Sect 2.4 - Introduction to Problem Solving

Concept #1 Problem-Solving Strategies

When solving application problems, it is important to be organized and systematic in our approach. The following is a set of guidelines to help us to do so:

Problem Solving

1) Read the problem - Overview - Identify the type of problem
2) Read the problem - Phrase by Phrase - Ask yourself what does each phrase means.
3) Read the problem - Begin to write things down. Label known and unknown values. Write all unknowns using one variable
4) Draw a chart, a diagram, or a picture. Label it appropriately. You may need to get a formula.
5) Write an equation in words.
6) Translate the problem into an equation and solve.
7) Check.
   i) Reality Check - does it make sense?
   ii) Did you answer the questions posed by the problem?
8) Interpret the results and write your final answer in words.

Concept #2 Translation involving linear equations

a) Translate the following into an equation and b) solve the equation:

Ex. 1 The product of 3 and a number added to 17 amounts to twice the number subtract 4. Find the number.

Solution:

a) Let n = the number

The product of 3 and a number added to 17: $17 + 3n$

Twice the number subtract 4: $2n - 4$

So, $17 + 3n$ amounts to $n - 4$: $17 + 3n = 2n - 4$

b) $17 + 3n = 2n - 4$ (subtract 2n from both sides)

$\underline{\quad - 2n = -2n}$

$17 + n = -4$ (subtract 17 from both sides)

$\underline{-17 = -17}$

$n = -21$

The number is – 21.

Ex. 2 The sum of three times a number and two-thirds is the same as the difference of one-eighth of a number and seven. Find the number.

Solution:

a) Let n = the number

Three times a number: $3n$

The sum of three times a number and two-thirds: $(3n + \frac{2}{3})$

One-eighth of a number: $\frac{1}{8}n$

The difference of one-eighth of a number and seven: $(\frac{1}{8}n - 7)$

The sum of three times a number and two-thirds is the same as the difference of one-eighth of a number and seven:

$$3n + \frac{2}{3} = \frac{1}{8}n - 7$$

b) $3n + \frac{2}{3} = \frac{1}{8}n - 7$ (the L.C.D. = 24, multiply both sides by 24)

$24(3n + \frac{2}{3}) = 24(\frac{1}{8}n - 7)$ (distribute)

$24(3n) + \frac{24}{1}\left(\frac{2}{3}\right) = \frac{24}{1}\left(\frac{1}{8}n\right) - 24(7)$ (reduce)

$24(3n) + \frac{8}{1}\left(\frac{2}{1}\right) = \frac{3}{1}\left(\frac{1}{1}n\right) - 24(7)$

$72n + 16 = 3n - 168$ (subtract 3n from both sides)

$$\begin{array}{r} 72n + 16 = 3n - 168 \\ \underline{-\,3n \qquad = -\,3n} \\ 69n + 16 = -\,168 \\ \underline{-\,16 = -\,16} \\ 69n = -\,184 \end{array}$$

(divide both sides by 69)

$\frac{69n}{69} = \frac{-184}{69}$ (divide both sides by 69 and reduce)

$n = -\frac{8}{3}$

The number is $-\frac{8}{3}$.

Ex. 3 Twice the total of a number and – 3.2 is four less the product of three and the number. Find the number.

Solution:

a) Let n = the number

Total of a number and – 3.2: $(n + (-3.2))$

Twice the total of a number and – 3.2: $2(n + (-3.2))$
The product of three and the number: $3n$
Four less the product of three and the number: $4 - 3n$
Twice the total of a number and – 3.2 is four less the product of three and the number: $2(n + (-3.2)) = 4 - 3n$

b) $2(n + (-3.2)) = 4 - 3n$ (distribute "{clear parentheses}")
$2n - 6.4 = 4 - 3n$ (add 3n to both sides)
$\underline{+\,3n \quad = \quad +\,3n}$
$5n - 6.4 = 4$ (add 6.4 to both sides)
$\underline{\quad +\,6.4 = +\,6.4}$
$5n = 10.4$

$\dfrac{5n}{5} = \dfrac{10.4}{5}$ (divide both sides by 5)
$n = 2.08$
The number is 2.08.

