Analysis of the Modified MOS Wilson Current Mirror: A Pedagogical Exercise in Signal Flow Graphs, Mason's Gain Rule, and Driving-Point Impedance Techniques

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***Abstract*—A pedagogical analysis of the modified MOS Wilson current mirror using signal flow graphs (SFGs), Mason's gain rule, and driving-point impedance (DPI) techniques is presented as an exercise for undergraduate electrical engineering students learning to analyze transistor-level circuits with multiple-feedback loops. While students often prefer the SFG representation for single feedback loops, they often abandon it in favor of the more familiar nodal analysis methods for multiple loops. Yet these methods can be long and cumbersome and contribute little to intuition. In an attempt to preserve the intuitive grasp of tradeoffs, this paper presents an exercise of several well-established analytical techniques for generating and analyzing SFGs. The modified Wilson current mirror is used to compare three analytical approaches: 1) fundamental laws with brute-force algrebra; 2) fundamental laws with Mason's gain rule; and 3) DPI technique with Mason's gain rule. The concepts reinforced in this paper include: 1) tradeoffs between gain and other quantities such as output resistance or bandwidth; 2) how Mason's gain rule simplifies the analysis of closed-loop gain; and 3) how DPI techniques simplify the generation of SFGs.**

***Index Terms*—Driving-point impedance (DPI), feedback, Mason's gain rule, signal-flow graphs (SFGs), small-signal analysis, transistors.**

I. Introduction

When teaching the fundamentals of transistor-level circuit analysis to junior and senior level students, it is helpful to tie concepts together into unifying frameworks in order to avoid giving the impression the subject is full of disjointed concepts. For instance, the topic of feedback acts as a foundation for understanding why the output resistance of a transistor increases when a resistor is placed between small-signal ac ground and the source of a metal-oxide semiconductor field-effect transistor (MOSFET) or emitter of a bipolar junction transistor (BJT), a fact that is usually studied before feedback. Also introduced before feedback is the closed-loop transconductance gain for these configurations, which has much in common with the output resistance, but the relationship may be overlooked or underemphasized. To the student, gain and output resistance are unrelated items, contributing to the difficulty in understanding. However, when feedback later ties them together in one unifying framework, many students are enlightened as the subject matter is suddenly reduced to something more rational.

In [1], Sedra and Smith do a good job of making the connection between the *amount of feedback* and the corresponding decreases in gain and changes in input and output resistances. In the chapter on feedback they lead the student into an implicit understanding of some tradeoffs; e.g., that a decrease in gain due to feedback leads to a change in some other quantity by the same amount. Fortunately, the students can verify the direct relationship between these quantities using fundamental analytical techniques such as the short-circuit and test-voltage methods, thus reinforcing this understanding. But in addition to establishing a direct relationship between these quantities, the results suggest that the knowledge that is embodied by the methods themselves is somewhat redundant and carrying out each one constitutes unnecessary work, which may be simplified by a basic understanding feedback. Many find the unifying framework satisfying, as it maintains a degree of coherence in a topic that is potentially very illusive.

All is well until the student encounters a circuit with multiple feedback loops like the modified Wilson current mirror [2]. Here, the output resistance given by Sedra and Smith is not quite like that of the well-known cascode structure that is understood from the given lessons on feedback. The immediate question from students, and understandably so, is Why? It is the objective of this paper to answer this question and propose a pedagogical exercise for understanding and gaining confidence in signal flow graph (SFG) generation and the calculation of closed-loop quantities such as gain and output resistance. The objective is to reinforce the notion that generation and analysis of SFGs should rarely be abandoned, Mason's gain rule [3] extends the analytical framework presented by Sedra and Smith, and the SFG may be generated systematically, using the driving-point impedance (DPI) method [4], [5]. Several concepts may be reinforced from such an analysis: 1) that the conservational laws of feedback apply to circuits with multiple feedback loops; 2) that Mason's gain rule can simplify such analyses; and 3) that DPI techniques may be used to systematically generate SFGs.

