Do not award half marks.
In all cases give credit for appropriate alternative answers.

Question 1 Compulsory

(a) Use De Morgan's laws to write negations of the following statements.

(i) John is less than six feet tall and he weighs more than 200 pounds. [2]

(ii) The bus was late or Tom's watch was slow. [2]

(i) John is more than six feet tall or he weighs less than 200 pounds

(ii) The bus was not late and Tom's watch was not slow

In each subpart one mark for correctly negating the statements and one mark for interchanging "and" and "or".

(b) Let the set A be given by

$A=\{1, 2, 3, 4\}$

and let the relation R on A be defined by

$R = \{(1,1), (2,1), (3,2), (4,3)\}$

(i) Write down the members of R^{-1} [1]

(ii) Write down the members of the composite relation $R \circ R$ [2]

(i) $R^{-1} = \{(1,1), (1,2), (2,3), (3,4)\}$ [1]

(ii) $\{(1,1),(2,1),(3,1),(4,2)\}$

Two marks. Deduct one mark for each error to a maximum of two

(c) Give a direct proof that the sum of two odd numbers is an even number. [3]

Let the first odd number be 2n+1 and let the second odd number be 2m+1 where n and m are integers [1]

the sum of the two odd numbers = 2n+1 + 2m +1
= 2(n+m+1) [1]
implies the result is even. [1]

(d) Write down the converse and contrapositive of the following statement.

"If a graphics driver is not available, then my program cannot run." [3]

converse:
If my program cannot run, then a graphics driver is not available [1]

contrapositive:
If my program can run, then a graphics driver is available
Correctly negates both statements – [1]
Interchanges order of statements – [1]

(e) Let the matrix A be given by

$$A = \begin{pmatrix} 3 & 0 & -1 & 2 \end{pmatrix}$$

(i) Write down A^t [1]

(ii) Calculate $A^t A$ [2]

(iii) Calculate $A A^t$ [2]

(i) $$\mathbf{A^t} = \begin{pmatrix} \mathbf{3} \\ \mathbf{0} \\ -\mathbf{1} \\ \mathbf{2} \end{pmatrix}$$

1m for correct transpose.

(ii) $$A^t A = \begin{pmatrix} 3 \\ 0 \\ -1 \\ 2 \end{pmatrix} \begin{bmatrix} 3 & 0 & -1 & 2 \end{bmatrix} = \begin{pmatrix} 9 & 0 & -3 & 6 \\ 0 & 0 & 0 & 0 \\ -3 & 0 & 1 & -2 \\ 6 & 0 & -2 & 4 \end{pmatrix}$$

2m for correct answer; Award 1 mark if at least 12 entries are correct.

(iii) $$AA^t = \begin{pmatrix} 3 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ -1 \\ 2 \end{pmatrix} = 14$$

One mark for correct answer, one mark if candidate shows an understanding of the multiplication to be performed,

(f) An urn contains 5 red balls and 7 blue balls.

(i) What is the probability of drawing 3 red balls in a row if a ball is put back into the urn after it is drawn? [2]

(ii) What is the probability of drawing 3 red balls if a ball is **not** put back into the urn after it is drawn? [2]

(i) $5^3 / 12^3$ [1] = 125/1728 [1]

(ii) (5 X 4 X 3) / (12 X 11 X 10) [1] = 1/22 [1]

(g) How many bit strings of length eight have exactly three 1's? [2]

8! / (3! 5!) = 56 One mark for method, one mark for correct answer.

(h) Mike is giving a party and wants to set out 15 assorted cans of soft drinks for his guests. He shops at a store that sells 5 different types of soft drinks. In how many ways can Mike select the cans of soft drinks? [2]

$^{15+5-1}C_{15}$ [1] = 3,876 [1]

(i) Draw the complete graph K_4 and write down the adjacency matrix of this graph. [4]

Award two marks for a completely correct graph, award one mark if at most two edges are missing.

Award two marks for all rows of matrix correct, award one mark if at least 12 entries are correct.

$$\begin{array}{c c} & \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Do not award half marks.
In all cases give credit for appropriate alternative answers.

Question 2

(a) Define a simple graph. [1]

A simple graph is a graph consists of graphs that do not have any loops or parallel edges. One mark.