Concept #3: Consecutive Integer Problems

Consecutive integers follow each other, like 8, 9, and 10 or – 4, – 3, and – 2. Notice the second integer is one more than the first, the third is two more than the first, the fourth is three more than the first, etc. If we let x represent the first integer, then (x + 1) will represent the next consecutive integer, (x + 2) the consecutive integer after that, and so on. Consecutive even or odd integers are every other integers like 8, 10, and 12 or – 9, – 7, and – 5. Notice the second even or odd integer is two more than the first, the third is four more than the first, the fourth is six more than the first, etc. If we let y represent the first even (or odd) integer, then (y + 2) will represent the next consecutive even (or odd) integer, (y + 4) the consecutive even (or odd) integer after that, and so on. Let us try some examples:

a) Set-up the equation and b) then solve:

Ex. 4 If the sum of three consecutive integers is 132, find the integers.
Solution:
a) Let f = the first consecutive integer
then f + 1 = the second consecutive integer
and f + 2 = the third consecutive integer

The sum of three consecutive integers is 132:

$f + (f + 1) + (f + 2) = 132$

b) $f + (f + 1) + (f + 2) = 132$ (combine like terms)

$3f + 3 = 132$ (subtract 3 from both sides)

$$\begin{array}{r}3f + 3 = 132\\ \underline{\quad -3 = -3}\\ 3f \quad = 129\end{array}$$

$\dfrac{3f}{3} = \dfrac{129}{3}$ (divide both sides by 3)

$f = 43$

Replace f by 43 in the expressions f + 1 and f + 2 to find the other integers:

$f + 1 = 44$

$f + 2 = 45$

The integers are 43, 44, and 45.

Ex. 5 The three numbers of the combination for a lock are consecutive odd integers. If three times the first number is seventeen more than twice the third number, find the combination for the lock.

<u>Solution:</u>

a) Let f = first odd integer

$f + 2$ = second odd integer

$f + 4$ = third odd integer

Now, let's look at the second sentence:

"three times the first number is seventeen more than twice the third number:"

3•(first number) = 2•(third number) + 17

Replace the (first number) by f and (third number) by (f + 4):

$3f = 2(f + 4) + 17$ (distribute "{clear parentheses}")

$3f = \mathbf{2}f + \mathbf{2}\bullet 4 + 17$ (multiply)

$3f = 2f + 8 + 17$ (combine like term)

$3f = 2f + 25$ (subtract 2f from both sides)

$$\begin{array}{r}3f = 2f + 25\\ \underline{-2f = -2f\quad}\\ f = 25\end{array}$$

Replace f by 25 in the expressions f + 2 and f + 4 to find the other integers:

$f + 2 = 25 + 2 = 27$ and $f + 4 = 25 + 4 = 29$.

So, the combination is 25 - 27 - 29.

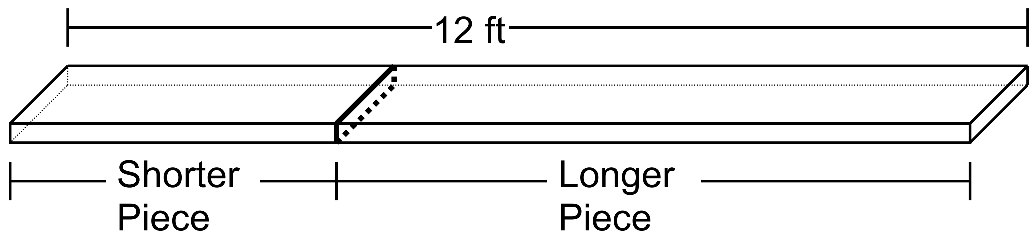
Concept #4 Applications of Linear Equations

Now, we will examine more complex applications. The key is to follow our steps for setting up and solving the problems.

Ex. 6 A 12-foot board is cut into two pieces. If the longer piece is three feet less than twice the shorter piece, find the length of each piece.

<u>Solution :</u>

Let us first draw a picture and label it:



Thus, the shorter piece plus the longer piece is 12 ft.

Since the length of the longer piece is described in terms of the length of the shorter piece, we will let a variable equal to the length of the shorter piece and write an expression for the longer piece using that variable.

Let s = the length of the shorter piece.

