Sect 4.3 – Multiplication of Decimals

Objective a: Understanding how to multiply decimals.

In the first section of this chapter, we saw how to write decimals as fractions and vice-versa. Although we will not normally do a multiplication of decimals problems this way, we can convert the decimals into fractions and multiply the fractions. We can then rewrite our answer as a decimal.

Simplify:

Ex. 1 $0.03 \bullet 0.007$

Solution:

Since 3 is in the hundredths place, we can write 0.03 as $\frac{3}{100}$.

Since 7 is in the thousandths place, we can write 0.007 as $\frac{7}{1000}$.

So, $0.03 \bullet 0.007 = \frac{3}{100} \bullet \frac{7}{1000} = \frac{21}{100{,}000}$.

Since 100,000 has five zeros, we need to have five places to the right of the decimal point. So, $\frac{21}{100{,}000} = 0.00021$.

Notice that 0.03 had two digits to the write of the decimal point and 0.007 had three digits to the right of the decimal point. The answer, 0.00021 had 2 + 3 = 5 digits to the right of the decimal point. Recall that when we multiply fractions, we not only multiply the numerators, but also multiply the denominators. Because of this fact, when we multiply decimals, we count the total number of digits to the right of the decimal point of the decimal. This will give us the total number of digits we need to the right of the decimal point in our answer. So, we multiply the decimals as if we are multiplying whole numbers. Then, we make sure that the answer has the same number of digits to the right of the decimal point as the total.

Simplify and then estimate the answer by front-end rounding:

Ex. 2 $7.5 \bullet 2.07$

Solution:

Multiply as if the numbers were whole numbers:

$$\begin{array}{r} {\scriptstyle 3} \\ 7.5 \\ \times\ 2.07 \\ \hline 525 \end{array} \quad \Rightarrow \quad \begin{array}{r} {\scriptstyle 1} \\ 7.5 \\ \times\ 2.07 \\ \hline 525 \\ +\ 15000 \\ \hline \end{array}$$

Since 7.5 has one digit to the right of the decimal point and 2.07 has two digits to the right of the decimal

$$\Rightarrow \quad \begin{array}{r} 7.5 \\ \times\ 2.07 \\ \hline 525 \\ +\ 15000 \\ \hline 15525 \end{array} \qquad \Rightarrow \quad \begin{array}{r} 7.5 \\ \times\ 2.07 \\ \hline 525 \\ +\ 15000 \\ \hline 15.525 \end{array}$$

our answer will have 1 + 2 = 3 places to the right of the decimal point.

So, 7.5 •2.07 = 15.525. *Estimate*: 8•2 = 16.*

* - The estimation technique we are doing is called front-end rounding. To estimate by front-end rounding, we round each number to its largest place value (left-most digit).

Ex. 3 The product of 1.03 and 0.006.

Solution:

Multiply as if the numbers were whole numbers:

$$\begin{array}{r} \textbf{\textit{1}} \\ 1.03 \\ \times\ 0.006 \\ \hline 618 \end{array}$$

Since 1.03 has two digits to the right of the decimal point and 0.006 has three digits to the right of the decimal point, then the answer will need five digits to the right of the decimal point. Since our answer only has three digits, we will need to insert two zeros to the left of 618. Thus, our answer is: 0.00618. *Estimate: 1•0.006 = 0.006.*

Ex. 4 7.82 times 0.435.

Solution:

Multiply as if the numbers were whole numbers:

$$\begin{array}{r} \textbf{\textit{4 1}} \\ 7.82 \\ \times\ 0.435 \\ \hline 3910 \end{array} \qquad \Rightarrow \quad \begin{array}{r} \textbf{\textit{2}} \\ 7.82 \\ \times\ 0.435 \\ \hline 3910 \\ 23460 \end{array}$$

$$\Rightarrow \quad \begin{array}{r} \textbf{\textit{3}} \\ 7.82 \\ \times\ 0.435 \\ \hline 3910 \\ 23460 \\ +\ 312800 \end{array} \qquad \Rightarrow \quad \begin{array}{r} 7.82 \\ \times\ 0.435 \\ \hline 3910 \\ 23460 \\ +\ 312800 \\ \hline 340170 \end{array}$$

We need a total of 2 + 3 = 5 digits to the right of the decimal point. So, we place the decimal point between the 3 and the 4. After the decimal point is placed, we can then drop the last 0:

3.40170 = 3.4017. *Estimate: 8•0.4 = 3.2*

Ex. 5 20.6 times 5.023.