II. Brute-Force Analysis: Short-Circuit and Test Voltage Methods

The modified NMOS Wilson current mirror is shown in Fig. 1. It consists of four transistors, two of which are gate-drain connected (M2 and M3). As all current mirrors are ideally

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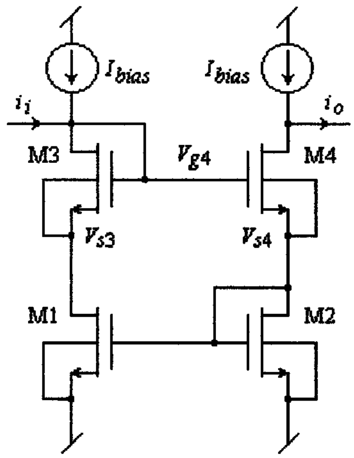

Fig. 1. The NMOS modified Wilson current mirror.

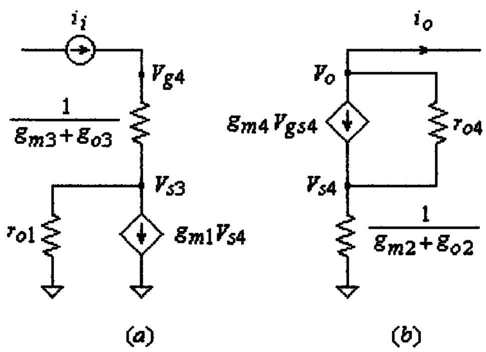

Fig. 2. Small-signal circuits of the modified Wilson current mirror: (a) left side (M1 and M3) and (b) right side (M2 and M4).

designed to do, input current flowing down the left side is amplified (or attenuated) and mirrored to the right side.

For small amplitude ac signals, the left and right sides may be modeled as shown in Fig. 2. Transistor M3 is approximated as a resistor with a value of $(g_{m3} + g_{o3})^{-1}$, where g_{m3} and g_{o3} are the transconductance and output conductance of M3, respectively, and M1 is modeled as a voltage controlled current source with a transconductance gain of g_{m1} and output resistance, r_{o1} as shown in Fig. 2(a). Likewise, M4 and M2 may be modeled as a nonideal transconductor and source degeneration transresistance, respectively, as shown in Fig. 2(b). Although shown to be two separate circuits, they are, of course coupled.

Several feedback loops may be identified, two of which are most significant: 1) the source degeneration loop that exists due to the transresistance that results from M2 being gate-drain connected between the source of M4 and ac ground and 2) the loop that is made by all four transistors. Another less significant loop exists due to the finite output resistance of M4. Because there are multiple feedback loops, Mason's gain rule is required to analyze the circuit from within a feedback framework. The output resistances of the gate-drain connected transistors, M2 and M3 may be neglected for simplicity because the transconductance is much greater than the output conductance, but the exact expression is carried through in this analysis for the sake of completeness.

Next, the closed-loop gain and output resistance will be found using two brute-force techniques that do not make explicit use of the concepts of feedback: the short-circuit method for gain and the test-voltage method for output resistance.

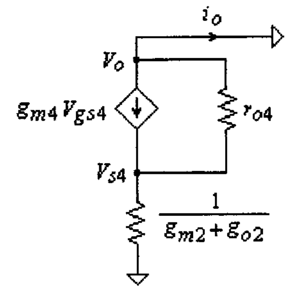

Fig. 3. The current mirror with ac grounded output for finding closed-loop current gain. The left side, which is the same as that shown in Fig. 2(a), is not shown.

A. The Short-Circuit Method for Finding Closed-Loop Gain

The short-circuit method for finding thetransconductor gain involves grounding the output and finding the ratio of output to input current. By shorting the output to ac ground, the contribution from the finite output resistance, which could distort the measurement of the feedforward action, is eliminated. Fig. 3 shows the grounding of the output on the right side of the current mirror for this purpose.

