Sect 5.2 - More Properties of Exponents

Concept #1 Power Rule for Exponents

Let us consider the next example:

Simplify the following:

Ex. 1 $(x^4)^5$

Solution:

Write in expanded form and use the product rule
$(x^4)^5 = (x^4)\bullet(x^4)\bullet(x^4)\bullet(x^4)\bullet(x^4) = x^{4+4+4+4+4} = x^{20}$. Notice that this is the same multiplying the exponents 4 and 5. This introduces our next property:

The Power Rule for Exponents

If m and n are positive integers and a is a non-zero real number, then $\mathbf{(a^m)^n = a^{mn}}$ (Property #3). In words, when raising a power by a power, multiply the exponents and keep the same base.

Simplify the following:

Ex. 2a $(x^5)^8$ Ex. 2b $(y^7)^{13}$

Ex. 2c $(5^{18})^{10}$ Ex. 2d $(t^3)^7\bullet(t^4)^6$

Solution:

a) $(x^5)^8 = x^{5\cdot 8} = x^{40}$.

b) $(y^7)^{13} = y^{7\cdot 13} = y^{91}$.

c) $(5^{18})^{10} = 5^{18\cdot 10} = 5^{180}$.

d) $(t^3)^7\bullet(t^4)^6 = t^{3\cdot 7}\bullet t^{4\cdot 6} = t^{21}\bullet t^{24} = t^{21+24} = t^{45}$.

Ex. 3a 3^4 Ex. 3b $x^3\bullet x^4$ Ex. 3c $(x^3)^4$

Solution:

It is important not to confuse these different situations:

a) The base is 3 and the exponent is 4, so $3^4 = 3\bullet 3\bullet 3\bullet 3 = 81$. Notice 3 is not even an exponent.

b) We are multiplying powers with the same base, so $x^3\bullet x^4 = x^{3+4} = x^7$. (property #1)

c) We are raising a power by a power, so $(x^3)^4 = x^{3\cdot 4} = x^{12}$. (property #3)

Concept #2 The Power of a Product and Quotient Rules

Let us examine the next set of examples:

Simplify the following:

Ex. 4a $(ab)^3$

Solution:

a) Write in expanded form and simplify:

$(ab)^3 = (ab)(ab)(ab)$

$= a^{1+1+1}b^{1+1+1} = a^3b^3$.

Ex. 4b $\left(\frac{a}{b}\right)^4$

Solution:

b) Write in expanded form and simplify:

$\left(\frac{a}{b}\right)^4 = \frac{a}{b}\bullet\frac{a}{b}\bullet\frac{a}{b}\bullet\frac{a}{b}$

$= \frac{a^{1+1+1+1}}{b^{1+1+1+1}} = \frac{a^4}{b^4}$.

Notice that when we raise a product and/or quotient to a power, we raise each factor to that power. This shows our next two properties:

Power of a Product and/or Quotient Rules

If n is a positive integer, then

$(xyz)^n = x^ny^nz^n$ (Property #4)

and if $b \neq 0$, then

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (Property #5)

In words, when we raise a product and/or quotient to a power, we raise each factor to that power.

Simplify the following:

Ex. 5a $(4x^2y)^3$

Ex. 5b $\left(\frac{5x^2}{y^3}\right)^4$

Solution:

a) Using the Power of a Product Rule, we raise the 4, x^2, and y to the third power:

$(4x^2y)^3$ (#4 power of a product rule)

$= (4)^3(x^2)^3(y)^3$ (#3 power rule)

$= (4)^3x^6y^3$ (simplify)

$= 64\,x^6y^3$

b) Using the Power of a Quotient Rule, we raise the numerator and the denominator to the fourth power:

$\left(\frac{5x^2}{y^3}\right)^4$ (#5 power of a quotient rule)

$= \dfrac{\left(5x^2\right)^4}{\left(y^3\right)^4}$ (#4 power of a product rule in the numerator)

$= \dfrac{(5)^4(x^2)^4}{(y^3)^4}$ (#3 power rule)

