New Signal Processing Methods for Improved Performance in Single-Channel Feedforward Active Noise Control Systems

by

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To

ALLAMA GHULAM AHMAD PARWEZ

For his great research in Islam
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This thesis is a study of active noise control (ANC) systems with the view point of adaptive signal processing. The discussion is restricted to the single-channel feedforward ANC systems. A literature survey shows that the filtered-x least mean square (FxLMS) algorithm is the most popular adaptive algorithm for ANC systems, due mainly to its simplicity and robust performance. However, the convergence speed of the FxLMS algorithm is slow. Furthermore, its performance is degraded when there is a large measurement noise in the reference and error signals.

To solve these problems, this thesis proposes an adaptive filtering with averaging based algorithm for ANC systems. This algorithm uses a similar structure as that of the FxLMS algorithm based ANC systems. The proposed algorithm, filtered-x adaptive filtering with averaging (FxAFA) algorithm, uses averages of both data and correction terms to find the updated values of the tap weights of the ANC controller. The computer simulations show that the proposed algorithm gives somewhat faster convergence as compared with the FxLMS algorithm and achieves better performance in the presence of the measurement noise. The comparison with the FxRLS algorithm shows that the proposed FxAFA algorithm is a better choice for low computational complexity and a stable performance.

This thesis also addresses the issue of the secondary path modeling during the online operation of ANC systems. The online secondary path modeling is needed in the situation when the secondary path is time varying. In this thesis two new methods have been proposed to achieve improved performance in ANC systems with online secondary path modeling. In contrast to the existing improved methods, which are comprised of three adaptive filters, these proposed methods consist of two adaptive filters.
In the first proposed method, the error signal used in the adaptation of the secondary path modeling filter is also used as an error signal for the ANC control filter. Furthermore, the ANC control filter is adapted using the FxAFA algorithm. The main features of the second proposed method are that 1) the modified FxLMS (MFxLMS) algorithm is used in adapting the noise control filter and 2) a variable step size (VSS) based LMS algorithm is used in the secondary path modeling filter. The step size is varied in accordance with the power of the residual error signal [the desired response for the modeling filter]. It is found that the desired response for the modeling filter is corrupted by a noise which is decreasing in nature, (ideally) converging to zero. Hence a small step size is used initially and later its value is increased accordingly. Extensive computer simulations have been carried out, which show the effectiveness of the methods proposed in this thesis.

**Key words:** Adaptive filters, LMS algorithm, variable step size, active noise control, averaging, FxLMS algorithm, modified FxLMS algorithm, online secondary path modeling.
PREFACE

The subject of ‘adaptive signal processing’ constitute an important part of the statistical signal processing. Widrow developed the foundations of the adaptive filters in 1960s by introducing the famous least mean square (LMS) algorithm. He also described the concept of adaptive noise cancelation using LMS algorithm. After this early investigation, later adaptive filters have been successfully applied in such diverse fields as, communications, control, radar, sonar, seismology, and biological engineering, among others.

This thesis provides an outline of active noise control (ANC) systems and studies adaptive signal processing used in ANC systems. The concept of active noise control is based on the simple principle of destructive interference of propagating acoustic waves. The idea that acoustic wave interference can be controlled to produce zones of quietness was first proposed by P. Lueg in 1936 for an analogue ANC system. However success with the early analogue controllers was very limited and in the resent years powerful digital signal processing (DSP) devices and algorithms have made possible the development of real time ANC system with wide range of applications including air conditioning ducts, cars, aircrafts, and so on. The most popular adaptive signal processing algorithm used for ANC applications is the filtered-x least mean square (FxLMS) algorithm which is a modified version of the LMS algorithm.

This thesis combines the concepts of the FxLMS algorithm and adaptive filtering with averaging, and proposes a new algorithm (FxAFA – filtered-x adaptive filtering with averaging algorithm) for ANC systems. The proposed algorithm outperforms the conventional FxLMS based ANC systems under the situation of large measurement noise. The effectiveness of the proposed algorithm in ANC systems with online secondary path modeling, is also demonstrated.

This thesis also proposes a new variable step size (VSS) LMS algorithm for online secondary path modeling in ANC systems. Here step size is varied on the basis of power of desired response of the modeling filter. It is different from the normalized-LMS (NLMS) algorithm, where step size is varied with the power of reference signal. It is also different from the other VSS algorithms where initially
a large step size is selected for fast convergence and finally a small value is used for small misadjustment. The proposed VSS LMS algorithm stems from the fact that desired response for the modeling filter is corrupted by a noise which is decreasing in nature, (ideally) converging to zero. Infact, initially this interference may be so large that the online secondary path modeling may become unstable in the worst case. The proposed VSS LMS algorithm, in contrast to the existing VSS algorithms, uses a small step size initially and later its value is increased in accordance with the decrease in the residual noise.

A brief summary of the contents of this thesis is given below.

