Sect 5.5 - Addition and Subtraction of Polynomials

Concept #1 Introduction to Polynomials

Before we begin discussing polynomials, let's review some items from chapter 1 with the following example:

Complete the following chart for the expression given below:

Ex. 1 $-3.6x^6 - 7x^4 + 8.9x^3 - \frac{3}{2}x - 9$

Term	Coefficient
-9	
	-7
$8.9x^3$	
	$-\frac{3}{2}$
$-3.6x^6$	

Solution:
Since -9 is a constant term, its coefficient is -9.
The term with the coefficient of -7 is $-7x^4$.
The coefficient of $8.9x^3$ is 8.9.
The term with the coefficient of $-\frac{3}{2}$ is $-\frac{3}{2}x$.
The coefficient of $-3.6x^6$ is -3.6. The chart should look like:

Term	Coefficient
-9	$\mathbf{-9}$
$\mathbf{-7x^4}$	-7
$8.9x^3$	**8.9**
$\mathbf{-\frac{3}{2}x}$	$-\frac{3}{2}$
$-3.6x^6$	$\mathbf{-3.6}$

A **Polynomial** is an algebraic expression where the powers of the variables are whole numbers (0, 1, 2, 3, ...). There can be no variables in the denominator. We can sub-classify polynomials by the number of terms that they have.

A **Monomial** is a polynomial that has one term.
A **Binomial** is a polynomial that has two terms.
A **Trinomial** is a polynomial that has three terms.

Determine if the following is a polynomial. If so, state whether it is a monomial, binomial, or trinomial:

Ex. 2a $x^5 - \frac{8}{9}x^4y + \frac{5}{6}xy^2$

Ex. 2b $x^{2/3} - 8y - 11$

Ex. 2c $-\frac{7x^3y}{3}$

Ex. 2d $\frac{35x^2y^3}{11z^2}$

Ex. 2e $4.95x^2 - 9$

Solution:

a) The powers of the variables are 5, 4, 1, 1, and 2 which are all whole numbers. So, the expression is a polynomial. Since it has three terms, it is a trinomial.

b) Since 2/3 is a not a whole number, then the expression is not a polynomial.

c) The powers of the variables are 3, and 1 which are all whole numbers. So, the expression is a polynomial. Since it has one term, it is a monomial.

d) Since z^2 is in the denominator, then the expression is not a polynomial.

e) The powers of the variable is 2 which is a whole number. So, the expression is a polynomial. Since it has two terms, it is a binomial.

The **Degree of a Term** or monomial is the sum of the powers of the variables in that term.

Find the degree of the each term:

Ex. 3a $-7x^5$

Ex. 3b $6x^2y^3z$

Ex. 3c 1.4

Ex. 3d 7x

Ex. 3e $9^2x^3y^4$

Ex. 3f $\frac{2}{3}$

Ex. 3g $-x$

Solution:

a) Since the power is 5, then the degree of the term is 5.

b) Since the powers are 2, 3, and 1, and the sum is: 2 + 3 + 1 = 6, then the degree of the term is 6.

c) Since $1.4 = 1.4x^0$, then the degree of the term is 0. In fact, the degree of a constant term is always zero.

d) Since the power of x is 1, then the degree of the term is 1.

e) Since $9^2x^3y^4 = 81x^3y^4$ and 3 + 4 = 7, then the degree of the term is 7.

f) $\frac{2}{3}$ is a constant term, so the degree of the term is 0.

g) Since the power of x is 1, then the degree of the term is 1.

The **Degree of a Polynomial** is the highest degree of the terms of the polynomial.

Find the degree of each polynomial:

Ex. 4a $1.5x^2 – 0.7x^3y^3 + 0.3x$

Ex. 4b $a^3b – 6 + 2a^2b^3 – 2^9$

Ex. 4c $9x – \frac{1}{2}$

Ex. 4d $12L^7m + 10L^4m^3 – Lm$

Solution:

a) The degree of the terms are 2, 6, and 1 respectively. Since 6 is the highest, then the degree of the polynomial is 6.

b) The degree of the terms are 4, 0, 5, and 0 respectively. Since 5 is the highest, then the degree of the polynomial is 5.

c) The degree of the terms are 1 and 0 respectively. Since 1 is the highest, then the degree of the polynomial is 1.

d) The degree of the terms are 8, 7, and 2 respectively. Since 8 is the highest, then the degree of the polynomial is 8.

Concept #2 Applications of Polynomials

Evaluating polynomials works the same as evaluating expressions; we replace the variables by the numbers given and follow the order of operations to simplify.

Solve the following:

Ex. 5 The number of new homes built per month for the months of July of 2004 through July of 2005 can be modeled by:

$$h = \frac{2}{3}t^3 - 13.55245t^2 + 92.016127t + 1036.51049$$

where h is the number of new homes built (in thousands of homes) in a month and t is the time in months after June of 2004.
Find the number of new homes built in a) August of 2004 and b) January of 2005. Round to the nearest tenth of a thousand.

