Sect 2.6 - Formulas and Applications of Geometry:

Concept #1 Solving Literal Equations for a particular variable.

Now, we will examine solving formulas for a particular variable. Sometimes it is useful to take a formula that is solved for one variable and rewrite it so it is solved for another variable. To see how this is done, we will first work a problem with numbers that we can plug in and solve. We will then use that as a blueprint for solving a formula (or literal equation)
for a particular variable.

Solve:

Ex. 1a $A = Lw$ (area of a rectangle); $A = 45\ ft^2$ and $w = 9$ ft

Solution:

Substitute the given values for A and w and solve:

$A = Lw$

$45 = L(9)$ (rewrite the right side)

$\frac{45}{9} = \frac{9L}{9}$ (divide both sides by the number in front of L)

$5 = L$ So, L = 5 ft.

Ex. 1b $A = Lw$ for L

Solution:

Follow the steps we performed in Ex. 1a after substituting:

$A = Lw$ (rewrite the right side)

$\frac{A}{w} = \frac{wL}{w}$ (divide both sides by the number in front of L)

$\frac{A}{w} = L$ So, $L = \frac{A}{w}$.

Ex. 2a $i = prt$ (simple interest); $r = 0.09$, $t = 1.5$ & $i = \$945$

Solution:

Substitute the given values and solve:

$i = prt$

$945 = p(0.09)(1.5)$ (multiply)

$\frac{945}{0.135} = \frac{0.135p}{0.135}$ (divide by the number in front of p)

$7000 = p$ So, p = \$7000

Ex. 2b $i = prt$ for p

Solution:

Follow the steps we performed in Ex. 2a after substituting:

$i = prt$ (multiply)

$\frac{i}{(rt)} = \frac{(rt)p}{(rt)}$ (divide by the number in front of p)

$\frac{i}{rt} = p$ So, $p = \frac{i}{rt}$

Ex. 3a $T_e = K_e + P_e$ (Total Energy); $P_e = 9152$ J and $T_e = 15612$ J.

Solution:

Substitute the given values and solve:

$T_e = K_e + P_e$

$15612 = K_e + 9152$ (subtract the constant term from both sides)

$$\begin{array}{l} 15612 = K_e + 9152 \\ \underline{-9152 = -9152} \\ 6460 = K_e \end{array}$$

So, $K_e = 6460$ J

Ex. 3b $T_e = K_e + P_e$ for K_e

Solution:

Follow the steps we performed in Ex. 3a after substituting:

$T_e = K_e + P_e$ (subtract the constant term from both sides)

$$\begin{array}{l} T_e = K_e + P_e \\ \underline{-P_e = -P_e} \\ T_e - P_e = K_e \end{array}$$

So, $K_e = T_e - P_e$

Ex. 4a $P = \frac{F}{A}$ (pressure); $A = 33\ m^2$ and $P = 202{,}000\ N/m^2$

Solution:

Substitute the given values and solve:

$P = \frac{F}{A}$

$202{,}000 = \frac{F}{33}$ (multiply both sides by the

$33(202{,}000) = \frac{F}{33}(33)$ number below F)

$6{,}666{,}000 = F$ So, $F = 6{,}666{,}000$ N

Ex. 4b $P = \frac{F}{A}$ for F.

Solution:

Follow the steps we performed in Ex. 4a after substituting:

$P = \frac{F}{A}$ (multiply both sides by the number below F)

$A(P) = \frac{F}{A}(A)$

$AP = F$ So, $F = AP$

Ex. 5 $V = \pi r^2 h$ for h (volume of the cylinder)

Solution:

Since h is multiplied by πr^2, we need to divide both sides by πr^2:

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

So, $h = \frac{V}{\pi r^2}$

Ex. 6 $x = a - y$ for a

Solution:

Add y to both sides:

$$\begin{aligned} x &= a - y \\ + y &= + y \\ \hline x + y &= a \end{aligned}$$

So, $a = x + y$

Ex. 7 $R = \frac{V}{I}$ for V (Ohm's Law)

Solution:

Multiply both sides by I to solve for V:

$R = \frac{V}{I}$

$I(R) = I\left(\frac{V}{I}\right)$

$IR = V$ So, $V = IR$

Ex. 8 $R_s = R_1 + R_2 + R_3$ for R_1 (resistors in series)

Solution:

Isolate the term with R_1 by itself and solve:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \quad \text{(subtract } R_2 + R_3 \text{ from both sides)} \\ - R_2 - R_3 &= \quad - R_2 - R_3 \\ \hline R_s - R_2 - R_3 &= R_1 \end{aligned}$$

So, $R_1 = R_s - R_2 - R_3$

Ex. 9 $A = P(1 + rt)$ for r

Solution:

This formula is more complicated then the others we have solved so far. To see how to do this problem, let us pretend for the moment that A = 7, P = 5, and t = 3 and see how we would solve that equation:

$A = P(1 + rt)$ (plug in the numbers)

$7 = 5(1 + r(3))$ (distribute the 5)

$7 = 5 + 15r$ (subtract 5 from both sides)

$\underline{-5 = -5}$

$2 = 15r$

$\frac{2}{15} = \frac{15r}{15}$ (divide both sides by 15)

$r = \frac{2}{15}$

Now, let us follow the same steps to solve $A = P(1 + rt)$ for r:

$A = P(1 + \mathbf{r}t)$ (distribute the P)

$A = P + (Pt)\mathbf{r}$ (subtract P from both sides)

$\underline{-P = -P}$

$A - P = (Pt)\mathbf{r}$

$\frac{A - P}{Pt} = \frac{(Pt)\mathbf{r}}{Pt}$ (divide both sides by Pt)

$r = \frac{A - P}{Pt}$

Ex. 10 Solve $7x + 5y = 15$ for y

Solution:

$7x + 5\mathbf{y} = 15$ (subtract 7x from both sides)

$\underline{-7x \quad = -7x}$

$5\mathbf{y} = -7x + 15$

$\frac{5\mathbf{y}}{5} = \frac{-7x + 15}{5}$ (divide both sides by 5)

$y = \frac{-7x+15}{5}$ or $y = -\frac{7}{5}x + 3$

Note, when dividing an expression by a number, we must divide each term by that number. Thus, $\frac{-7x+15}{5} \neq -7x + 3$ since we must also divide $-7x$ by 5.

Ex. 11 Solve $F = \frac{9}{5}C + 32$ for C

Solution:

$F = \frac{9}{5}\mathbf{C} + 32$ (multiply both sides by 5 to clear fractions)

$5(F) = 5\left(\frac{9}{5}\mathbf{C} + 32\right)$ (distribute)

$5F = \frac{5}{1}\left(\frac{9}{5}\mathbf{C}\right) + 5(32)$ (reduce)

$5F = \frac{1}{1}\left(\frac{9}{1}\mathbf{C}\right) + 5(32)$ (multiply)

$$\begin{array}{r} 5F = 9\mathbf{C} + 160 \\ \underline{-160 = \quad -160} \\ 5F - 160 = 9\mathbf{C} \end{array}$$ (subtract 160 from both sides)

$\frac{5F - 160}{9} = \frac{9\mathbf{C}}{9}$ (divide both sides by 9)

$C = \frac{5F-160}{9}$ or $C = \frac{5(F-32)}{9}$ (since 160 ÷ 5 = 32)

or $C = \frac{5}{9}(F - 32)$

Ex. 12 Solve P = 2L + 2w for w

Solution:

This is a formula for the perimeter of a rectangle.

$$\begin{array}{r} P = 2L + 2\mathbf{w} \\ \underline{-2L = -2L} \\ P - 2L = 2\mathbf{w} \end{array}$$ (subtract 2L from both sides)

$\frac{P - 2L}{2} = \frac{2\mathbf{w}}{2}$ (divide both sides by 2)

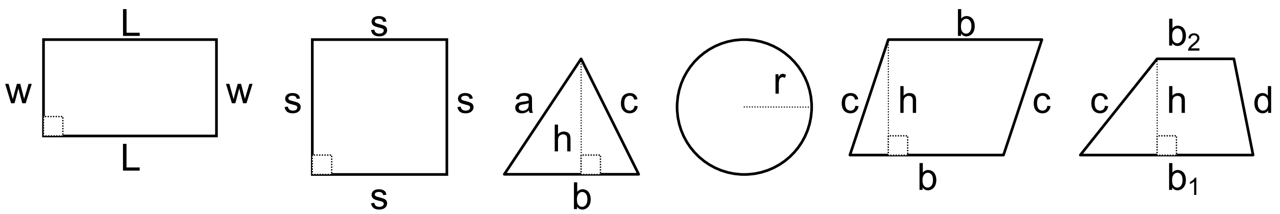
$w = \frac{P-2L}{2}$ or $w = \frac{P}{2} - L$

Concept #2Geometry Applications

Before we start working on geometry applications, we need to review some formulas from geometry. When performing computations involving π, we will usually use either 3.14, $\frac{22}{7}$, or the π value from a scientific calculator.