The longer piece is 3 ft less than twice s, so

$2s - 3$ = the length of the longer piece

The shorter piece + the longer piece = 12:

$(s) \quad + \quad (2s - 3) \quad = 12$

So, the equation is: $s + 2s - 3 = 12$ (combine like terms)

$3s - 3 = 12$ (add three to both sides)

$\underline{\quad + 3 = + 3\quad}$

$\frac{3s}{3} = \frac{15}{3}$ (divide by 3)

$s = 5$ feet.

$2s - 3 = 2(5) - 3 = 10 - 3 = 7$ feet

The shorter piece is 5 feet and the longer piece is 7 feet.

Ex. 7 A study found that the number of business bankruptcies in 2003 was 19,000 more than eight times what the government data showed. If the study showed that there were 315,000 business bankruptcies in 2003, how many bankruptcies did the government data show? (Source: Jim Hopkin's article: "Study puts bankruptcy rates higher," June 15, 2005 USAToday).

Solution:

Notice the number of bankruptcies the study showed is described in terms of the number of bankruptcies the government showed.

Let g = the number of bankruptcies the government data showed.

The study showed 19,000 more than eight times what the government data showed: $8 \bullet g + 19{,}000$

Thus, our equation is:

$$315{,}000 = 8g + 19{,}000 \quad \text{(subtract 19,000 from both sides)}$$
$$-19000 = \quad -19000$$
$$\frac{296{,}000}{8} = \frac{8g}{8} \quad \text{(divide both sides by 8)}$$
$$37{,}000 = g$$

The government data showed 37,000 business bankruptcies in 2003.

Ex. 8 A Double Quarter Pounder® with Cheese with a Large French Fries has 1250 calories (Source: www.Mcdonalds.com). If the number of calories from a Double Quarter Pounder® with Cheese has fifty calories less than half of three orders of the Large French Fries, find the number of calories each has?

Solution:

The first sentence tells us that:

Double Quarter Pounder with cheese + Large French Fries = 1250.

Let f = the number calories that Large French Fries has.

Fifty calories less than half of three orders of the Large French Fries:

$$\frac{1}{2}(3f) - 50 = 1.5f - 50$$

So, 1.5f – 50 = the number calories that Double Quarter Pounder with cheese has.

Double Quarter Pounder with cheese + Large French Fries = 1250.

$$1.5f - 50 \quad + \quad f \quad = 1250$$

So, our equation is:

$$1.5f - 50 + f = 1250 \quad \text{(combine like terms)}$$
$$2.5f - 50 = 1250 \quad \text{(add 50 to both sides)}$$
$$+50 \quad +50$$
$$\frac{2.5f}{2.5} = \frac{1300}{2.5} \quad \text{(divide by 2.5)}$$
$$f = 520$$
$$1.5f - 50 = 1.5(520) - 50 = 780 - 50 = 730$$

A Large French Fries has 520 calories and a Double Quarter Pounder® with Cheese has 730 calories.

Ex. 9 Juan buys three CDs for every DVD he purchases. If the cost of a CD is \$15.99 and the cost of DVD is \$19.99, how many CDs and DVDs does he purchase if he spends \$271.84?

Solution:

Let D = the number of DVDs he buys.

Now, write the number of CDs is terms of the number of DVDs:

"Juan buys three CDs for every DVD he purchases"

Thus, if Juan buys 1 DVD, he will buy 3 CDs. If he buys 2 DVDs, he will buy 6 CDs. If he buys D number of DVDs, he will buy $3 \bullet D$ number of CDs.

3D = the number of CDs he buys.

Since the total spent for each type of item is the price times the number purchased, then the amount he spends on DVDs is $19.99 \bullet D$ and the amount he spends on CDs is $15.99 \bullet 3D = 47.97D$. The total amount spent is:

amount spent on DVDs + amount spent on CDs = 271.84

$$19.99D \quad + \quad 47.97D \quad = 271.84$$

Thus, our equation is:

$$19.99D + 47.97D = 271.84 \quad \text{(combine like terms)}$$

$$\frac{67.96D}{67.96} = \frac{271.84}{67.96} \quad \text{(divide by 67.96)}$$

$$D = 4$$

Also, $3D = 3(4) = 12$.

Juan bought 4 DVDs and 12 CDs.