Sect 2.3 - Linear Equations: Clearing Fractions & Decimals

Concept #1 & 2 Solving Linear Equations with Fractions

Solve the following:

Ex. 1 $\frac{7}{12} = \frac{3}{8} - \frac{5}{6}x$

Solution:

Notice there is only a variable term on the right side so we need to get the constant terms to the left side.

$\frac{7}{12} = \frac{3}{8} - \frac{5}{6}x$ (L.C.D. of 8 and 12 is 24)

$\frac{14}{24} = \frac{9}{24} - \frac{5}{6}x$ (subtract $\frac{9}{24}$ from both sides)

$-\frac{9}{24} = -\frac{9}{24}$

$\frac{5}{24} = -\frac{5}{6}x$

Now, multiply by the reciprocal of $-\frac{5}{6}$ which is $-\frac{6}{5}$.

$-\frac{6}{5}\left(\frac{5}{24}\right) = -\frac{6}{5}\left(-\frac{5}{6}x\right)$ (reduce)

$-\frac{1}{1}\left(\frac{1}{4}\right) = x$ (multiply)

$-\frac{1}{4} = x$

In the procedure outlined in the last section, it mentioned clearing fractions and decimals. Since we are solving an equation, we are allowed to multiply both sides of an equation by any non-zero number. So, what we could do instead is to find the L.C.D. of all the denominators and then use the multiplication property of equality to multiply both sides by the L.C.D. This will clear the fractions and give us a much easier equation to solve. Let us try this with example #1:

$\frac{7}{12} = \frac{3}{8} - \frac{5}{6}x$ (L.C.D. is 24, multiply both sides by 24)

$\frac{24}{1}\left(\frac{7}{12}\right) = \frac{24}{1}\left(\frac{3}{8} - \frac{5}{6}x\right)$ (distribute)

$\frac{24}{1}\left(\frac{7}{12}\right) = \frac{24}{1}\left(\frac{3}{8}\right) - \frac{24}{1}\left(\frac{5}{6}x\right)$ (reduce)

$\frac{2}{1}\left(\frac{7}{1}\right) = \frac{3}{1}\left(\frac{3}{1}\right) - \frac{4}{1}\left(\frac{5}{1}x\right)$ (simplify)

$14 = 9 - 20x$

$$\begin{aligned} 14 &= 9 - 20x \\ -9 &= -9 \\ \hline 5 &= -20x \end{aligned}$$ (subtract 9 from both sides)

$$\frac{5}{-20} = \frac{-20x}{-20}$$ (divide both sides by – 20 & reduce)

$$-\frac{1}{4} = x$$

Notice that we do get the same result.

Solve the following:

Ex. 2 $\frac{2}{3}x - \frac{3}{4} = \frac{7}{12}x + 2$

Solution:

The L.C.D. of the denominators is 12. So, We will use the multiplication property of equality to multiply both sides by 12:

$$\frac{12}{1}\left(\frac{2}{3}x - \frac{3}{4}\right) = \frac{12}{1}\left(\frac{7}{12}x + 2\right)$$ (distribute)

$$\frac{12}{1}\left(\frac{2}{3}x\right) - \frac{12}{1}\left(\frac{3}{4}\right) = \frac{12}{1}\left(\frac{7}{12}x\right) + \frac{12}{1}\left(2\right)$$ (reduce)

$$\frac{4}{1}\left(\frac{2}{1}x\right) - \frac{3}{1}\left(\frac{3}{1}\right) = \frac{1}{1}\left(\frac{7}{1}x\right) + \frac{12}{1}\left(\frac{2}{1}\right)$$ (multiply)

$$\begin{aligned} 8x - 9 &= 7x + 24 \\ -7x \quad &= -7x \\ \hline x - 9 &= 24 \end{aligned}$$ (subtract 7x from both sides)

$$\begin{aligned} +9 &= +9 \\ \hline x &= 33 \end{aligned}$$ (add 9 to both sides)

Ex. 3 $\frac{4x-5}{9} - 4 = \frac{2x+5}{6}$

Solution:

First, clear fractions by multiplying both sides by 18:

$$\frac{18}{1}\left(\frac{4x-5}{9} - 4\right) = \frac{18}{1}\left(\frac{2x+5}{6}\right)$$ (distribute on the left side)

$$\frac{18}{1}\left(\frac{4x-5}{9}\right) - 18(4) = \frac{18}{1}\left(\frac{2x+5}{6}\right)$$ (reduce)

$$\frac{2}{1}\left(\frac{4x-5}{1}\right) - 18(4) = \frac{3}{1}\left(\frac{2x+5}{1}\right)$$