This method is more or less a brute-force way of calculating the closed-loop gain, regardless of how one goes about it. Using either straightforward substitution or matrix techniques, the node voltage method can require quite a few calculations. Using the circuit in Fig. 2(a), two independent node-voltage equations may be written: one at V_{g4} (1) and the other at V_{s3} (2). The transconductances of M1 and M3 are denoted by g_{m1} and g_{m3}, respectively, and the output conductance of M1 is denoted by g_{o1}

$$i_i + (V_{s3} - V_{g4})(g_{m3} + g_{o3}) = 0 \tag{1}$$

and

$$i_i - V_{s4}g_{m1} - V_{s3}g_{o1} = 0. \tag{2}$$

By substitution (1) and (2) are combined to obtain (3), which expresses V_{g4} in terms of the input current, transconductance, and output resistance of M1 and M3, and the source terminal voltage of M4

$$V_{g4} = r_{o1}\left[i_i\left[1 + \frac{g_{o1}}{g_{m3} + g_{o3}}\right] - g_{m1}V_{s4}\right]. \tag{3}$$

Making the assumption $g_{o1} \ll g_{m3} + g_{o3}$, which is neither unrealistic nor necessary, the effect of M3 is eliminated altogether

$$V_{g4} \approx r_{o1}(i_i - g_{m1}V_{s4}). \tag{4}$$

Moving on to the right side of Fig. 3, two more independent equations may be written using the node voltage method and Ohm's law

$$i_o + g_{m4}(V_{g4} - V_{s4}) - g_{o4}V_{s4} = 0; \quad V_{s4} = \frac{i_o}{g_{m2} + g_{o2}} \tag{5}$$

where the output is shorted to ac ground to obtain the short-circuit current gain.

Combining (4) and (5) yields a good approximation of the closed-loop current gain, H_{cl}

$$H_{cl} \equiv \left.\frac{i_o}{i_i}\right|_{V_o=0} = \frac{-g_{m4}r_{o1}}{1+\dfrac{g_{m4}(1+g_{m1}r_{o1})+g_{o4}}{g_{m2}+g_{o2}}}. \tag{6}$$

It is instructive to note that the denominator of the closed-loop current gain is unity plus a combination of feedback and feedforward terms, which will be shown to be consistent with Mason's gain rule in the next section. In the meantime, it is sufficient to note that the numerator is the open-loop current gain

$$H_{ol} \equiv \left.\frac{i_o}{i_i}\right|_{V_{s4},\,V_o=0} = -g_{m4}r_{o1} \tag{7}$$

i.e., the current gain when V_{s4} is ac grounded, again neglecting g_{o1} in comparison to $g_{m3}+g_{o3}$.

B. The Test-Voltage/Test-Current Methods for Finding Closed-Loop Output Resistance

The test-voltage/test-current methods are two brute-force methods for finding closed-loop *output resistance*. Without feedback, these methods are quite simple. However, when feedback is present, they constitute more effort than Mason's gain rule and are prone to error. As will be shown in the next section, knowing the open-loop output resistance and feedback structure can be used to obtain a good approximation of the closed-loop output resistance by inspection. But first, a review of the brute-force methods is in order.

The test-voltage method requires that 1) all independent sources be zeroed[1] and 2) a test voltage be applied to the node under consideration, to calculate the resulting test current. Alternately, the test-current method requires that a current be applied to the node under consideration to calculate the resulting voltage. In either case, the ratio of test voltage to test current determines the resistance looking into the node.

Fig. 4 shows the test-voltage method applied to the right side of the current mirror. Node voltage and Ohm's law generates two equations

$$\begin{aligned} i_{test} &= g_{m4}(V_{g4}-V_{s4}) + (V_{test}-V_{s4})g_{o4} \\ V_{s4} &= \frac{i_{test}}{g_{m2}+g_{o2}} \end{aligned} \tag{8}$$

and the closed-loop output resistance may be obtained from (4) and (8) where $i_i = 0$

$$R_{of} \equiv \frac{V_{test}}{i_{test}} = r_{o4}\left(1+\frac{g_{m4}(1+g_{m1}r_{o1})+g_{o4}}{g_{m2}+g_{o2}}\right). \tag{9}$$

It should be noted that the closed-loop output resistance is the open-loop[2] output resistance, r_{o4} times the same quantity that the open-loop current gain was *divided* by to obtain the closed-loop current gain. This result reinforces the fact that the effects of multiple feedback loops are conservational. Whereas gain was divided, output resistance was multiplied,[3] by the same quantity.

