Sect 2.2 - Solving Linear Equations

Concept #1 Solving Linear Equations Involving Multiple Steps

In solving an equation involving more than one step, our goal is to isolate the variable terms on one side and the constant terms on the other side. We will do this by using the addition and subtraction properties first. Once we get the variable and constant terms isolated on opposite sides, we then use the multiplication and division properties to solve. Let's try some examples:

Solve and check the following:

Ex. 1 $-3x + 8 = -15$

Solution:

First subtract 8 from both sides, and then divide both sides by – 3.

$-3x + 8 = -15$

$\underline{\quad -8 = -8 \quad}$

$-3x = -23$

$\frac{-3x}{-3} = \frac{-23}{-3}$

$x = \frac{23}{3} = 7\frac{2}{3}$

Check:

$-3x + 8 = -15$

$-3(\frac{23}{3}) + 8 = -15$

$-23 + 8 = -15$

$-15 = -15$

True

Ex. 2 $-8.2 + 3.6x = 2.3x - 10.8$

Solution:

$-8.2 + 3.6x = 2.3x - 10.8$ (subtract 2.3x from both sides)

$\underline{\quad -2.3x = -2.3x \quad}$

$-8.2 + 1.3x = -10.8$ (add 8.2 to both sides)

$\underline{+8.2 \quad = +8.2}$

$1.3x = -2.6$

$\frac{1.3x}{1.3} = \frac{-2.6}{1.3}$ (divide both sides by 1.3)

$x = -2$

Check:

$-8.2 + 3.6x = 2.3x - 10.8$

$-8.2 + 3.6(-2) = 2.3(-2) - 10.8$

$-8.2 - 7.2 = -4.6 - 10.8$

$-15.4 = -15.4$ True

Ex. 3 $-\frac{6}{7}x + 36 = 92 - \frac{2}{7}x$

Solution:

$-\frac{6}{7}x + 36 = 92 - \frac{2}{7}x$ (add $\frac{2}{7}x$ to both sides)

$+\frac{2}{7}x \quad = \quad +\frac{2}{7}x$

$-\frac{4}{7}x + 36 = 92$ (subtract 36 from both sides)

$-36 = -36$

$-\frac{4}{7}x = 56$ (multiply both sides by the reciprocal of $-\frac{4}{7}$)

$-\frac{7}{4}\left(-\frac{4}{7}x\right) = -\frac{7}{4}\left(\frac{56}{1}\right)$

$x = -\frac{7}{1}\left(\frac{14}{1}\right) = -98$

Check:

$-\frac{6}{7}x + 36 = 92 - \frac{2}{7}x$

$-\frac{6}{7}\left(\frac{-98}{1}\right) + 36 = 92 - \frac{2}{7}\left(\frac{-98}{1}\right)$

$-\frac{6}{1}\left(\frac{-14}{1}\right) + 36 = 92 - \frac{2}{1}\left(\frac{-14}{1}\right)$

$84 + 36 = 92 + 28$

$120 = 120$ True

Ex. 4 $7x - 3 = 5x$

Solution:

To get the variable terms isolated on the opposite side from the constant term, we need to subtract 7x from both sides:

$7x - 3 = 5x$

$-7x \quad = -7x$

$-3 = -2x$

Now, divide by – 2

$\frac{-3}{-2} = \frac{-2x}{-2}$

$1.5 = x$

Check:

$7x - 3 = 5x$

$7(1.5) - 3 = 5(1.5)$

$10.5 - 3 = 7.5$

$7.5 = 7.5$

True

Concept #2 Steps to Solve a Linear Equation in One Variable.

When working with equations, we may need to simplify each side of the equation before solving. This may include:

i) Using the Distributive Property to Clear Parentheses
ii) Clearing Fractions and Decimals
iii) Combining Like Terms

Thus, we can outline a procedure for solving linear equations in one variable:

Procedure for Solving Equations:

1) Simplify each side of the equation. This includes:
   i) Using the Distributive Property to Clear Parentheses
   ii) Clearing Fractions and Decimals
   iii) Combining Like Term
2) Use the addition and subtraction properties of equality to isolate the variables terms on one side, the constant terms on the other side.
3) Use the multiplication and division properties of equality to make the coefficient of the variable term equal to one.
4) Check the solution by plugging in your answer into the original equation and simplifying each side to see if you have the same number on both sides.

Solve the following:

Ex. 5 $-3x + 4 + 5x = 2x - (3x + 8)$

Solution:

First, let us simplify the left side:

$-3x + 4 + 5x$ (combine like terms)

$= 2x + 4$

Now, let us simplify the right side:

$2x - (3x + 8)$ (distribute {"clear parentheses"})

$2x \mathbf{-1}(3x) + \mathbf{(-1)}(8)$ (multiply)

$2x - 3x - 8$ (combine like terms)

$= -x - 8$

Thus, our equation becomes:

$2x + 4 = -x - 8$ (add x to both sides)

$+x \quad = +x$

$3x + 4 = -8$ (subtract 4 from both sides)

$-4 = -4$

$3x = -12$

$\frac{3x}{3} = \frac{-12}{3}$ (divide both sides by 3)

$x = -4$

Ex. 6 $4 - 3(2x - 5) = \frac{1}{8}x - 11 + \frac{1}{4}x$

Solution:

First, let us simplify the left side:

$4 - 3(2x - 5)$ (distribute {"clear parentheses"})