$2(4x - 5) - 18(4) = 3(2x + 5)$ (distribute)

$\mathbf{2}(4x) - \mathbf{2}(5) - 18(4) = \mathbf{3}(2x) + \mathbf{3}(5)$ (multiply)

$8x - 10 - 72 = 6x + 15$ (combine like terms)

$8x - 82 = 6x + 15$

$$\begin{array}{ll} 8x - 82 = 6x + 15 & \text{(subtract 6x from both sides)} \\ \underline{-6x \quad = -6x} & \\ 2x - 82 = 15 & \text{(add 82 to both sides)} \\ \underline{\quad + 82 = + 82} & \\ \quad 2x = 97 & \end{array}$$

$$\frac{2x}{2} = \frac{97}{2} \qquad \text{(divide by 2)}$$

$$x = 48.5$$

Keep in mind we can always check the answer by plugging our answer for the variable into the original equation and then simplify each side to see if we get the same number on both sides. Let us see how the check would work for this example:

Check

$$\frac{4x-5}{9} - 4 = \frac{2x+5}{6} \qquad \text{(replace x by 48.5)}$$

$$\frac{4(48.5)-5}{9} - 4 = \frac{2(48.5)+5}{6} \qquad \text{(multiply)}$$

$$\frac{194-5}{9} - 4 = \frac{97+5}{6} \qquad \text{(add and subtract)}$$

$$\frac{189}{9} - 4 = \frac{102}{6} \qquad \text{(divide)}$$

$$21 - 4 = 17 \qquad \text{(subtract)}$$

$$17 = 17$$

Concepts 1 & 3 Solving Linear Equations with Decimals

In much the same way as we cleared fractions, we can clear decimals by multiplying both sides of the equation by the appropriate power of ten.

Ex. 4 $0.35x - 0.65 = x + 8.125$

Solution:

Since 8.125 has the most digits to the right of the decimal point and the last digit is in the thousandths place, we will multiply both sides by 1000:

$$\mathbf{1000}(0.35x - 0.65) = \mathbf{1000}(x + 8.125) \qquad \text{(distribute)}$$

$$\mathbf{1000}(0.35x) - 1000(0.65) = \mathbf{1000}(x) + 1000(8.125) \qquad \text{(multiply)}$$

$$\begin{array}{ll} 350x - 650 = 1000x + 8125 & \text{(subtract 350x from both sides)} \\ \underline{-350x \qquad = -350x \qquad\quad} & \\ \quad -650 = 650x + 8125 & \text{(subtract 8125 from both sides)} \\ \underline{\quad -8125 = \qquad\quad -8125} & \\ \quad -8775 = 650x & \end{array}$$

$$\frac{-8775}{650} = \frac{650x}{650} \qquad \text{(divide both sides by 650)}$$

$-13.5 = x$

Ex. 5 $0.4p - (0.11p + 6) = 0.6(p - 2)$

Solution:

Simplify each side of the equation:

$0.4p - (0.11p + 6) = 0.6(p - 2)$ (distribute "{clear parentheses}")

$0.4p - 0.11p - 6 = 0.6p - 1.2$ (combine like terms)

$0.29p - 6 = 0.6p - 1.2$

Since 0.29 has the most digits to the right of the decimal point and the last digit is in the hundredths place, we will multiply both sides by 100:

$\mathbf{100}(0.29p - 6) = \mathbf{100}(0.6p - 1.2)$ (distribute)

$\mathbf{100}(0.29p) - 100(6) = \mathbf{100}(0.6p) - \mathbf{100}(1.2)$ (multiply)

$29p - 600 = 60p - 120$

$$\begin{array}{rcl} 29p - 600 & = & 60p - 120 \\ \underline{-29p \qquad} & = & \underline{-29p \qquad} \end{array} \qquad \text{(subtract 29p from both sides)}$$

$$\begin{array}{rcl} -600 & = & 31p - 120 \\ \underline{+120} & = & \underline{\qquad +120} \\ -480 & = & 31p \end{array} \qquad \text{(add 120 to both sides)}$$

$$\frac{-480}{31} = \frac{31p}{31} \qquad \text{(divide both sides by 31)}$$

$-\frac{480}{31} = p$