[1]Replaced by their internal impedance.

[2]Open-loop is defined as V_{s4} being ac grounded.

[3]Multiplied rather than divided due to series sampling.

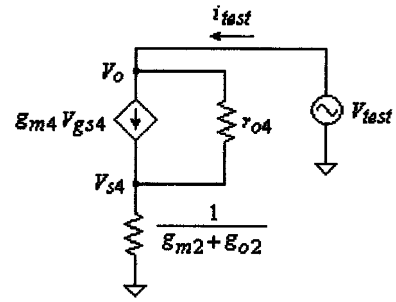

Fig. 4. The current mirror with a test voltage applied to the output for finding closed-loop output resistance. Additionally, the current input is zeroed, making V_{g4} equal to V_{s3}.

The above analyses require a considerable amount of effort for analyzing such a simple circuit, and several steps were omitted in order to emphasize the flow. When performing substitutions and rewriting equations, errors can render the results inaccurate. There is a better way—signal flow graphs and Mason's gain rule.

III. Feedback Analysis Using Signal Flow Graphs and Mason's Gain Rule

In the simplest case, with only one feedback loop, the *amount of feedback* of a system is given by $1+AB$ where A is the *open-loop gain* of the amplifier and B is the *feedback factor*. Faced with multiple loops, one turns to Mason's gain rule [3], which requires an SFG from which the closed-loop gain may be obtained in a perfunctory manner. It is this feature that makes Mason's gain rule so powerful; a form of knowledge is encapsulated in the rule, which saves time.

Mason's gain rule [3], [6] states that the closed-loop gain equals a weighted-sum of forward path gains divided by Δ, the graph determinant

$$\text{Gain}_{\text{closed loop}} = \Delta^{-1}\sum_{\forall i} P_i\Delta_i \tag{10}$$

where P_i is the ith forward path gain and Δ_i is the corresponding path cofactor. The graph determinant is defined as unity minus the sum of all loop gains, plus the sum of all combinations of two nontouching loop gain products, minus the sum of all combinations of three nontouching loop gain products, etc. The ith cofactor equals the determinant, excluding any loops that touch the ith forward path. For simple graphs the path cofactors are often unity.

Generation of the SFG and application of Mason's gain rule for a given circuit may proceed in six steps.

1) Identify and write the independent equations that describe the circuit using: 1) Ohm's law; 2) Kirchhoff's current law (KCL); and/or 3) Kirchhoff's voltage law (KVL).
2) Construct the SFG from the independent equations.
3) Find the forward path gain—the direct path gains from input to output weighted by the path cofactors. If there are more than one input to the SFG, superimpose each stand-alone contribution (an example is shown in the next section).
4) Find Δ, defined as unity minus the (sum of all loop gains, *minus* the sum of all product combinations of two non-

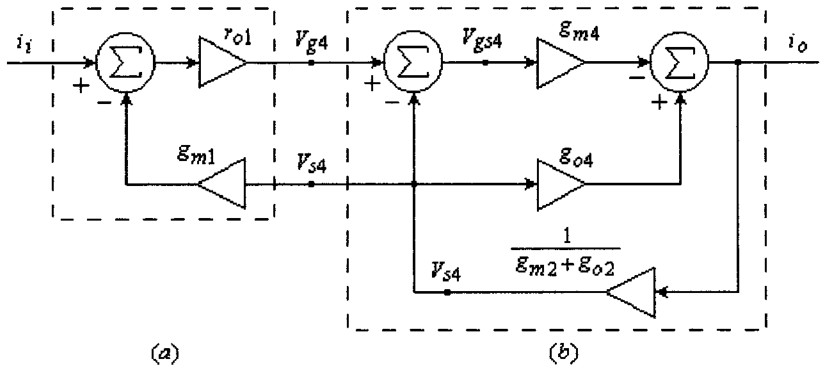

Fig. 5. Approximate SFG of the modified Wilson current mirror in terms of low-level small-signal primitives: (a) left side (M1 and M3) and (b) right side (M2 and M4).

touching loop gains, *plus* the sum of all product combinations of three nontouching loop gains, *minus* … etc.).

