Math1107B Practice Test #1 **Jan 28th 2005**

1. Does the following system have a solution? Why?

$$x_1 - x_2 + 3x_3 - 7x_4 = 1$$
$$-2x_1 + 2x_2 - 6x_3 + 14x_4 = 3$$
$$x_1 + x_2 + x_3 + x_4 = 5$$

2. Consider the linear system:

$$2x_1 + x_2 + 3x_3 = 1$$
$$3x_1 - x_2 + x_3 = 5$$

(a) Write down the corresponding augmented matrix.

(b) Obtain the echelon form.

(c) Identify the pivot positions.

(d) Is the system consistent? Why?

(e) Identify the basic variables.

(f) Identify the free variables if any.

(g) How many solutions does this system have? Why?

(h) Obtain reduced row echelon form of the augmented matrix.

(i) Write down in parametric form the set of all solutions of the system.

3. For what values of g and h is $\begin{bmatrix} g \\ h \end{bmatrix}$ in the span of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$?

1. No, the system has no solution since upon row reduction we obtain:

$$\begin{bmatrix} 1 & -1 & 3 & -7 & 1 \\ -2 & 2 & -6 & 14 & 3 \\ 1 & 1 & 1 & 1 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3 & -7 & 1 \\ 0 & 0 & 0 & 0 & 5 \\ & & & & \end{bmatrix}$$

2. (a) The augmented matrix is: $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 3 & -1 & 1 & 5 \end{bmatrix}$.

(b) The echelon form is: $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & -5 & -7 & 7 \end{bmatrix}$.

(c) The pivot positions are (1,1) and (2,2).

(d) The system is consistent since the every row of the coefficient matrix has a pivot.

(e) There are two basic variables x_1 and x_2.

(f) There is one free variable x_3.

(g) There are infinitely many solutions since there is a free variable.

(h) The reduced echelon form is $\begin{bmatrix} 1 & 0 & 4/5 & 6/5 \\ 0 & 1 & 7/5 & -7/5 \end{bmatrix}$.

(i) The solutions in parametric form are:

$$\begin{aligned} x_1 &= -4/5x_3 + 6/5 \\ x_2 &= -7/5x_3 - 7/5 \\ &x3 \text{ is free.} \end{aligned} \qquad v = \begin{bmatrix} 6/5 \\ -7/5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4/5 \\ -7/5 \\ 1 \end{bmatrix}$$

3. This is the same asking whether the system $\begin{bmatrix} 1 & -3 & g \\ 1 & 1 & h \end{bmatrix}$ has a solution. By row reducation $\begin{bmatrix} 1 & -3 & g \\ 0 & 4 & h-g \end{bmatrix}$. But this always has a solution. So, the two vectors span all of $\mathbb{R}^2$ irrespective of g and h.