that 3 and 17 are factors.
51 is not divisible by 4, 5, or 6.
7 does not go into 51 evenly.
So, the factors of 51 are 1, 3, 17, and 51.

Some Additional Divisibility Tests for Whole Numbers

E) **Four**: If the last two digits of a number is divisible by 4, then the number is divisible by 4.

F) **Six**: If a number is divisible by 2 and 3, it is divisible by 6.

G) **Eight**: If the last three digits of a number is divisible by 8, then the number is divisible by 8.

H) **Nine:** If the sum of the digits of a number is divisible by nine, then the number is divisible by 9.