5) Divide the forward path gain by Δ to obtain the closed-loop gain.
6) Use Δ to determine other closed-loop quantities in the circuit using the respective open-loop quantities.[4]

These steps may be demonstrated as follows. The independent equations obtained in the previous section for the left side [see (1), (2), and (4)], may be used to construct Fig. 5(a). The independent equations obtained for the right side (5) may be used to construct Fig. 5(b). Using the entire SFG in Fig. 5, the denominator may be constructed by inspection. In this case, Δ is found by subtracting three loop gains from unity

$$\begin{aligned}\Delta &= 1 - \left(-\frac{r_{o1}g_{m4}g_{m1}}{g_{m2}+g_{o2}} - \frac{g_{m4}}{g_{m2}+g_{o2}} - \frac{g_{o4}}{g_{m2}+g_{o2}}\right)\\ &= 1 + \frac{g_{m4}(1+g_{m1}r_{o1})+g_{o4}}{g_{m2}+g_{o2}}. \end{aligned} \tag{11}$$

Realizing that the open-loop current gain is the gain of the path directly from the input to the output, the closed-loop gain is given by

$$H_{cl} = \frac{P_1\Delta_1}{\Delta} = \frac{-g_{m4}r_{o1}}{1+\dfrac{g_{m4}(1+g_{m1}r_{o1})+g_{o4}}{g_{m2}+g_{o2}}} = \frac{H_{ol}}{\Delta} \tag{12}$$

where $P_1 = -r_{o1}g_{m4}$ and Δ_1 is unity. Notice that this equation is identical to that obtained by the short-circuit method in (6).

Knowing that the effects of feedback are conservational, the closed-loop output resistance may now be written by inspection. Knowing that the open-loop gain was divided by Δ, the closed-loop output resistance is found by multiplying the open-loop output resistance by Δ

$$R_{of} = R_{ol}\Delta = r_{o4}\left(1+\frac{g_{m4}(1+g_{m1}r_{o1})+g_{o4}}{g_{m2}+g_{o2}}\right) \tag{13}$$

which agrees with the test-voltage method (10), but required no algebra.

Arriving at (12) and (13) by Mason's gain rule is simpler than the brute-force methods. The savings come about from Mason's gain rule, rather than carrying out the algebra directly. However, the same independent equations are still required to *construct* the SFG, and the appropriate method for determining each equation (KCL, KVL, nodal analysis, or Ohm's law) is still a matter of choice and experience. In other words, the SFG is not yet generated in a perfunctory manner.

IV. Systematic Generation of an Equivalent Signal Flow Graph Using the Driving-Point Impedance Technique

In the previous section, Fig. 5 was generated by node voltage and other equations for the modified Wilson current mirror, which then required some algebraic manipulation to obtain the simplest flow graph. The architecture uniquely determined whether KVL, KCL, or nodal analysis was most appropriate for a given instance. In order to assimilate this part of the analysis, it is helpful to employ the DPI technique for systematic generation the SFG, which is equivalent in terms of closed-loop gain.

There is one tradeoff to this approach which, when properly understood, is not unreasonable. To understand the implications of such systematically generated graphs, one must first look at the SFG that was *not* obtained by the DPI technique in Fig. 5. The process of writing nodal equations and using one's own knowledge of the architecture to perform the algebraic manipulation does the following: 1) simplifies the flow so as not to express unnecessary node voltages (V_{s3}, for example) and 2) keeps the SFG "atomic."[5] For example, each gain block in Fig. 5 was one of the following fundamental small-signal circuit parameters: g_{m1}, g_{m2}, g_{m4}, r_{o1}, g_{o2}, and g_{o4}. The advantage of such a SFG is that the determinant, Δ, is the celebrated "amount of feedback" quantity which may be used to obtain either the closed-loop gain or output resistance directly from the corresponding open-loop quantities. In short, this buys a form of signal flow that is fundamental, at the expense of having to perform additional substitution and algebraic manipulation.