Solution:

a) Since August of 2004 is two months after June of 2004, then t = 2.
Replace t by 2 and simplify:

$$h = \frac{2}{3}t^3 - 13.55245t^2 + 92.016127t + 1036.51049$$
$$= \frac{2}{3}(2)^3 - 13.55245(2)^2 + 92.016127(2) + 1036.51049$$
$$= \frac{2}{3}(8) - 13.55245(4) + 92.016127(2) + 1036.51049$$
$$= \frac{16}{3} - 54.2098 + 184.032254 + 1036.51049$$
$$= 1171.66627733... \approx 1171.7 \text{ thousand new homes}$$

In August of 2004, there were approximately 1171.7 thousand (1,171,700) new homes built.

b) Since January of 2005 is seven months after June of 2004, then t = 7.
Replace t by 7 and simplify:

$$h = \frac{2}{3}t^3 - 13.55245t^2 + 92.016127t + 1036.51049$$
$$= \frac{2}{3}(7)^3 - 13.55245(7)^2 + 92.016127(7) + 1036.51049$$
$$= \frac{2}{3}(343) - 13.55245(49) + 92.016127(7) + 1036.51049$$
$$= \frac{686}{3} - 664.07005 + 644.112889 + 1036.51049$$
$$= 1245.21999567... \approx 1245.2 \text{ thousand new homes}$$

In January of 2005, there were approximately 1245.2 thousand (1,245,200) new homes built.

Ex. 6 The total revenue (in hundreds of dollars) that a manufacturer of computer system receives is given by $R = -0.004c^2 + 3c$ where c is the number of computer systems produced.

a) Find the total revenue if 375 computer systems are produced.

b) Does increasing the number of computer systems being produced to 400 increase the revenue?

Solution:

a) Since 375 computer systems are produced, we can find the total revenue by finding plugging c = 375:
$R = -0.004(375)^2 + 3(375)$
$= -0.004(140625) + 3(375)$
$= -562.5 + 1125$
= 562.5 hundreds of dollars or $56,250.

b) If 400 computer systems are produced, we can find the total revenue by plugging in c = 400.
$R = -0.004(400)^2 + 3(400)$
$= -0.004(160000) + 3(400)$
$= -640 + 1200$
= 560 hundreds of dollars or $56,000 which is less than $56,250. No, the revenue does not increase.

Concept #3 Addition of Polynomials

Adding and subtract polynomials works exact the same way as adding and subtracting algebraic expressions. We first distribute if necessary and then combine like terms.

Simplify the following:

Ex. 7a $1.8x - 9.5x$ Ex. 7b $(3x)^4 - (2x^2)^2$

Solution:

a) Combine like terms: $1.8x - 9.5x = -7.7x$.

b) First apply the exponents and then combine like terms:
$(3x)^4 - (2x^2)^2 = (3)^4(x)^4 - (2)^2(x^2)^2 = 81x^4 - 4x^4 = 77x^4$.

Ex. 8 $(3.5x^2 - 8.4x + 3.2) + (-9.2x^2 - 2.3x - 9.1)$

Solution:

Simply combine like terms:
$(3.5x^2 - 8.4x + 3.2) + (-9.2x^2 - 2.3x - 9.1)$
$= 3.5x^2 - 9.2x^2 - 8.4x - 2.3x + 3.2 - 9.1$
$= -5.7x^2 - 10.7x - 5.9$

Ex. 9 $3(9x - 5y) + 2(-4x - 3z) + 3(2y - z)$

Solution:

First, distribute:

$3(9x - 5y) + 2(-4x - 3z) + 3(2y - z)$

$= 27x - 15y - 8x - 6z + 6y - 3z$ Now, combine like terms:

$= 27x - 8x - 15y + 6y - 6z - 3z$

$= 19x - 9y - 9z$

Ex. 10 $\left(-\frac{4}{3}x + \frac{2}{3}xy\right) + \left(-\frac{5}{6}x + \frac{9}{7}xy\right)$

Solution:

Group like terms:

$\left(-\frac{4}{3}x + \frac{2}{3}xy\right) + \left(-\frac{5}{6}x + \frac{9}{7}xy\right)$

$= -\frac{4}{3}x - \frac{5}{6}x + \frac{2}{3}xy + \frac{9}{7}xy$ (LCD's are 6 and 21 respectively)

$= -\frac{8}{6}x - \frac{5}{6}x + \frac{14}{21}xy + \frac{27}{21}xy$

$= -\frac{13}{6}x + \frac{41}{21}xy$

Concept #4 Subtraction of Polynomials

Recall that the opposite of a number a is – a. This can also be applied to polynomials. The opposite of polynomial A is – A

Find the opposite of the following:

Ex. 11a $7y$

Ex. 11b $3xy - 6x + y$

Ex. 11c $\frac{10}{3}a^5 + 4a^4 - \frac{1}{6}a^3 + \frac{4}{21}a^2$

Solution:

a) $-(7y) = -7y$

b) $-(3xy - 6x + y) = -3xy + 6x - y$

c) $-\left(\frac{10}{3}a^5 + 4a^4 - \frac{1}{6}a^3 + \frac{4}{21}a^2\right) = -\frac{10}{3}a^5 - 4a^4 + \frac{1}{6}a^3 - \frac{4}{21}a^2$

To subtraction two polynomials, we add the opposite of the polynomial to the right of the operation and then combine like terms.