(b) (i) Draw the graph $K_{4,2}$ [2]

Award two marks for a correct graph; award one mark if one edge is missing; award no marks otherwise

(ii) Write down the distance matrix for the graph $K_{4,2}$ [2]

For example

$$\begin{pmatrix} 0 & 2 & 2 & 2 & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 & 1 \\ 2 & 2 & 0 & 2 & 1 & 1 \\ 2 & 2 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 & 2 & 0 \end{pmatrix}$$

Award two marks for a correct answer. Award one mark if there are between one and four errors. Award marks for alternative ordering of vertices.

(c) (i) How many vertices of $K_{m,n}$ have degree m, and how many vertices of $K_{m,n}$ have degree n? [2]

n vertices have degree m. [1]
m vertices have degree n. [1]

(ii) Write down the sum of the degrees of the vertices of $K_{m,n}$ [1]

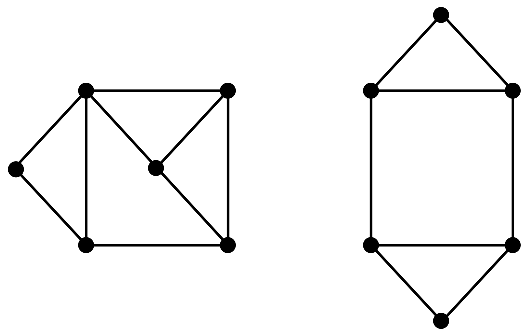
total degree of $K_{m,n}$ = 2mn. [1]

(d) Suppose that you are attempting to show that two graphs are not isomorphic. List **three** structural properties that differ between non-isomorphic graphs. [3]

the number of vertices
the number of components
the number of edges
the degree sequence
the length of the shortest path between pairs of vertices with a given degree
the length of the longest path in the graph

any three;1 point for 1m; up to max 3m

(e) (i) Show that the graphs G and G’ are not isomorphic by finding **two** structural properties that they do not share.



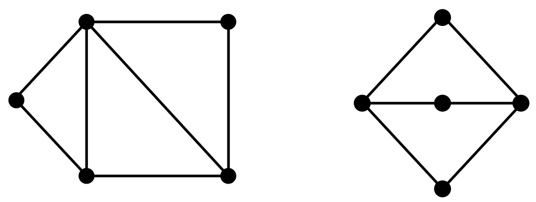
G *G’*

[2]

G has nine edges, but G' has only eight edges.
G has a vertex of degree four, but G' does not.

One mark for each reason to a maximum of two.

(ii) Show that the graphs H and H’ are not isomorphic by finding **two** structural properties that they do not share.



H *H’*

[2]

H has a vertex of degree four, but H' does not.
H has 7 edges, H’ has 6 edges

One mark for each reason to a maximum of two.

Do not award half marks.
In all cases give credit for appropriate alternative answers.

Question 3

(a) A function is defined from set A to set B. Set A has m elements, and set B has n elements. How many one-to-one functions are there in the following cases?

(i) $m = n$ [2]

Each element of A must be mapped to exactly one element of B; no element of B may be mapped to from more than one element of A. Thus, there are n different possibilities for the image of the first element of A, n -1 different possibilities for the image of the second element of A, and so on.
Therefore the answer is n(n - 1)(n -2)...1 = n!.

Award one mark for getting the right argument, and one mark for getting the right answer

(ii) $m > n$ [2]

There are insufficient elements for us to construct any one-to-one functions. Therefore the answer is 0.
Award one mark for getting the right argument, and one mark for getting the right answer

(b) A travel agent offers a holiday to California that consists of a visit to both Los Angeles and San Francisco. In Los Angeles the travel agent offers 6 trips to attractions, from which the traveller must choose 3, and may not choose any trip more than once. In San Francisco the traveller must choose 4 trips from a range of 8 trips, without repeating any trip.

(i) In how many ways may the trips in Los Angeles be chosen? [2]

(ii) In how many ways may the trips in San Francisco be chosen? [2]

(iii) In how many ways may the trips for the whole holiday be chosen? [1]

(i) $^6C_3=20$ **One mark for method, one mark for correct answer**

(ii) $^8C_4=70$ **One mark for method, one mark for correct answer**

(iii) 20*70=1400 One mark.

(c) (i) Prove, using the laws of set theory, that for all sets A, B and C

$$(A \cup B) - C = (A - C) \cup (B - C)$$

Identify the laws used at each step. [3]

(A U B) – C = (AU B) $\cap$ C^C alter repr. [1]
= (A$\cap$$C^C$) U (B$\cap$$C^C$) distributive law [1]
= (A-C) U (B-C) alter rep.[1]

Deduct a maximum of one mark for not stating the laws used.