Alternately, the DPI technique allows one to obtain the SFG in a more perfunctory manner. However, the determinant of the resulting graph will not always be "atomic." Although the closed-loop gain obtained by Mason's gain rule may require some manipulation to reduce it to a form in which the open-loop

[4]However, some knowledge of the effects of feedback is required to determine whether to multiply or divide by Δ.

[5]In terms of fundamental, low-level, small-signal ac circuit parameters; granular.

gain and the atomic determinant are evident, the algebraic manipulation is shifted to single equations, rather than searching for the appropriate substitutions and then simplifying. Furthermore, some degree of algebraic processing is built-in to the DPI SFG, which occasionally eliminates the need for algebraic manipulation altogether.

The DPI technique involves two equations at every dependent node [5]. One is written for the short-circuit current (SCC), and another for the DPI. The SCC is the sum of all currents entering the node when shorted to ac ground. Thus, a current leaving the node would be negative. To make the equations suitable for generating the SFG, neighboring connections may be represented as voltage-controlled current sources (VCCSs) with proportionality constants equal to the respective admittance, since the node is hypothetically ac grounded.[6] The resulting dependent current sources may then be combined to generate the SCC. This eliminates the computational "back action," or "loading effects" that are present in the node voltage method.[7] Subsequently, the local DPI is determined by taking the parallel combination of all impedances leading out of the node. Finding the DPI is very straightforward due to the inclusion of only *local* admittances and gives one an immediate feel for the gain leading into the node. Finding the SCC is also straightforward due to being solely determined by local admittances and neighboring node voltages. The resulting equations end up being relatively simple and may be used to construct an SFG by multiplying the SSC by the DPI to obtain the given node voltage[8]

$$V_{\text{node}} = \text{SCC}_{\text{node}}\text{DPI}_{\text{node}}. \tag{14}$$

Once the SFG has been obtained, approximations may be made as appropriate and Mason's gain rule may be applied.

Four equations may be written for Fig. 2(a)—two equations for each node. The SCC at V_{g4} is the sum of the small-signal input current and the current contribution from V_{s3}. The corresponding DPI is the parallel combination of all resistances leading out of V_{g4}

$$\text{SCC}_{Vg4} = i_i + (g_{m3} + g_{o3})V_{s3}$$

and

$$\text{DPI}_{Vg4} = \frac{1}{g_{m3} + g_{o3}}. \tag{15}$$

The SCC at V_{s3} is the current contribution of V_{g4}, minus the voltage-controlled current of M1. The corresponding DPI is the parallel combination of all resistances leading out of V_{s3}

$$\text{SCC}_{Vs3} = (g_{m3} + g_{o3})V_{g4} - g_{m1}V_{s4}$$

and

$$\text{DPI}_{Vs3} = \frac{1}{g_{m3} + g_{o3} + g_{o1}}. \tag{16}$$

An SFG may be constructed from these equations, as shown in Fig. 6, where $V_{g4} = \text{SCC}_{Vg4}DPI_{Vg4}$ and $V_{s3} = \text{SCC}_{Vs3} * \text{DPI}_{Vs3}$. Four more equations may be written for Fig. 2(b). The SCC at V_{s4} is the current contribution from $g_{o4}V_o$ plus $g_{m4}V_{gs4}$, with V_{s4} ac grounded, and the DPI is the reciprocal of conductances leading out of V_{s4}, where g_{m4} is the conductance looking into the source of M4

$$\text{SCC}_{Vs4} = g_{m4}V_{g4} + g_{o4}V_o$$

and

$$\text{DPI}_{Vs4} = \frac{1}{g_{m4} + g_{m2} + g_{o4}}. \tag{17}$$

The SCC at V_o is the current contribution from V_{s4} minus the voltage-controlled current of M4. The corresponding DPI is the parallel combination of resistances leading out of V_o

$$\begin{aligned}\text{SCC}_{Vo} &= g_{o4}V_{s4} - g_{m4}V_{gs4}\\ &= V_{s4}(g_{m4} + g_{o4}) - V_{g4}g_{m4}\end{aligned}$$

and

$$\text{DPI}_{Vo} = r_{o4}. \tag{18}$$

These equations may be used to construct an SFG of the right side of the modified Wilson current mirror, where $V_o = \text{SCC}_{Vo}\text{DPI}_{Vo}$ and $V_{s4} = \text{SCC}_{Vs4}\text{DPI}_{Vs4}$ (Fig. 7).