Ex. 12 $\left(\frac{1}{6}a^2 + \frac{4}{3}ab - \frac{5}{4}b^2\right) - \left(\frac{4}{3}a^2 - \frac{5}{9}ab + \frac{3}{8}b^2\right)$

Solution:

First, distribute the minus sign to each term in the second polynomial:

$\left(\frac{1}{6}a^2 + \frac{4}{3}ab - \frac{5}{4}b^2\right) - \left(\frac{4}{3}a^2 - \frac{5}{9}ab + \frac{3}{8}b^2\right)$

$= \frac{1}{6}a^2 + \frac{4}{3}ab - \frac{5}{4}b^2 - \frac{4}{3}a^2 + \frac{5}{9}ab - \frac{3}{8}b^2$

Now, combine like terms:

$= \frac{1}{6}a^2 - \frac{4}{3}a^2 + \frac{4}{3}ab + \frac{5}{9}ab - \frac{5}{4}b^2 - \frac{3}{8}b^2$

$= \frac{1}{6}a^2 - \frac{8}{6}a^2 + \frac{12}{9}ab + \frac{5}{9}ab - \frac{10}{8}b^2 - \frac{3}{8}b^2$

$= -\frac{7}{6}a^2 + \frac{17}{9}ab - \frac{13}{8}b^2.$

Ex. 13 $(7x - 8x^2 + 4) - (-3x^3 + 4x^2 - 9)$

Solution:

First, distribute the minus sign to each term in the second polynomial:

$(7x - 8x^2 + 4) - (-3x^3 + 4x^2 - 9)$

$= 7x - 8x^2 + 4 + 3x^3 - 4x^2 + 9$

Now, combine like terms:

$= 7x - 8x^2 - 4x^2 + 4 + 9 + 3x^3$

$= 7x - 12x^2 + 13 + 3x^3 = 3x^3 - 12x^2 + 7x + 13.$

Notice that we rewrote our answer so the term with the highest power comes first, the term with the second highest power comes next, and so on. This is called writing the polynomial **in order of descending powers**. Since the $3x^3$ is the term with the highest degree, it is called the **leading term**. Since 3 is the coefficient, it is called the **leading coefficient**.

In arithmetic, if we want to add 2567 and 3462, we usually write the addition vertically being careful to align the matching place values. We can do the same thing with polynomials except we will be lining up like terms. It is important that the polynomials are written in order of descending powers before we perform the operation. Let’s redo example #13 vertically.

Ex. 14 $(7x - 8x^2 + 4) - (-3x^3 + 4x^2 - 9)$

Solution:

First, rewrite the polynomials in order of descending powers:

$(7x - 8x^2 + 4) = -8x^2 + 7x + 4$

$(-3x^3 + 4x^2 - 9) = -3x^3 + 4x^2 - 9$

Next, we add the opposite of $-3x^3 + 4x^2 - 9$

$$\begin{array}{r} -8x^2 + 7x + 4 \\ \underline{+3x^3 - 4x^2 \quad\quad + 9} \\ 3x^3 - 12x^2 + 7x + 13 \end{array}$$

Ex. 15 $(8.2x^3 - 7.5x^2 + 1.3x - 6.8) - (4.8x^3 + 9.7x^2 - 8.2x - 7)$

Solution:

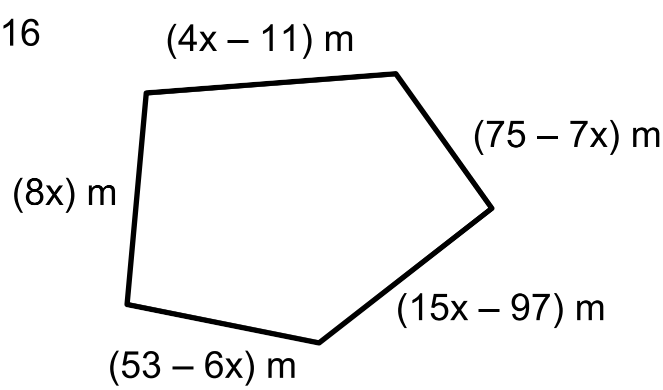
Change the signs of the polynomial on the bottom and combine like terms:

$$\begin{array}{r} 8.2x^3 - 7.5x^2 + 1.3x - 6.8 \\ \underline{-4.8x^3 - 9.7x^2 + 8.2x + 7} \\ 3.4x^3 - 17.2x^2 + 9.5x + 0.2 \end{array}$$

Concept #5 Polynomials and Applications to Geometry

Find the perimeter of the following:

Ex. 16



Solution:

To find the perimeter of an object, add up all the sides:

$(4x - 11) + (75 - 7x) + (15x - 97) + (53 - 6x) + (8x)$

$= 4x - 7x + 15x - 6x + 8x - 11 + 75 - 97 + 53$

$= 14x + 20$

Thus, the perimeter is $(14x + 20)$ m.