$= \dfrac{(5)^4 x^8}{y^{12}}$ (simplify)

$= \dfrac{625x^8}{y^{12}}$

Concept #3 Simplify Expressions with exponents

Simplify the following:

Ex. 6a $3\bullet(-2x^2y^3)^4$ Ex. 6b $\left(\dfrac{-7x^3y^4}{6a^2b^5}\right)^3$

Solution:

a) $3\bullet(-2x^2y^3)^4 = 3\bullet(-2)^4(x^2)^4(y^3)^4 = 3\bullet(16x^{2\cdot 4}y^{3\cdot 4}) = 48x^8y^{12}$.

b) $\left(\dfrac{-7x^3y^4}{6a^2b^5}\right)^3 = \dfrac{(-7)^3(x^3)^3(y^4)^3}{(6)^3(a^2)^3(b^5)^3} = \dfrac{-343x^9y^{12}}{216a^6b^{15}} = -\dfrac{343x^9y^{12}}{216a^6b^{15}}$.

Ex. 7a $(-4px^2y^4)^2(-p^3xy^4)^3$ Ex. 7b $\left(\dfrac{-4t^5t^4v^3}{6t^2t^5v}\right)^3$

Solution:

Think of your order of operations

1) Parenthesis - nothing to simplify inside
2) Exponents - $1^{st} \rightarrow$ power of a product/quotient rule
   $2^{nd} \rightarrow$ power rule
3) Multiplication and Division from left to right - product rule

They will apply in algebra as well.

a) $(-4px^2y^4)^2(-p^3xy^4)^3$ (#4 power of a product rule)

$= (-4)^2(p)^2(x^2)^2(y^4)^2(-1)^3(p^3)^3(x)^3(y^4)^3$ (#3 power rule)

$= 16p^2x^4y^8(-p^9x^3y^{12}) = -16p^{11}x^7y^{20}$. (#1 product rule)

b) $\left(\frac{-4t^5t^4v^3}{6t^2t^5v}\right)^3$ (parenthesis - #1 product rule)

$= \left(\frac{-4t^9v^3}{6t^7v}\right)^3$ (parenthesis - #2 quotient rule)

$= \left(-\frac{2}{3}t^2v^2\right)^3$ (#4 power of a product rule)

$= \left(-\frac{2}{3}\right)^3(t^2)^3(v^2)^3$ (#3 power rule)

$= -\frac{8}{27}t^6v^6$

Ex. 8 $\frac{(x^2y)^4(x^3y^5)^2}{(x^4z)^3}$

Solution:

$\frac{(x^2y)^4(x^3y^5)^2}{(x^4z)^3}$ (#4 power of a product rule)

$= \frac{(x^2)^4(y)^4(x^3)^2(y^5)^2}{(x^4)^3(z)^3}$ (#3 power rule)

$= \frac{x^8y^4x^6y^{10}}{x^{12}z^3}$ (commutative property of multiplication)

$= \frac{x^8x^6y^{10}y^4}{x^{12}z^3}$ (#1 product rule in the numerator)

$= \boxed{\quad}$ (#2 quotient rule)

$= \frac{x^2y^{14}}{z^3}$

The rules for exponents work for the operations of multiplication and division only. They do not work for addition or subtraction (i.e., $(x + y)^2 \neq x^2 + y^2$ and $(x - y)^3 \neq x^3 - y^3$).

<u>Show that the following for x = 5 and y = 3:</u>

Ex. 9a $(x + y)^2 \neq x^2 + y^2$

Solution:

Replace x by 5 and y by 3:

$(x + y)^2 \neq x^2 + y^2$

$((5) + (3))^2 \neq (5)^2 + (3)^2$

$(8)^2 \neq 25 + 9$

$64 \neq 34$

Ex. 9b $(x - y)^2 \neq x^2 - y^2$

Solution:

Replace x by 5 and y by 3:

$(x - y)^2 \neq x^2 - y^2$

$((5) - (3))^2 \neq (5)^2 - (3)^2$

$(2)^2 \neq 25 - 9$

$4 \neq 16$