At this point, Figs. 6 and 7 could be joined and Mason's gain rule applied, but these SFGs may be simplified. The SFG in Fig. 6 is a dual-input single-output graph with one feedback loop, which may be compressed. The resulting feedforward graph will be easier to deal with in the final application of Mason's gain rule. To rearrange it, V_{g4} may be expressed as a superposition of the stand-alone contributions from i_i and V_{s4}. Each term is the product of one input and the stand-alone closed-loop gain of that input, which may also be obtained by Mason's gain rule

$$\begin{aligned}\left.\frac{V_{g4}}{V_{s4}}\right|_{i_i=0} &= \frac{\text{Fwd.Gain}_{V_{s4}}}{\Delta_{V_{s4}}}\\ &= \frac{-g_{m1}\text{DPI}_{V_{s3}}\text{DPI}_{V_{g4}}(g_{m3} + g_{o3})}{1 - \text{DPI}_{V_{g4}}\text{DPI}_{V_{s3}}(g_{m3} + g_{o3})^2}\\ &= -g_{m1}r_{o1}\end{aligned} \tag{19}$$

and

$$\begin{aligned}\left.\frac{V_{g4}}{i_i}\right|_{V_{s4}=0} &= \frac{\text{Fwd.Gain}_{i_i}}{\Delta_{i_i}}\\ &= \frac{\text{DPI}_{V_{g4}}}{1 - \text{DPI}_{V_{g4}}\text{DPI}_{V_{s3}}(g_{m3} + g_{o3})^2} \approx r_{o1}\end{aligned} \tag{20}$$

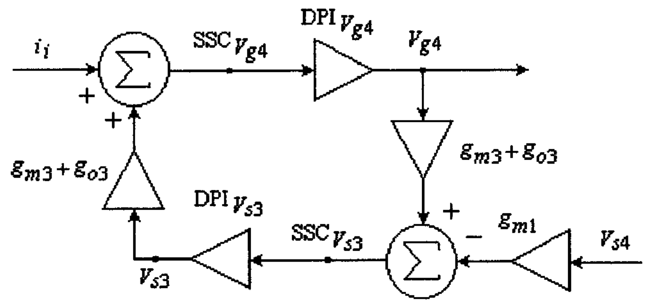

Fig. 6. DPI-generated SFG of the left side.

[6]A concept similar to current leading into the virtual short of an operational amplifier via a resistor.

[7]In fact, the DPI method may be derived from the node voltage method by separating the back action from the forward action.

[8]This equation reduces to the familiar $V_{out} = V_{in}g_mR_{out}$ for a single-stage amplifiers where $V_{in}g_m$ represents SCC and R_{out} represents DPI.

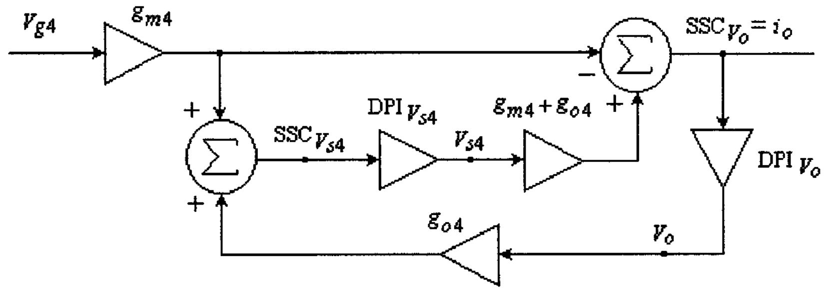

Fig. 7. DPI-generated SFG of the right side.