Solution:

Multiply as if the numbers were whole numbers:

$$\begin{array}{r} {\scriptstyle 1} \\ 20.6 \\ \times\ 5.023 \\ \hline 618 \end{array} \quad \Rightarrow \quad \begin{array}{r} {\scriptstyle 1} \\ 20.6 \\ \times\ 5.023 \\ \hline 618 \\ 4120 \end{array}$$

$$\Rightarrow \begin{array}{r} {\scriptstyle 3} \\ 20.6 \\ \times\ 5.023 \\ \hline 618 \\ 4120 \\ +\ 1030000 \end{array} \quad \Rightarrow \quad \begin{array}{r} 20.6 \\ \times\ 5.023 \\ \hline 618 \\ 4120 \\ +\ 1030000 \\ \hline 1034738 \end{array}$$

We need a total of 1 + 3 = 4 digits to the right of the decimal point. So, we place the decimal point between the 3 and the 4:

103.4738. *Estimate: 20•5 = 100.*

Objective b: Multiplying by a power of 10 and by a power of 0.1

Simplify:

Ex. 6a 1.987•10

Solution:

First, multiply 1987 by 10. Then, the answer will need three digits to the right of the decimal point:

1987•10 = 19870 so 1.987•10 = 19.870 = 19.87.

Ex. 6b 1.987•100

Solution:

First, multiply 1987 by 100. Then, the answer will need three digits to the right of the decimal point:

1987•100 = 198700 so 1.987•100 = 198.700 = 198.7.

Ex. 6c 1.987•1000

Solution:

First, multiply 1987 by 1000. Then, the answer will need three digits to the right of the decimal point:

1987•1000 = 1987000 so 1.987•1000 = 1987.000 = 1987.

Notice the pattern here. Multiplying by 10 moved the decimal point one place to the right. Multiplying by 100 moved the decimal point two places to the right. The number of zeros after the one gives you the number of places the decimal point moves.

Ex. 6d 1.987•10,000,000

Solution:

Since 10 million has seven zeros, move the decimal point seven places to the right: 1.9870000 = 19,870,000

Ex. 7a 1.987•0.01

Solution:

Since 0.01 = 1/100, the decimal point will move two places to the **left**: 001.987 = 0.01987.

Ex. 7b 1.987•0.001

Solution:

Since 0.001 = 1/1000, the decimal point will move three places to the **left**: 0001.987 = 0.001987.

Ex. 8a 6.34×10^4

Solution:

Since 10^4 = 10,000 has four zeros, move the decimal point four places to the **right**: 6.3400 = 63,400.

Ex. 8b 8.26×0.000001

Solution:

Since 0.000001 = $\frac{1}{1000000}$, the decimal point will move six places to the **left**: 0000008.26 = 0.00000826.

Ex. 8c Write 78.4 million in standard notation:

Solution:

78.4 million = 78.4 • 1,000,000. We need to move the decimal point six places to the right: 78400000 = 78,400,000

Objective c: Applications

Solve the following:

Ex. 9 According to the EPA (www.epa.gov), each gallon used by a motor vehicle produces approximately 19.4 pounds of carbon dioxide. If Juan used 14.7 gallons of gas to drive 516 miles to El Paso, how many pounds (to the nearest pound) of carbon dioxide did his car produce?

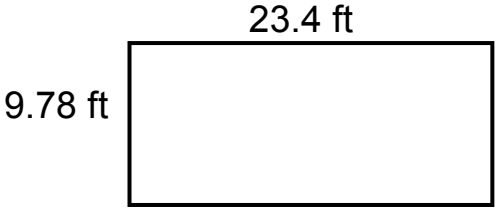
Solution:

We need to take the number of pounds of carbon dioxide produced per gallon and multiply it by the number of gallons used:

$$\begin{array}{r} {\scriptstyle 6\,2} \\ 19.4 \\ \times\ 14.\mathbf{7} \\ \hline 1358 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 3\,1} \\ 19.4 \\ \times\ 1\mathbf{4}.7 \\ \hline 1358 \\ 7760 \end{array} \Rightarrow \begin{array}{r} 19.4 \\ \times\ 1\mathbf{4}.7 \\ \hline 1358 \\ 7760 \\ 19400 \end{array} \Rightarrow \begin{array}{r} 19.4 \\ \times\ 1\mathbf{4}.7 \\ \hline 1358 \\ 7760 \\ +\ 19400 \\ \hline 285.18 \end{array}$$

The car produced approximately 285 pounds of carbon dioxide.

Ex. 10 Find the area of the following:



Solution:

The area equals the length times width:

$$\begin{array}{r} {\scriptstyle 2\,3} \\ 23.4 \\ \times\ 9.78 \\ \hline 1872 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 2\,2} \\ 23.4 \\ \times\ 9.78 \\ \hline 1872 \\ 16380 \end{array} \Rightarrow \begin{array}{r} {\scriptstyle 3\,3} \\ 23.4 \\ \times\ 9.78 \\ \hline 1872 \\ 16380 \\ 210600 \end{array} \Rightarrow \begin{array}{r} 23.4 \\ \times\ 9.78 \\ \hline 1872 \\ 16380 \\ +\ 210600 \\ \hline 228.852 \end{array}$$

The area is 228.852 ft^2.