The **perimeter** (P) (or **circumference** (C) for a circle) of a geometric figure is the distance around the outside of the figure. The **area** (A) of a geometric figure is the number of square units that can be enclosed inside of a figure. Here are some common figures and formulas:

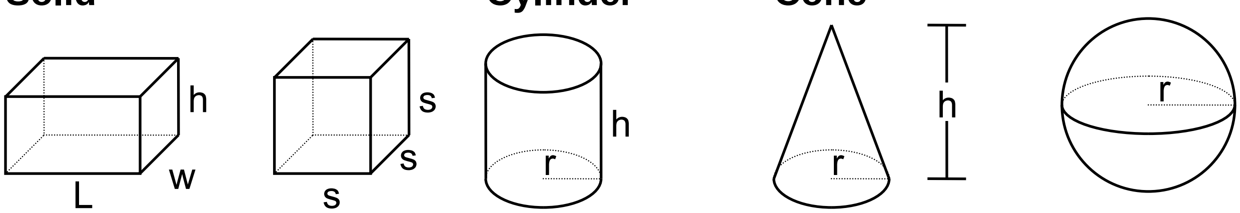
Rectangle	**Square**	**Triangle**	**Circle**	**Parallelogram**	**Trapezoid**



$P = 2L + 2w$	$P = 4s$	$P = a + b + c$	$C = 2\pi r$	$P = 2b + 2c$	$P = b_1 + c + b_2 + d$
$A = Lw$	$A = s^2$	$A = \frac{1}{2}bh$	$A = \pi r^2$	$A = bh$	$A = \frac{1}{2}(b_1 + b_2)h$

The **volume** (V) of a solid is the number of cubic units that can be enclosed within a solid. Here are some common figures and formulas:

Rectangular Solid	**Cube**	**Right Circular Cylinder**	**Right Circular Cone**	**Sphere**



$V = Lwh$	$V = s^3$	$V = \pi r^2 h$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

The sum of the measures of **Complementary Angles** is 90˚.
The sum of the measures of **Supplementary Angles** is 180˚.
The sum of the measures of the angles of any triangle is 180˚.

<u>Solve the following:</u>

Ex. 13 If circumference of a circle is 158.256 m, find the area ($\pi \approx 3.14$).

<u>Solution:</u>

$C = 2\pi r$ (plug in the values)
$158.256 = 2(3.14)r$ (multiply)
$158.256 = 6.28r$

$\frac{158.256}{6.28} = \frac{6.28r}{6.28}$ (divide by 6.28)

$25.2 \text{ m} = r$

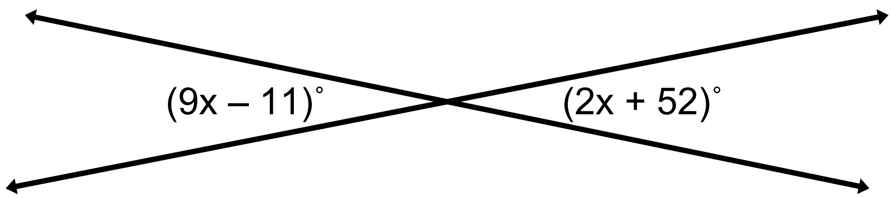
Now, we can find the area.

$A = \pi r^2$ (replace r by 25.2)
$A = (3.14)(25.2)^2$ (exponents)
$A = (3.14)(635.04)$ (multiply)

$A = 1994.0256 \text{ m}^2$

Thus, the area is 1994.0256 m^2.

Ex. 14 Two angles are called vertical angles if they are a pair of non-adjacent angles in the intersection of two lines. Use the fact that vertical angles are equal to find the value of x and the measures of the angles in the following diagram.



Solution:

Since the angles are equal, set the two expressions equal and solve. Then find the measure of the angles:

$9x - 11 = 2x + 52$ (subtract 2x from both sides)

$\underline{-2x \quad = -2x}$

$7x - 11 = 52$ (add 11 to both sides)

$\underline{+11 = +11}$

$7x = 63$

$\frac{7x}{7} = \frac{63}{7}$ (divide by 7)

$x = 9$

Thus, $9x - 11 = 9(9) - 11 = 81 - 11 = 70$

and $2x + 52 = 2(9) + 52 = 18 + 52 = 70$

So, x = 9 and the angles are 70°.

Ex. 15 A rectangular garden has a perimeter of 144 feet. If the length is twenty-four feet less than twice the width, find the dimensions of the garden.

Solution:

Since the length is described in terms of the width, we want to let the variable w represent the width and write the length in terms of w:

Let w = the width of the rectangle

Since "length is twenty-four feet less than twice the width," then

2w – 24 = the length of the rectangle

The formula for the perimeter of a rectangle is:

$P = 2L + 2w$

Replace P by 144 feet and L by (2w – 24):

$144 = 2(2w - 24) + 2w$ (distribute)

$144 = \mathbf{2}(2w) - \mathbf{2}(24) + 2w$ (multiply)

$144 = 4w - 48 + 2w$ (combine like terms)

$144 = 6w - 48$

Now, solve for w:

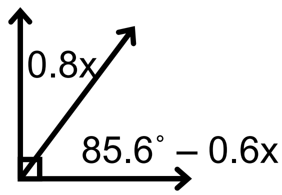
$$144 = 6w - 48 \qquad \text{(add 48 to both sides)}$$
$$\underline{+48 = \quad +48}$$
$$\frac{192}{6} = \frac{6w}{6} \qquad \text{(divide by 6)}$$
$$32 = w$$

Plug 32 into the expression $2w - 24$ to find the length:

$$2(32) - 24 = 64 - 24 = 40.$$

So, the length of the rectangle is 40 feet and the width is 32 feet.