(ii) Let U be the universal set and let A, B and C be sets. Draw a Venn diagram showing the region $(A \cup B) - C$ [3]

One mark for 3 overlapping sets within a universal set.

Two marks for a completely correct region. Deduct one mark for each extra or omitted region of the diagram to a maximum of two marks.

Do not award half marks.
In all cases give credit for appropriate alternative answers.

Question 4

(a) The sequence of numbers a_0, a_1, a_2, a_3 $\ldots$ is defined by

$$a_0 = 0$$
$$a_{n+1} = 2a_n + 3 \quad \text{for} \quad n = 0,1,2,3,\ldots,$$

Use mathematical induction to prove that for all integers $n = 0,1,2,3,\ldots,$

$$a_n = 3(2^n - 1)$$ [6]

Step 1 : show true for n=0.[1]
a0=0 and $3(2^0$-1)=0, so true for n=0. [1]

Step 2 : assume true for n=k, prove true for n=k+1. [1]
a_{k+1}=$2a_k$+3 [1]
=$6(2^n$-1)+3
=$3(2^{n+1}$-1) [1]
which is the correct form for a_{k+1} [1]

(b) Given the set A ={a, b, c} and the relation R on A defined by

R = {(a, b), (b, b), (b, c), (c, b), (a, c)}

(i) Determine whether or not the relation R is irreflexive. [2]

(ii) Determine whether or not the relation R is symmetric. [2]

(iii) Determine whether or not the relation R is transitive. [2]

Justify your answers.

(i) Not irreflexive [1] since it has components (b,b) [1]

(ii) Not symmetric [1] since (a,b) but not (b,a) [1]

(iii) Not transitive [1] because (c,b) and (b,c) are in R but (c,c) isn't [1]

(c) Let the function $f : \Re \to \Re$ be defined by

$f(x) = 15 - x$

(i) Calculate the composite function $f(f(x))$ [2]

(ii) Using your answer to part (i) or otherwise write down the inverse function $f^{-1}(x)$ [1]

(i) f(f(x))=f(15-x) One mark
=15-(15-x)=x One mark

(ii) f(f(x))=x and so f^{-1}(x)=f(x)=15-x. One mark
Award one mark for any method of reaching the correct answer.

Do not award half marks.
In all cases give credit for appropriate alternative answers.

Question 5

(a) Box A contains 8 items of which 3 are defective and box B contains 5 items of which 2 are defective. An item is drawn at random from each box.

(i) What is the probability that both items are defective? [2]

3/8 * 2/5 [1] = 3/20 [1]

(ii) What is the probability that one item is defective and one item is not defective? [2]

5/8*2/5+3/8*3/5 [1] =19/40 [1]

(iii) What is the probability that at least one item is defective? [1]

5/8 [1]

(b) A fair die is rolled once. The random variable *X* is the square of the number showing on the uppermost face. For example, if the die shows the number 3 then the random variable *X* takes the value $3^2 = 9$.

(i) Write down $P(X = k)$ for *k*=1, 4, 9, 16, 25, 36 [3]

(ii) Calculate the expected value of *X*, E(*X*) [2]

(i) P(X=k)=1/6 for k=1,4,9,16,25,36.

One mark for any demonstration of understanding that X takes values 1, 4, 9, 16, 25 and 36

One mark if at least one value of P(X=k) correct, two marks if all correct.

(iii) E(X)=(1+4+9+16+25+36)/6=91/6. One mark for method, one mark for correct answer.

(c) Consider the argument below.

"The interest rate will be high if and only if the stock market is strong. The stock market is weak therefore the interest rate is low."

Define the statements *p* and *q* by

p – the interest rate is high
q – the stock market is strong

(i) Write the argument above in symbolic form, using *p*, *q* and logical connectives. [2]

(ii) Construct a truth table to test the validity of the argument. [3]

(i) $[(p \leftrightarrow q) \wedge \sim q] \rightarrow \sim p$
Two marks for a completely correct answer. Deduct one mark for each error to a maximum of two.

(ii)

p	q	~p	~ q	p ↔ q	[(p ↔ q) ∧ ~ q	[(p ↔ q) ∧ ~ q] → ~p
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T
				[1]	[1]	[1]

- END OF PAPER -