$= 4 \mathbf{-} \mathbf{3}(2x) - \mathbf{(-} \mathbf{3})(5)$ (multiply)

$= 4 - 6x + 15$ (combine like terms)

$= -6x + 19$

Now let us simplify the right side:

$\frac{1}{8}x - 11 + \frac{1}{4}x$ (L.C.D. = 8)

$= \frac{1}{8}x - 11 + \frac{2}{8}x$ (combine like terms)

$= \frac{3}{8}x - 11$

Thus, our equation becomes:

$-6x + 19 = \frac{3}{8}x - 11$ (write 6 over 1)

$-\frac{6}{1}x + 19 = \frac{3}{8}x - 11$ (L.C.D. = 8)

$-\frac{48}{8}x + 19 = \frac{3}{8}x - 11$ (add $\frac{48}{8}x$ to both sides)

$+\frac{48}{8}x \quad = +\frac{48}{8}x$

$19 = \frac{51}{8}x - 11$ (add 11 to both sides)

$+11 = \quad +11$

$\frac{30}{1} = \frac{51}{8}x$ (write 30 over 1)

Now, multiply by the reciprocal of $\frac{51}{8}$ which is $\frac{8}{51}$.

$\frac{8}{51}\left(\frac{30}{1}\right) = \frac{8}{51}\left(\frac{51}{8}x\right)$ (30 and 51 have a common factor of 3)

$\frac{8}{17}\left(\frac{10}{1}\right) = x$ (multiply)

$\frac{80}{17} = x$

Ex. 7 $3(2.2x - 6.1) - 4.5x = 7.3x - 6.2 + 11.1x$

Solution:

First, let us simplify the left side:

$3(2.2x - 6.1) - 4.5x$ (distribute)

$= \mathbf{3}(2.2x) - \mathbf{3}(6.1) - 4.5x$ (multiply)

$= 6.6x - 18.3 - 4.5x$ (combine like terms)

$= 2.1x - 18.3$

Now, let us simplify the right side:
$7.3x - 6.2 + 11.1x$ (combine like terms)
$= 18.4x - 6.2$
Thus, our equation becomes:
$2.1x - 18.3 = 18.4x - 6.2$ (subtract 18.4x from both sides)
$\underline{-18.4x \quad = -18.4x}$
$-16.3x - 18.3 = -6.2$ (add 18.3 to both sides)
$\underline{+18.3 = +18.3}$
$-16.3x = 12.1$

$\frac{-16.3x}{-16.3} = \frac{12.1}{-16.3}$ (divide both sides by – 16.3)
(slide the decimal point over on place to write as a fraction)
$x = -\frac{121}{163}$

Ex. 8 $3x - 8 = -4(2x + 5) - 3(4x - 8)$

Solution:
The left side is already simplified, so we need to focus only on the right side:

$-4(2x + 5) - 3(4x - 8)$ (distribute)
$= \mathbf{-4}(2x) + \mathbf{-4}(5) \mathbf{-3}(4x) - \mathbf{-3}(8)$ (multiply)
$= -8x - 20 - 12x + 24$ (combine like terms)
$= -20x + 4$

Thus, our equation becomes:

$3x - 8 = -20x + 4$ (add 20x to both sides)
$\underline{+20x \quad = +20x}$
$23x - 8 = 4$ (add 8 to both sides)
$\underline{\quad +8 = +8}$
$23x = 12$

Now, divide by 23

$\frac{23x}{23} = \frac{12}{23}$
$x = \frac{12}{23}$

Concept #3 Conditional Equations, Identities, and Contradictions.

In the equations that we have solved thus far, there has been one solution. These types of equations are called **conditional equations**. However, some equations, called **contradictions**, have no solution, meaning that there is no real number that will satisfy the equation. There are also some equations, called **identities**, where every real number is a

solution to the equation. Examples one through eight are conditional equations. Let's try some more examples.

Solve the following:

Ex. 9 $6.3x - 8x - 10 = -2.2x - 5(2 - 0.1x)$

Solution:

Simplify the left side:

$6.3x - 8x - 10$ (combine like terms)

$= -1.7x - 10$

Now, simplify the right side

$-2.2x - 5(2 - 0.1x)$ (distribute)

$= -2.2x \mathbf{-5}(2) \mathbf{-} \mathbf{-5}(0.1x)$ (multiply)

$= -2.2x - 10 + 0.5x$ (combine like terms)

$= -1.7x - 10$

So, our equation becomes:

$-1.7x - 10 = -1.7x - 10$ (add 1.7x to both sides)

$+1.7x \quad = +1.7x$

$-10 = -10$

This is a true statement meaning that this equation is true for any real number. This equation is an identity since the two sides are identical after we simplified. The solution is all real numbers.

Ex. 10 $7(3x - 4) = 21x - 5$

Solution:

$7(3x - 4) = 21x - 5$ (distribute in the left side)

$\mathbf{7}(3x) - \mathbf{7}(4) = 21x - 5$ (multiply)

$21x - 28 = 21x - 5$ (subtract 21x from both sides)

$-21x \quad = -21x$

$-28 = -5$

This is a false statement of a contradiction. This equation is a contradiction and there is no solution.

Ex. 11 $4x + 5 = 3x + 5$

Solution:

$4x + 5 = 3x + 5$ (subtract 3x from both sides)

$-3x \quad = -3x$

$x + 5 = 5$ (subtract 5 from both sides)

$-5 = -5$

$x = 0$

This is a conditional equation. The solution is $x = 0$.