Ex. 16 Given that the two angles in the diagram below are complementary angles, find the measure of each angle.



<u>Solution:</u>

The two angles are $0.8x$ and $85.6^\circ - 0.6x$. Their sum is 90°, so we write:

$$0.8x + 85.6^\circ - 0.6x = 90^\circ \qquad \text{(combine like terms)}$$
$$0.2x + 85.6^\circ = 90^\circ \qquad \text{(subtract } 85.6^\circ \text{ from both sides)}$$
$$\underline{-85.6^\circ = -85.6^\circ}$$
$$\frac{0.2x}{0.2} = \frac{4.4^\circ}{0.2} \qquad \text{(divide by 0.2)}$$
$$x = 22^\circ$$

To find the angles, replace x by 22° in both $0.8x$ and $85.6^\circ - 0.6x$:

$$0.8x = 0.8(22) = 17.6^\circ \quad \text{and}$$
$$85.6^\circ - 0.6x = 85.6^\circ - 0.6(22^\circ) = 85.6^\circ - 13.2^\circ = 72.4^\circ.$$

So, the angles are 17.6° and 72.4°.

Ex. 17 Each of the two equal angles of an isosceles triangle are six degrees less than triple the third angle. Find the angles.

<u>Solution:</u>

Let T = the measure of the third angle.

Now, write each of the other two angles in terms of T:

"each of the two angles are six less than triple the third angle:"

$3T - 6^\circ$ = the measure of each of the other two angles.

The sum of the angles of a triangle is 180°:

$$\text{First angle} + \text{second angle} + \text{third angle} = 180^\circ$$
$$(3T - 6^\circ) + (3T - 6^\circ) + T = 180^\circ$$
$$3T - 6^\circ + 3T - 6^\circ + T = 180^\circ \qquad \text{(combine like terms)}$$
$$7T - 12^\circ = 180^\circ$$

$7T - 12^\circ = 180^\circ$ (add 12° from both sides)

$\underline{\ \ +12^\circ = +12^\circ}$

$\frac{7T}{7} = \frac{192^\circ}{7}$ (divide by 7)

$T = \frac{192}{7}^\circ = 27\frac{3}{7}^\circ$

$3T - 6^\circ = 3\left(\frac{192}{7}^\circ\right) - 6^\circ = \frac{576}{7}^\circ - 6^\circ = 82\frac{2}{7}^\circ - 6^\circ = 76\frac{2}{7}^\circ$

So, the angles are $76\frac{2}{7}^\circ$, $76\frac{2}{7}^\circ$, and $27\frac{3}{7}^\circ$.

Ex. 18 Find the measure of an angle whose supplement is six degrees more than two-thirds times the measure of the angle.

Solution:

Recall that two angles are called supplementary angles if the sum of their measures is 180˚. In other words, an angle plus its supplement is 180˚.

Let A = the measure of the angle

Then $\frac{2}{3}A + 6^\circ$ = the measure of the supplement

Since, Angle + Supplement = 180˚, then

$A + \frac{2}{3}A + 6^\circ = 180^\circ$ (clear fractions)

$3(A) + \frac{3}{1}\left(\frac{2}{3}A\right) + 3(6^\circ) = 3(180^\circ)$ (simplify)

$3A + 2A + 18^\circ = 540^\circ$ (combine like terms)

$5A + 18^\circ = 540^\circ$ (subtract 18° from both sides)

$\underline{\ \ -18^\circ = -18^\circ}$

$\frac{5A}{5} = \frac{522^\circ}{5}$ (divide by 5)

$A = 104.4^\circ$

So, the angle is 104.4˚.

Ex. 19 Using $\pi \approx 3.14$, the volume of a cylinder is approximately 361.571 ft^3. If the radius is 7 feet, find the height.

Solution:

$V = \pi r^2 h$ (plug in the numbers)

$(361.571) = (3.14)(7)^2 h$ (#2-exponents)

$361.571 = 3.14(49)h$ (#3-multiply)

$\frac{361.571}{153.86} = \frac{153.86h}{153.86}$ (divide both sides by 153.86)

$2.35 = h$

So, the height is 2.35 ft.