Chapter 1 is an overview of Active Noise Control systems with the view point of adaptive signal processing. Here basic concept of active noise control (ANC) is introduced. Only single-channel feedforward ANC are described. The development of FxLMS algorithm is given and an overview of various methods for ANC systems with online secondary path modeling is also presented.

Chapter 2 gives the development of Filtered-x Adaptive Filtering with Averaging (FxAFA) Algorithm in comparison to the FxLMS algorithm. Some theoretical comments on the properties of the FxAFA algorithm are also presented. It is also explained that why FxAFA algorithm is expected to give better performance than the FxLMS algorithm.

The rest of this thesis discusses ANC Systems With Online Secondary Path Modeling. The contents are organized in three parts, as described below.

Chapter 3 is Part-I: FxAFA Based method. The description is given in comparison with the Eriksson’s method, which is considered as a basic method for ANC systems with online secondary path modeling.

Chapter 4 is Part-II: Modified-FxLMS Based method. In the proposed method described in this Chapter, the control filter is adapted using modified-FxLMS algorithm, and a new variable step size LMS algorithm is used for online secondary path modeling.

Chapter 5 is Part-III: Simulation Results. This Chapter details the simulation results for ANC systems with online secondary path modeling. Extensive case studies have been performed, which show the effectiveness of the proposed methods [described in Chapter 3 and 4]. The simulations are carried out in both stationary and non-stationary environments.
Chapter 6 gives **Concluding Remarks and Future Recommendations**. The main achievements of the thesis are summarized in this chapter. The limitations of the proposed methods are also identified, suggesting directions for future work.

**Bibliography** lists the references cited in the thesis. For completeness, some extra references are also included.

**Appendices** give some useful information about the acoustic data used in, and the Matlab routines developed for the computer simulations. A list of publications, which resulted from work carried out in this thesis, is also given.

**About the Author** is an overview of educational and professional background of the author.
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Glossary

Symbols

- $t$: Continuous time
- $f_s$: Sampling frequency
- $n$: Discrete time index, so $n = tf_s$ where $n$ is integer
- $L$: Tap-weight length of FIR filter $W(z)$
- $M$: Tap-weight length of FIR filter $\hat{S}(z)$
- $\ast$: Linear convolution operation
- $\mu$: Step size (gain) parameter
- $\nabla$: Gradient operator
- $z^{-\Delta}$: Delay of $\Delta$ units
- $\sigma_x^2$: Variance of the signal $x(n)$
- $\lambda$: Forgetting factor
- $P_x$: Power of signal $x(n)$
- $\Re$: Reduction (in noise) at error microphone (in dB)
- $\Delta w$: Estimation error in $w(n)$ (in dB)
- $\Delta S$: Relative modeling error (in dB) between $\hat{S}(z)$ and $S(z)$

Signal Conventions

- $d(n)$: Desired or disturbance signal
- $r(n)$: Reference signal generated by noise source
- $v_1(n)$: Measurement noise associated with the reference microphone
- $v_2(n)$: Measurement noise associated with the error microphone
- $x(n)$: Reference signal measured by reference microphone:
  \[x(n) = r(n) + v_1(n)\]
  [Sometimes no measurement noise is assumed, and $x(n)$ is referred to as the reference signal. It will be clear from the context.]
e(n)  Error signal at the error microphone
y(n)  Controller output
v(n)  Excitation signal for the secondary path modeling filter
f(n)  Error signal for updating the modeling filter $\hat{S}(z)$
g(n)  Error signal for updating the control filter $W(z)$
X(z)  $z$-transform of the signal $x(n)$
W(z)  FIR filter representing controller transfer function
P(z)  Transfer function of the primary path
S(z)  Transfer function of the secondary path
$\hat{S}(z)$ FIR filter representing transfer function of the secondary path modeling filter
w(n)  Impulse response of the controller $W(z)$
p(n)  Impulse response of the primary path $P(z)$
s(n)  Impulse response of the secondary path $S(z)$
$\hat{s}(n)$ Impulse response of the secondary path modeling filter $\hat{S}(z)$
y′(n)  Canceling signal at the error microphone: $y'(n) = s(n) * y(n)$
$\hat{y}'(n)$ Estimate of canceling signal $y'(n)$: $\hat{y}'(n) = \hat{s}(n) * y(n)$
$\hat{x}'(n)$ Estimate of filtered reference signal: $\hat{x}'(n) = \hat{s}(n) * x(n)$
x(n)  $[x(n)x(n-1) \cdots x(n-l+1)]^T$ vector of $l$ recent samples of $x(n)$
w(n)  $[w_0(n)w_1(n) \cdots w_{L-1}(n)]^T$ tap-weight vector of FIR filter $W(z)$
$\hat{s}(n)$  $[\hat{s}_0(n)\hat{s}_1(n) \cdots \hat{s}_{M-1}(n)]^T$ tap-weight vector of FIR filter $\hat{S}(z)$