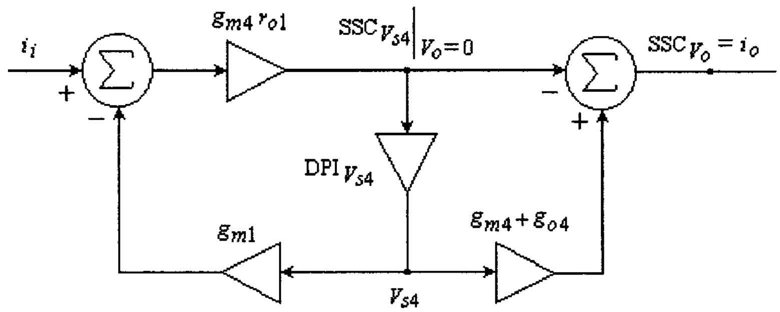

Fig. 8. Simplified DPI SFG of the modified Wilson current mirror. The last gain block was added to provide the correct polarity of the current mirror output as defined in Fig. 1.

where the same approximation made in earlier sections was made, i.e., $g_{m3} + g_{o3} \gg g_{o1}$. Finally

$$V_{g4} = i_i\left(\frac{V_{g4}}{i_i}\bigg|_{V_{s4}=0}\right) + V_{s4}\left(\frac{V_{g4}}{V_{s4}}\bigg|_{i_i=0}\right) \approx i_i r_{o1} - V_{s4} g_{m1} r_{o1}. \tag{21}$$

The right SFG is simplified by setting $V_o = 0$, which is necessary to obtain the short-circuit current gain. Now a new SFG may be constructed that combines both sides of the current mirror and is easy to work with (Fig. 8). Mason's gain rule then produces the following equation in a single shot:

$$\begin{aligned} H_{cl} &= \frac{\sum_{\forall i} P_i \Delta_i}{\Delta_{\mathrm{DPI}}} \\ &= \frac{g_{m4} r_{o1} \mathrm{DPI}_{V_{s4}}(g_{m4} + g_{o4}) - g_{m4} r_{o1}}{1 + g_{m4} r_{o1} \mathrm{DPI}_{V_{s4}} g_{m1}} \\ &= \frac{-g_{m4} r_{o1}}{1 + \dfrac{g_{m4}(1 + g_{m1} r_{o1}) + g_{o4}}{g_{m2} + g_{o2}}} = \frac{H_{ol}}{\Delta} \end{aligned} \tag{22}$$

which is identical to (6) and (12), yet was obtained in a more perfunctory manner. There are two forward path gains $P_1 = -g_{m4} r_{o1}$ and $P_2 = g_{m4} r_{o1} \mathrm{DPI}_{Vs4}(g_{m4} + g_{o4})$ in this graph, and both path cofactors are unity. It is instructive to note, however, that the determinant of the DPI SFG, Δ_{DPI}, is not the determinant of the simplified atomic SFG.

V. Conclusion

This paper presented an exercise for finding the closed-loop gain of transistor-level circuits with multiple feedback loops such as the modified MOS Wilson current mirror. Three methods were presented: 1) fundamental laws with brute-force substitution and algebraic manipulation; 2) SFG constructed from the fundamental laws followed by Mason's gain rule; and 3) SFG constructed from the DPI technique with Mason's gain rule. It was shown that the second method was easier than the first, since closed-loop gain and output resistance may be obtained graphically from the SFG. Yet the SFG in 2) was not obtained by inspection and therefore required algebraic manipulation to obtain the necessary independent equations. The third method was shown to be more systematic than the first and second because all parts were carried out in a perfunctory manner as the algebraic manipulation was shifted to single equations.

The results also reinforce the fact that the effects of feedback are conservational; i.e., that the loss of gain due to feedback improves some other quantity such as output resistance. In the case of the current mirror, information is conveyed in the current domain. In such a circuit, the output resistance should be high compared to the input resistance of the next stage. Therefore, the *desired* change in output resistance is an increase. Indeed, the effects of feedback were shown to increase the output resistance by Δ, which may be greater than unity. This demonstrates that the effects of feedback can be beneficial.

In summary, this exercise serves well as a supplement for teaching analysis of transistor-level circuits with multiple feedback loops and complements Sedra and Smith's chapter on feedback. Since Sedra and Smith present only single-loop feedback systems, this analysis could not have been carried out entirely within the context of feedback without introducing Mason's gain rule, and the DPI technique makes SFG generation more systematic and perfunctory.

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